Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/26-1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [385]. This is test number [26].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (385)	0.00 (0)
Mathematica	99.48 (383)	0.52 (2)
Maple	66.23 (255)	33.77 (130)
Fricas	56.62 (218)	43.38 (167)
Mupad	44.16 (170)	55.84 (215)
Maxima	43.12 (166)	56.88 (219)
Sympy	37.40 (144)	62.60 (241)
Giac	33.77 (130)	66.23 (255)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

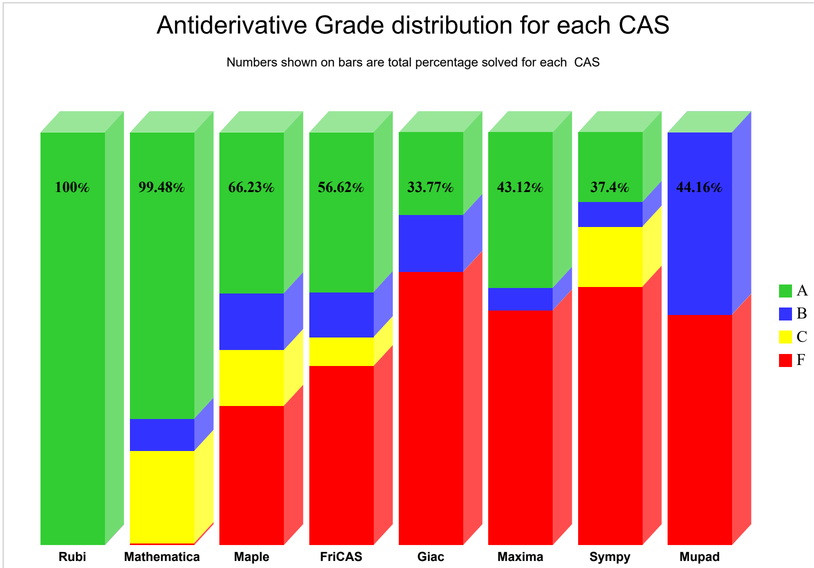
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

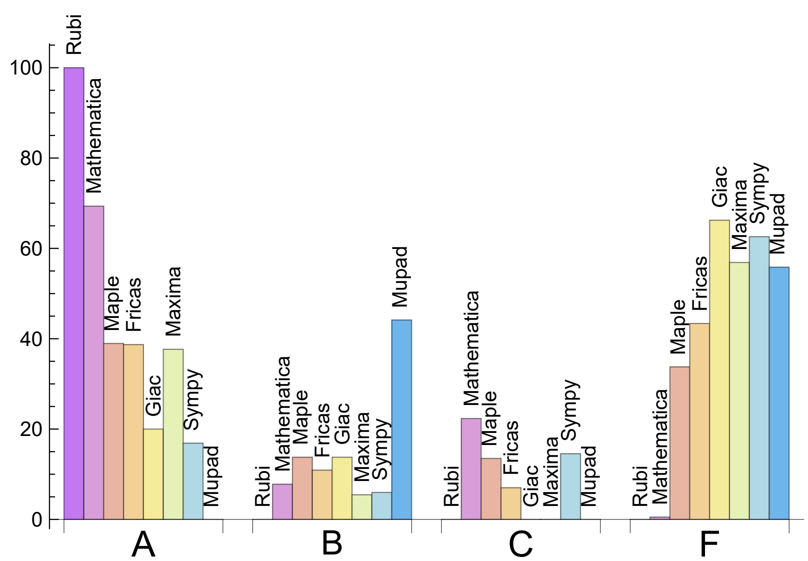
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	69.351	7.792	22.338	0.519
Maple	38.961	13.766	13.506	33.766
Fricas	38.701	10.909	7.013	43.377
Maxima	37.662	5.455	0.000	56.883
Giac	20.000	13.766	0.000	66.234
Sympy	16.883	5.974	14.545	62.597
Mupad	0.000	44.156	0.000	55.844

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	130	100.00	0.00	0.00
Fricas	167	56.29	41.92	1.80
Mupad	215	0.00	100.00	0.00
Maxima	219	99.54	0.46	0.00
Sympy	241	55.60	36.93	7.47
Giac	255	88.63	0.00	11.37

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Giac	0.31
Rubi	0.33
Fricas	2.21
Mathematica	3.22
Maple	3.74
Mupad	8.12
Sympy	11.60

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	171.64	1.01	122.00	1.00
Maxima	185.84	1.28	143.00	1.14
Mathematica	204.42	1.44	140.00	0.97
Maple	217.27	1.30	155.00	1.08
Giac	294.09	1.84	211.00	1.40
Sympy	381.10	3.33	149.50	1.19
Fricas	557.84	2.80	239.50	2.11
Mupad	1166.19	4.53	148.50	1.22

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

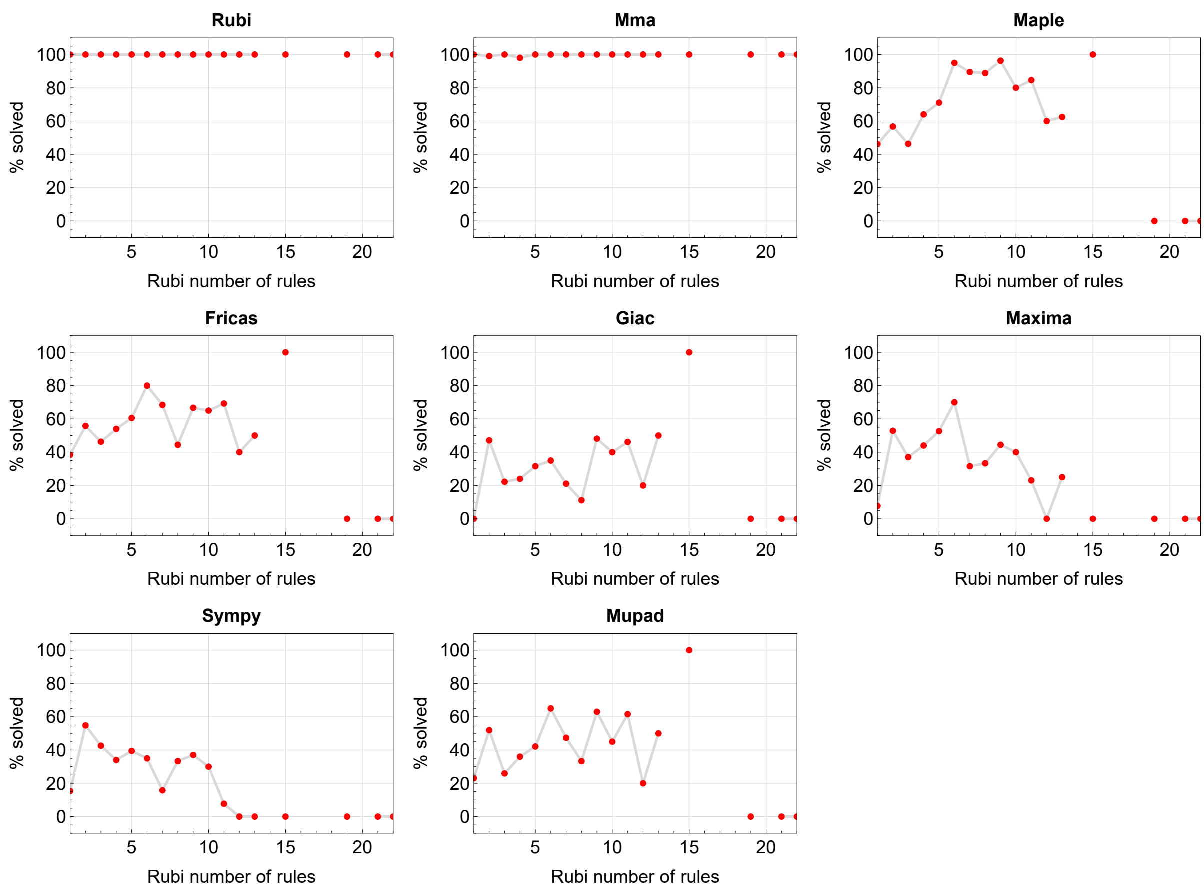


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

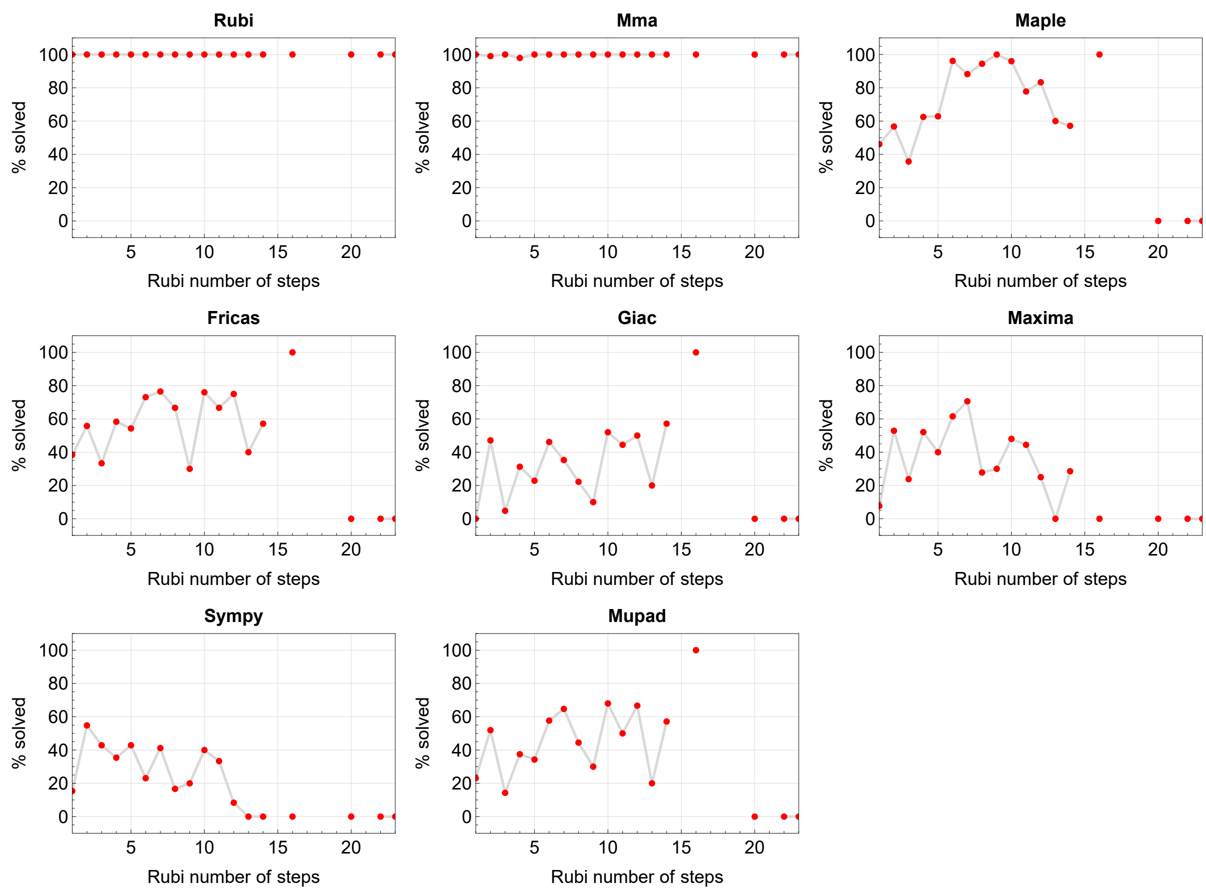


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

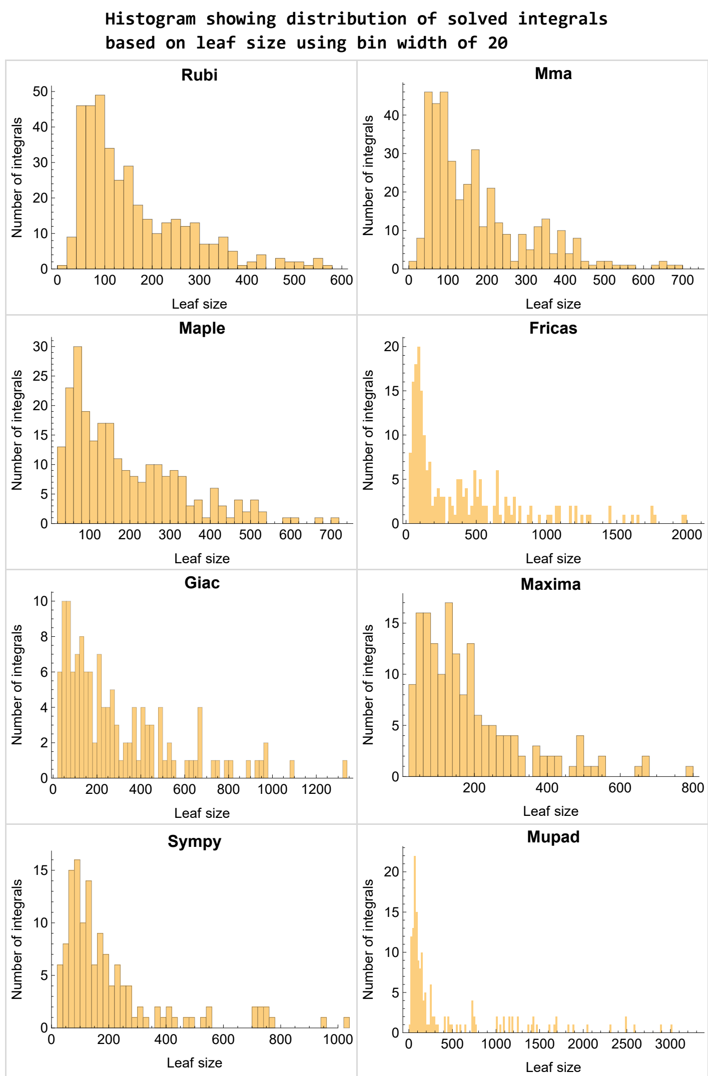


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

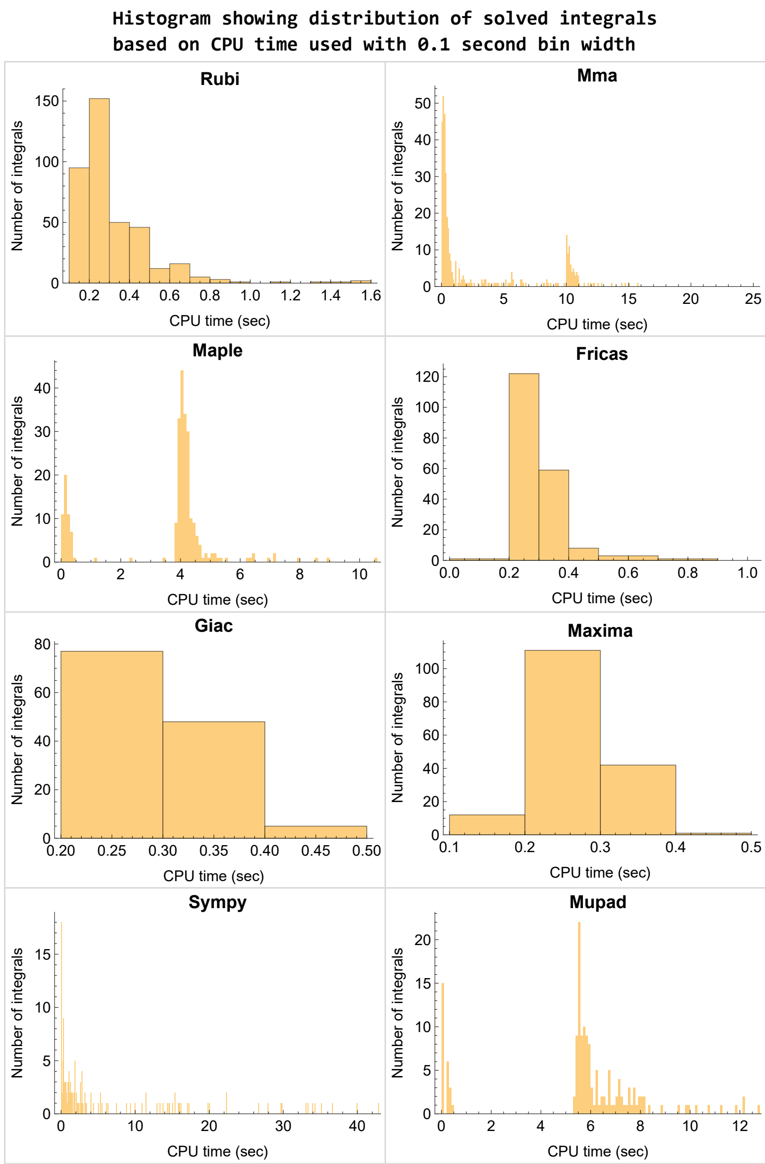


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

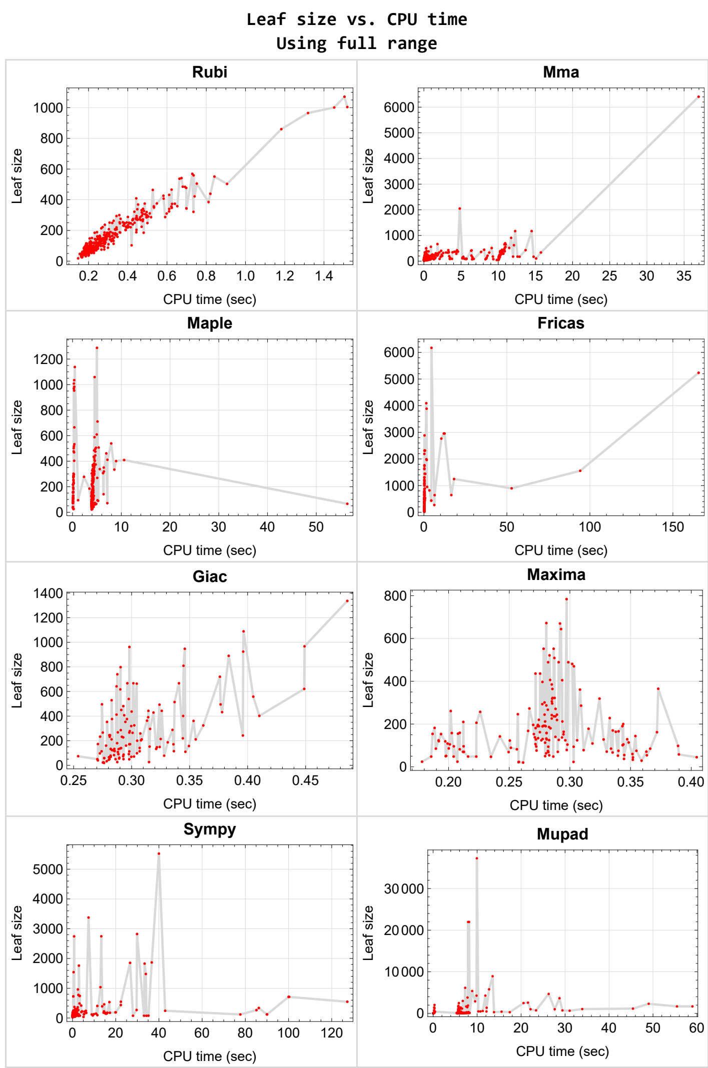


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {34, 35, 37, 38, 39, 79, 80, 83, 86, 87, 93, 94, 95, 96, 97, 98, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 199, 200, 201, 202, 203, 204, 211, 212, 213, 214, 215, 218, 221, 222, 269, 306, 307, 312, 317, 318, 319, 320, 336, 348, 350, 352, 355, 357, 384}

Maple {173, 174, 175, 176, 179, 180, 181, 184, 185, 186, 187}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

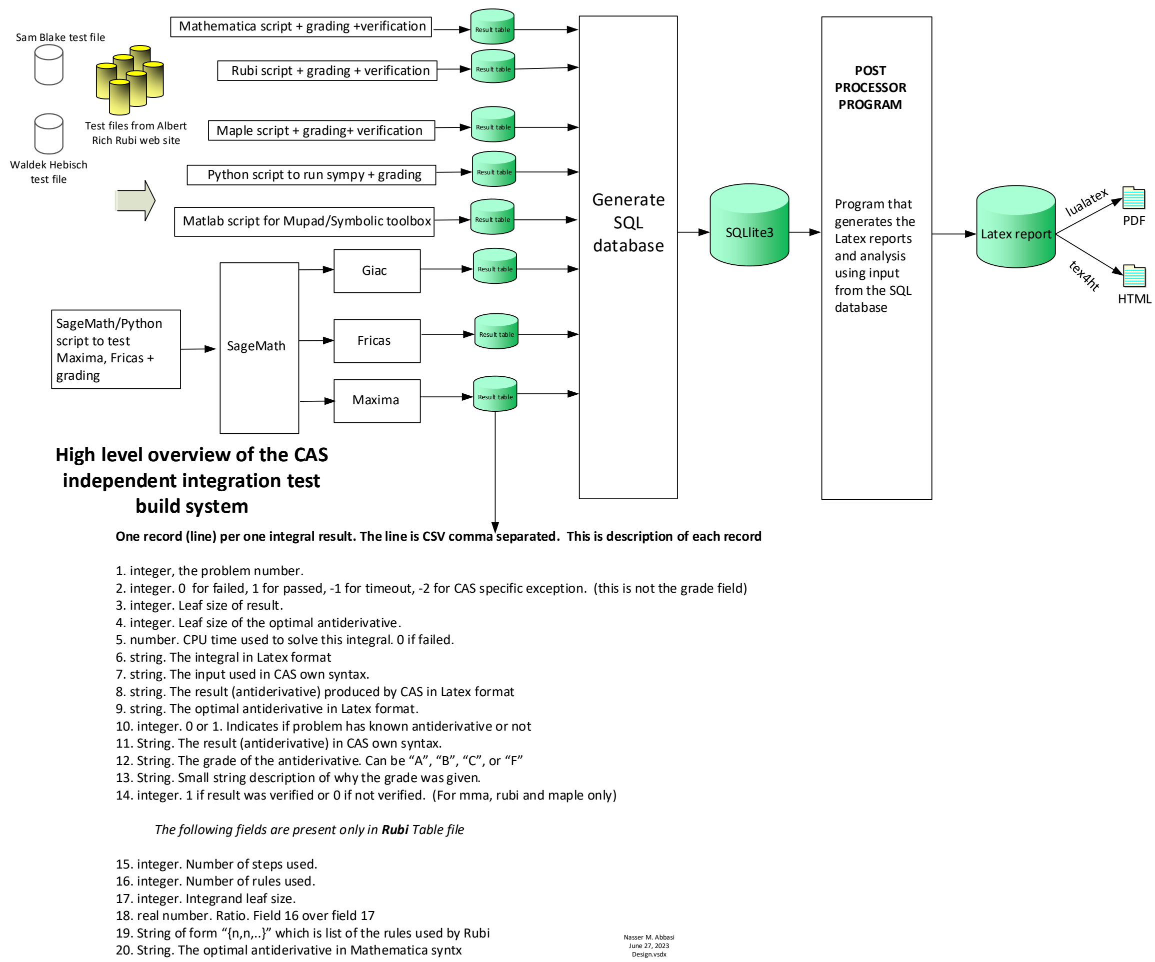
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 383, 385 }

B grade { 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142, 143, 144, 218, 221, 222, 269, 312, 317, 318, 319, 384 }

C grade { 34, 35, 37, 38, 39, 86, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 271, 273, 298, 299, 300, 306, 307, 323, 330, 331, 332, 333, 334, 335, 336, 379, 380 }

F normal fail { 328, 382 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 18, 19, 25, 26, 27, 28, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 56, 57, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 116, 145, 146, 147, 148, 149, 153, 154, 155, 156, 164, 165, 171, 172, 191, 192, 198, 210, 216, 217, 223, 224, 225, 226, 231, 232, 233, 234, 238, 239, 240, 241, 245, 246, 247, 248, 252, 270, 271, 272, 273, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 370, 371, 374, 376, 377, 378, 379 }

B grade { 29, 58, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 227, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 321, 322, 323, 324, 329, 330, 331, 340, 352, 363, 373, 375 }

C grade { 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 275, 354, 364, 372 }

F normal fail { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 199, 200, 201, 202, 203, 204, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 272, 273, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade { 7, 12, 13, 20, 21, 22, 23, 26, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 109, 110, 111, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade { 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192, 193, 194, 205, 206, 216, 380 }

F normal fail { 35, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 218, 219, 220, 221, 222, 269, 271, 274, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

F(-1) timedout fail { 34, 37, 38, 39, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217 }

F(-2) exception fail { 223, 281, 282 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 44, 45, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 85, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 241, 245, 248, 252, 253, 255, 259, 260, 261, 262, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379 }

B grade { 26, 27, 28, 32, 33, 40, 41, 42, 43, 46, 47, 56, 57, 58, 70, 71, 72, 73, 246, 247, 254 }

C grade { }

F normal fail { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { 240 }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 168, 169, 170, 171, 227, 248, 270, 275, 276, 277, 278, 279, 280, 340, 341, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 370, 378 }

B grade { 160, 161, 166, 167, 172, 229, 230, 236, 237, 243, 244, 247, 250, 251, 253, 254, 255, 258, 259, 260, 261, 262, 264, 265, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 342, 343, 344, 345, 347, 355, 356, 357, 365, 366, 367, 369, 371, 372, 373, 374, 375, 376, 377 }

C grade { }

F normal fail { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 332, 333, 334, 335, 336, 379, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { }

F(-2) exception fail { 224, 225, 226, 228, 231, 232, 233, 234, 235, 238, 239, 240, 241, 242, 245, 246, 249, 252, 256, 257, 263, 313, 314, 315, 321, 322, 323, 330, 331 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 316, 324, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 374, 376, 378, 379 }

C grade { }

F normal fail { }

F(-1) timedout fail { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 368, 370, 372, 373, 375, 377, 380, 381, 382, 383, 384, 385 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280 }

B grade { 30, 31, 60, 61, 234, 254, 261, 262, 270, 284, 285, 286, 287, 292, 293, 294, 321, 322, 323, 324, 330, 331, 332 }

C grade { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 134, 135, 139, 140, 141, 219, 220, 288, 289, 295, 298, 299, 300, 301, 308, 313, 314, 315, 316, 349, 351, 353, 354, 356, 359, 361, 363, 364, 366, 378, 379 }

F normal fail { 34, 35, 36, 37, 38, 39, 42, 43, 44, 51, 52, 73, 74, 75, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 113, 118, 125, 126, 127, 128, 129, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 223, 228, 229, 230, 235, 236, 242, 243, 249, 250, 252, 253, 256, 257, 259, 260, 263, 264, 266, 267, 268, 269, 271, 272, 273, 281, 282, 283, 296, 302, 305, 306, 307, 311, 319, 329, 337, 338, 339, 340, 341, 343, 344, 345, 346, 380, 381 }

F(-1) timedout fail { 18, 19, 25, 26, 32, 33, 45, 46, 47, 53, 54, 55, 62, 63, 76, 77, 78, 85, 98, 109, 110, 111, 112, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 130, 131, 132, 133, 136, 137, 138, 142, 143, 144, 145, 164, 165, 166, 167, 171, 172, 184, 189, 205, 218, 221, 222, 237, 244, 251, 258, 265, 274, 290, 291, 297, 342, 348, 350, 352, 355, 357, 358, 360, 362, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 385 }

F(-2) exception fail { 303, 304, 309, 310, 312, 317, 318, 320, 325, 326, 327, 328, 333, 334, 335, 336, 347, 384 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	94	96	96	104	97	87
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.03	0.93
time (sec)	N/A	0.243	0.024	5.121	0.212	0.271	0.025	0.273	0.055

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	80	74	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.14	1.06	0.94
time (sec)	N/A	0.216	0.019	3.997	0.213	0.278	0.022	0.254	0.035

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.200	0.011	3.866	0.223	0.275	0.021	0.270	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.175	0.008	0.231	0.178	0.292	0.018	0.288	0.037

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	131	128	42	128	369	71	133	123
N.S.	1	0.91	0.89	0.29	0.89	2.56	0.49	0.92	0.85
time (sec)	N/A	0.308	0.082	4.047	0.281	0.292	0.209	0.321	0.267

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.306	0.092	3.851	0.281	0.329	0.309	0.278	5.529

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	183	175	84	192	743	133	180	173
N.S.	1	0.93	0.89	0.43	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.338	0.131	3.854	0.279	0.321	0.398	0.287	5.537

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.274	0.022	3.915	0.192	0.285	0.030	0.282	5.334

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	90	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.10	1.11	0.91
time (sec)	N/A	0.236	0.014	3.916	0.209	0.282	0.025	0.289	0.045

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.197	0.009	3.872	0.186	0.276	0.020	0.281	0.048

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	189	505	156	211	152
N.S.	1	1.00	0.97	0.45	1.09	2.92	0.90	1.22	0.88
time (sec)	N/A	0.314	0.096	3.899	0.277	0.299	0.342	0.283	5.547

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	210	99	226	771	189	233	191
N.S.	1	1.00	1.03	0.49	1.11	3.80	0.93	1.15	0.94
time (sec)	N/A	0.409	0.212	3.911	0.285	0.336	0.653	0.273	5.593

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	221	217	131	267	1067	233	264	249
N.S.	1	0.86	0.84	0.51	1.03	4.14	0.90	1.02	0.97
time (sec)	N/A	0.434	0.272	3.890	0.282	0.323	1.018	0.301	5.591

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	253	201	364	873	371	391	250
N.S.	1	1.00	1.00	0.80	1.44	3.46	1.47	1.55	0.99
time (sec)	N/A	0.415	0.133	4.149	0.286	0.322	0.870	0.314	5.596

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	203	131	273	700	257	296	192
N.S.	1	1.00	0.98	0.63	1.31	3.37	1.24	1.42	0.92
time (sec)	N/A	0.362	0.096	3.923	0.267	0.316	0.495	0.316	5.592

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	190	507	156	211	152
N.S.	1	1.00	0.97	0.45	1.10	2.93	0.90	1.22	0.88
time (sec)	N/A	0.300	0.110	3.906	0.273	0.316	0.335	0.309	0.245

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	390	71	133	123
N.S.	1	0.90	0.89	0.29	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.295	0.065	4.166	0.272	0.319	0.211	0.285	5.533

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	293	254	0	278	1364
N.S.	1	0.86	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.404	0.121	4.026	0.277	0.351	0.000	0.298	12.176

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	307	336	246	489	432	0	443	2589
N.S.	1	0.89	0.97	0.71	1.41	1.25	0.00	1.28	7.48
time (sec)	N/A	0.499	0.217	4.037	0.299	4.481	0.000	0.314	21.570

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	313	304	509	1619	546	529	416
N.S.	1	1.00	0.98	0.95	1.59	5.06	1.71	1.65	1.30
time (sec)	N/A	0.519	0.259	3.925	0.287	0.318	127.217	0.283	0.443

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	260	218	397	1316	405	412	302
N.S.	1	1.00	0.97	0.82	1.49	4.93	1.52	1.54	1.13
time (sec)	N/A	0.447	0.225	3.930	0.277	0.312	11.467	0.287	5.744

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	227	153	306	1027	291	319	240
N.S.	1	1.00	0.97	0.65	1.31	4.39	1.24	1.36	1.03
time (sec)	N/A	0.421	0.164	3.899	0.285	0.291	1.307	0.300	0.308

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	205	101	220	768	189	227	191
N.S.	1	1.00	1.01	0.50	1.08	3.78	0.93	1.12	0.94
time (sec)	N/A	0.402	0.216	3.907	0.274	0.303	0.601	0.293	5.664

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.317	0.098	3.878	0.281	0.296	0.316	0.295	5.536

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	306	337	247	489	440	0	443	2492
N.S.	1	0.88	0.97	0.71	1.41	1.27	0.00	1.28	7.20
time (sec)	N/A	0.508	0.233	4.025	0.291	4.460	0.000	0.326	20.561

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	373	381	285	784	897	0	664	3637
N.S.	1	0.89	0.91	0.68	1.87	2.14	0.00	1.58	8.68
time (sec)	N/A	0.659	0.632	4.068	0.297	52.873	0.000	0.304	28.736

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	114	156	144	322	399	80	0	0
N.S.	1	1.02	1.39	1.29	2.88	3.56	0.71	0.00	0.00
time (sec)	N/A	0.203	0.518	4.067	0.286	0.350	1.874	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	92	140	118	244	363	76	0	0
N.S.	1	1.01	1.54	1.30	2.68	3.99	0.84	0.00	0.00
time (sec)	N/A	0.183	0.443	4.179	0.279	0.300	1.207	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	142	142	130	372	70	0	0
N.S.	1	1.00	1.67	1.67	1.53	4.38	0.82	0.00	0.00
time (sec)	N/A	0.182	0.433	3.931	0.278	0.280	2.808	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	50	44	190	0	27
N.S.	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57
time (sec)	N/A	0.164	0.306	4.057	0.198	0.287	19.872	0.000	5.492

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	40	37	85	69	709	0	44
N.S.	1	1.09	0.73	0.67	1.55	1.25	12.89	0.00	0.80
time (sec)	N/A	0.178	0.379	4.077	0.190	0.274	100.311	0.000	5.453

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	87	51	48	119	91	0	0	58
N.S.	1	1.18	0.69	0.65	1.61	1.23	0.00	0.00	0.78
time (sec)	N/A	0.194	0.515	3.935	0.197	0.289	0.000	0.000	5.494

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	114	62	59	153	113	0	0	73
N.S.	1	1.23	0.67	0.63	1.65	1.22	0.00	0.00	0.78
time (sec)	N/A	0.213	0.723	3.909	0.194	0.298	0.000	0.000	5.502

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	483	559	232	0	0	0	0	0	0
N.S.	1	1.16	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	10.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	464	537	217	0	0	0	0	0	0
N.S.	1	1.16	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	10.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	398	409	428	0	0	644	0	0	0
N.S.	1	1.03	1.08	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.473	3.288	0.000	0.000	16.545	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	452	464	153	0	0	0	0	0	0
N.S.	1	1.03	0.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	473	541	213	0	0	0	0	0	0
N.S.	1	1.14	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	10.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	492	569	240	0	0	0	0	0	0
N.S.	1	1.16	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	10.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	144	167	164	552	421	126	0	0
N.S.	1	1.04	1.20	1.18	3.97	3.03	0.91	0.00	0.00
time (sec)	N/A	0.240	0.763	4.129	0.287	0.443	5.341	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	122	165	144	436	399	121	0	0
N.S.	1	1.02	1.38	1.20	3.63	3.32	1.01	0.00	0.00
time (sec)	N/A	0.222	0.618	4.124	0.272	0.358	2.823	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	154	156	296	412	0	0	0
N.S.	1	1.01	1.36	1.38	2.62	3.65	0.00	0.00	0.00
time (sec)	N/A	0.219	0.678	4.200	0.285	0.356	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	137	128	180	521	0	0	0
N.S.	1	1.05	1.25	1.16	1.64	4.74	0.00	0.00	0.00
time (sec)	N/A	0.216	0.549	4.377	0.284	0.348	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	40	37	105	67	0	0	44
N.S.	1	1.07	0.53	0.49	1.38	0.88	0.00	0.00	0.58
time (sec)	N/A	0.197	0.456	4.126	0.201	0.443	0.000	0.000	5.484

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	118	51	48	155	91	0	0	56
N.S.	1	1.12	0.49	0.46	1.48	0.87	0.00	0.00	0.53
time (sec)	N/A	0.228	0.658	4.062	0.208	0.353	0.000	0.000	5.509

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	116	62	59	206	113	0	0	71
N.S.	1	1.18	0.63	0.60	2.10	1.15	0.00	0.00	0.72
time (sec)	N/A	0.234	0.883	3.993	0.223	0.344	0.000	0.000	5.518

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	143	73	70	257	135	0	0	86
N.S.	1	1.22	0.62	0.60	2.20	1.15	0.00	0.00	0.74
time (sec)	N/A	0.262	1.345	4.002	0.226	0.364	0.000	0.000	5.500

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	99	97	0	0	0	168	0	0
N.S.	1	1.05	1.03	0.00	0.00	0.00	1.79	0.00	0.00
time (sec)	N/A	0.222	8.400	0.000	0.000	0.000	2.098	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	93	85	0	0	0	126	0	0
N.S.	1	0.99	0.90	0.00	0.00	0.00	1.34	0.00	0.00
time (sec)	N/A	0.215	6.710	0.000	0.000	0.000	1.456	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	93	75	0	0	0	121	0	0
N.S.	1	0.99	0.80	0.00	0.00	0.00	1.29	0.00	0.00
time (sec)	N/A	0.214	10.047	0.000	0.000	0.000	1.565	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	10.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	10.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	10.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	98	95	0	0	0	0	0	0
N.S.	1	1.05	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	10.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	98	106	0	0	0	0	0	0
N.S.	1	1.05	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	10.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	151	208	180	406	482	170	0	0
N.S.	1	0.87	1.20	1.03	2.33	2.77	0.98	0.00	0.00
time (sec)	N/A	0.230	1.003	4.123	0.284	0.306	6.364	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	129	180	151	322	424	82	0	0
N.S.	1	0.91	1.28	1.07	2.28	3.01	0.58	0.00	0.00
time (sec)	N/A	0.215	0.828	4.081	0.287	0.314	1.825	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	107	163	225	244	362	78	0	0
N.S.	1	0.96	1.47	2.03	2.20	3.26	0.70	0.00	0.00
time (sec)	N/A	0.196	0.622	4.048	0.289	0.286	1.218	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	103	150	161	134	488	71	0	0
N.S.	1	1.04	1.52	1.63	1.35	4.93	0.72	0.00	0.00
time (sec)	N/A	0.198	0.529	3.985	0.274	0.309	2.835	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	51	54	190	0	33
N.S.	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70
time (sec)	N/A	0.164	0.374	3.904	0.198	0.267	20.029	0.000	5.459

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	86	87	709	0	87
N.S.	1	0.97	0.66	0.57	0.95	0.96	7.79	0.00	0.96
time (sec)	N/A	0.211	0.525	3.931	0.200	0.323	100.012	0.000	5.495

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	115	80	71	120	121	0	0	105
N.S.	1	0.95	0.66	0.59	0.99	1.00	0.00	0.00	0.87
time (sec)	N/A	0.227	0.755	3.930	0.192	0.301	0.000	0.000	5.514

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	142	100	90	154	155	0	0	132
N.S.	1	0.94	0.66	0.60	1.02	1.03	0.00	0.00	0.87
time (sec)	N/A	0.262	1.126	4.091	0.187	0.364	0.000	0.000	5.545

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	0.00
time (sec)	N/A	0.191	9.342	0.000	0.000	0.000	2.666	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	0.00
time (sec)	N/A	0.197	8.283	0.000	0.000	0.000	1.816	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.199	6.488	0.000	0.000	0.000	1.040	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.196	10.044	0.000	0.000	0.000	0.916	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.84	0.00	0.00
time (sec)	N/A	0.206	10.042	0.000	0.000	0.000	3.419	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	78	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.83	0.00	0.00
time (sec)	N/A	0.204	10.047	0.000	0.000	0.000	35.161	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	208	293	244	672	717	270	0	0
N.S.	1	0.79	1.12	0.93	2.56	2.74	1.03	0.00	0.00
time (sec)	N/A	0.313	1.615	4.262	0.281	0.300	29.662	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	187	256	203	552	634	131	0	0
N.S.	1	0.85	1.17	0.93	2.52	2.89	0.60	0.00	0.00
time (sec)	N/A	0.289	1.125	4.219	0.278	0.328	5.304	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	165	223	166	436	554	126	0	0
N.S.	1	0.94	1.27	0.95	2.49	3.17	0.72	0.00	0.00
time (sec)	N/A	0.265	0.994	4.194	0.275	0.329	2.814	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	199	170	301	652	0	0	0
N.S.	1	1.00	1.25	1.07	1.89	4.10	0.00	0.00	0.00
time (sec)	N/A	0.269	1.191	4.276	0.279	0.325	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	164	186	176	190	719	0	0	0
N.S.	1	1.08	1.22	1.16	1.25	4.73	0.00	0.00	0.00
time (sec)	N/A	0.263	1.101	4.129	0.274	0.401	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	84	73	71	109	103	0	0	148
N.S.	1	1.08	0.94	0.91	1.40	1.32	0.00	0.00	1.90
time (sec)	N/A	0.199	0.853	4.082	0.199	0.296	0.000	0.000	5.577

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	150	106	96	159	152	0	0	176
N.S.	1	0.86	0.61	0.55	0.91	0.87	0.00	0.00	1.01
time (sec)	N/A	0.270	1.140	4.204	0.209	0.363	0.000	0.000	5.575

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	186	138	126	210	200	0	0	217
N.S.	1	0.88	0.65	0.60	1.00	0.95	0.00	0.00	1.03
time (sec)	N/A	0.317	1.561	4.064	0.212	0.307	0.000	0.000	5.580

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	207	169	156	261	246	0	0	257
N.S.	1	0.82	0.67	0.62	1.03	0.97	0.00	0.00	1.02
time (sec)	N/A	0.360	2.260	4.213	0.202	0.342	0.000	0.000	5.595

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	143	177	0	0	0	418	0	0
N.S.	1	1.06	1.31	0.00	0.00	0.00	3.10	0.00	0.00
time (sec)	N/A	0.267	12.595	0.000	0.000	0.000	4.007	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	141	176	0	0	0	270	0	0
N.S.	1	1.06	1.32	0.00	0.00	0.00	2.03	0.00	0.00
time (sec)	N/A	0.269	11.999	0.000	0.000	0.000	2.604	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	139	179	0	0	0	131	0	0
N.S.	1	1.06	1.37	0.00	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.265	9.279	0.000	0.000	0.000	1.465	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	135	104	0	0	0	126	0	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.275	15.098	0.000	0.000	0.000	1.518	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	12.873	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	149	171	0	0	0	0	0	0
N.S.	1	1.01	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	14.757	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	121	120	110	182	166	0	0	271
N.S.	1	1.11	1.10	1.01	1.67	1.52	0.00	0.00	2.49
time (sec)	N/A	0.234	1.403	4.119	0.189	0.292	0.000	0.000	5.859

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	327	655	461	0	643	0	0	0
N.S.	1	0.99	1.98	1.39	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.510	10.833	6.922	0.000	6.475	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	273	276	443	402	0	535	0	0	0
N.S.	1	1.01	1.62	1.47	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.364	10.506	4.573	0.000	0.765	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	237	423	339	0	469	0	0	0
N.S.	1	1.02	1.82	1.45	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.291	4.463	4.159	0.000	0.303	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	255	168	0	0	0	0	0
N.S.	1	1.00	1.72	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	1.909	4.216	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	189	328	243	0	0	0	0	0
N.S.	1	1.06	1.83	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	3.232	4.091	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	249	364	269	0	0	0	0	0
N.S.	1	1.10	1.61	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	4.085	5.065	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	322	420	321	0	0	0	0	0
N.S.	1	1.15	1.50	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	6.425	4.423	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	10.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	10.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	10.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	0
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	0
N.S.	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	10.776	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	351	341	698	496	0	819	0	0	0
N.S.	1	0.97	1.99	1.41	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	0.491	10.938	4.674	0.000	3.351	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	300	510	435	0	631	0	0	0
N.S.	1	1.00	1.69	1.45	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.375	9.120	4.355	0.000	0.516	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	185	319	229	0	0	0	0	0
N.S.	1	1.02	1.75	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	2.605	4.162	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	214	336	261	0	0	0	0	0
N.S.	1	0.99	1.55	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	2.987	4.193	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	263	370	303	0	0	0	0	0
N.S.	1	1.01	1.42	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	4.585	4.461	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	338	443	371	0	0	0	0	0
N.S.	1	1.04	1.37	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	8.108	4.551	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	0
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	10.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	0
N.S.	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	10.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	0
N.S.	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	0
N.S.	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	0
N.S.	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.976	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	541	505	1171	711	0	1555	0	0	0
N.S.	1	0.93	2.16	1.31	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.792	12.279	5.124	0.000	94.183	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	458	432	908	609	0	1246	0	0	0
N.S.	1	0.94	1.98	1.33	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.645	11.742	4.972	0.000	18.185	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	391	375	651	505	0	954	0	0	0
N.S.	1	0.96	1.66	1.29	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.496	11.002	4.803	0.000	1.954	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	222	344	259	0	0	0	0	0
N.S.	1	1.02	1.59	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	3.845	4.321	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	251	366	297	0	0	0	0	0
N.S.	1	0.94	1.37	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	4.298	4.320	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	294	407	307	0	0	0	0	0
N.S.	1	0.96	1.33	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	6.353	4.374	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	369	428	426	0	0	0	0	0
N.S.	1	0.98	1.14	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	13.679	4.462	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	463	466	337	475	0	0	0	0	0
N.S.	1	1.01	0.73	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.632	15.745	4.687	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	285	0	0	0	0	0	0
N.S.	1	1.00	4.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	10.467	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	407	0	0	0	0	0	0
N.S.	1	1.00	6.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	10.624	0.000	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	418	0	0	0	0	0	0
N.S.	1	1.00	7.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.679	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	0
N.S.	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	10.918	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	0
N.S.	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	11.481	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	161	90	0	0	0	0	0	0
N.S.	1	1.04	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	5.738	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	161	90	0	0	0	0	0	0
N.S.	1	1.04	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	5.460	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	5.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	3.783	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	3.504	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	5.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	3.489	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	5.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	5.681	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	160	90	0	0	0	0	0	0
N.S.	1	1.05	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	5.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	160	90	0	0	0	0	0	0
N.S.	1	1.05	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	5.720	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	175	106	0	0	0	121	0	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.72	0.00	0.00
time (sec)	N/A	0.339	0.210	0.000	0.000	0.000	90.092	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	75	0	0
N.S.	1	1.01	1.07	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.221	0.143	0.000	0.000	0.000	34.200	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	0.442	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	288	137	0	0	0	0	0	0
N.S.	1	0.97	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	8.446	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	175	106	0	0	0	121	0	0
N.S.	1	0.99	0.60	0.00	0.00	0.00	0.69	0.00	0.00
time (sec)	N/A	0.336	5.186	0.000	0.000	0.000	89.965	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.220	0.140	0.000	0.000	0.000	33.081	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	203	0	0	0	34	0	41
N.S.	1	1.00	4.61	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.170	0.179	0.000	0.000	0.000	5.070	0.000	5.426

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.178	0.562	7.138	0.000	0.303	0.000	0.000	5.925

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.254	0.024	3.955	0.204	0.266	0.033	0.305	5.409

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	76	74	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.09	1.06	0.94
time (sec)	N/A	0.237	0.017	3.905	0.212	0.246	0.025	0.293	5.394

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.199	0.013	4.159	0.204	0.257	0.022	0.293	0.050

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.174	0.008	0.178	0.207	0.282	0.024	0.315	0.038

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	220	196	42	212	560	87	245	720
N.S.	1	0.99	0.88	0.19	0.95	2.51	0.39	1.10	3.23
time (sec)	N/A	0.411	0.133	4.324	0.294	0.283	0.284	0.281	0.235

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	247	212	65	236	648	112	266	740
N.S.	1	1.01	0.87	0.27	0.96	2.64	0.46	1.09	3.02
time (sec)	N/A	0.424	0.168	4.087	0.288	0.280	0.399	0.275	5.688

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	272	243	84	271	739	151	286	762
N.S.	1	1.00	0.89	0.31	0.99	2.71	0.55	1.05	2.79
time (sec)	N/A	0.454	0.208	3.934	0.288	0.301	0.522	0.291	5.934

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	160	158	158	185	173	146
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.20	1.12	0.95
time (sec)	N/A	0.322	0.034	3.949	0.203	0.275	0.031	0.271	5.647

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.295	0.024	3.876	0.276	0.260	0.036	0.289	5.590

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.236	0.019	4.004	0.256	0.350	0.028	0.298	0.046

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.198	0.010	3.949	0.216	0.270	0.026	0.286	0.044

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	78	286	1092	187	353	1081
N.S.	1	1.00	0.91	0.31	1.13	4.32	0.74	1.40	4.27
time (sec)	N/A	0.386	0.106	3.986	0.309	0.307	0.584	0.295	5.840

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	298	101	319	1210	219	376	1254
N.S.	1	1.00	1.02	0.35	1.10	4.16	0.75	1.29	4.31
time (sec)	N/A	0.521	0.175	3.965	0.324	0.283	2.016	0.296	5.950

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	311	319	131	361	1294	264	407	1401
N.S.	1	0.89	0.91	0.38	1.03	3.71	0.76	1.17	4.01
time (sec)	N/A	0.533	0.212	4.014	0.308	0.306	85.141	0.294	6.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	322	201	489	2190	435	617	1822
N.S.	1	1.00	0.97	0.61	1.47	6.60	1.31	1.86	5.49
time (sec)	N/A	0.494	0.210	4.205	0.282	0.295	22.351	0.290	5.745

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	271	131	385	1642	303	481	1433
N.S.	1	1.00	0.94	0.45	1.34	5.70	1.05	1.67	4.98
time (sec)	N/A	0.407	0.163	3.980	0.285	0.299	1.296	0.293	0.236

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	78	287	1093	187	353	1081
N.S.	1	1.00	0.91	0.31	1.13	4.32	0.74	1.40	4.27
time (sec)	N/A	0.374	0.103	4.109	0.281	0.287	0.530	0.279	5.706

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	219	196	42	212	560	87	245	720
N.S.	1	0.98	0.88	0.19	0.95	2.51	0.39	1.10	3.23
time (sec)	N/A	0.399	0.115	3.941	0.280	0.290	0.309	0.281	0.223

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	426	340	226	365	1171	0	437	6153
N.S.	1	0.95	0.76	0.50	0.81	2.61	0.00	0.97	13.70
time (sec)	N/A	0.601	0.126	4.086	0.373	0.392	0.000	0.300	7.296

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	485	498	262	481	2955	0	667	21975
N.S.	1	0.95	0.97	0.51	0.94	5.76	0.00	1.30	42.84
time (sec)	N/A	0.714	0.307	4.097	0.302	12.016	0.000	0.296	8.177

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	391	304	644	2884	0	798	2490
N.S.	1	1.00	0.96	0.75	1.58	7.09	0.00	1.96	6.12
time (sec)	N/A	0.600	0.366	4.025	0.293	0.401	0.000	0.290	5.856

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	341	219	521	2315	0	642	2043
N.S.	1	1.00	0.96	0.61	1.46	6.48	0.00	1.80	5.72
time (sec)	N/A	0.540	0.306	3.987	0.283	0.331	0.000	0.287	0.318

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	301	151	405	1741	337	496	1616
N.S.	1	1.00	0.95	0.48	1.28	5.49	1.06	1.56	5.10
time (sec)	N/A	0.489	0.243	4.011	0.294	0.323	86.251	0.274	5.719

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	297	101	319	1210	219	376	1254
N.S.	1	1.00	1.02	0.35	1.10	4.16	0.75	1.29	4.31
time (sec)	N/A	0.519	0.178	4.264	0.285	0.319	1.147	0.283	0.304

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	247	212	65	236	648	112	266	740
N.S.	1	1.01	0.87	0.27	0.96	2.64	0.46	1.09	3.02
time (sec)	N/A	0.430	0.167	4.001	0.287	0.293	0.457	0.298	5.701

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	484	499	263	470	2955	0	667	21975
N.S.	1	0.94	0.97	0.51	0.92	5.76	0.00	1.30	42.84
time (sec)	N/A	0.723	0.309	4.121	0.303	12.457	0.000	0.302	7.956

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	551	561	298	670	5234	0	967	37266
N.S.	1	0.92	0.94	0.50	1.12	8.78	0.00	1.62	62.53
time (sec)	N/A	0.862	0.945	4.195	0.292	165.512	0.000	0.449	9.949

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	321	324	290	350	0	0	0	0	0
N.S.	1	1.01	0.90	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	10.698	6.409	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	277	272	341	306	0	0	0	0	0
N.S.	1	0.98	1.23	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	10.352	6.264	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	232	155	259	0	0	0	0	0
N.S.	1	0.97	0.65	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	10.183	4.139	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	0
N.S.	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	10.224	4.233	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	281	269	381	301	0	0	0	0	0
N.S.	1	0.96	1.36	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	10.263	4.152	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	334	340	422	361	0	0	0	0	0
N.S.	1	1.02	1.26	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.628	10.716	4.184	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	926	1004	346	319	0	0	0	0	0
N.S.	1	1.08	0.37	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.563	10.473	6.444	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	881	965	161	273	0	0	0	0	0
N.S.	1	1.10	0.18	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.372	10.186	4.142	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	742	860	161	191	0	0	0	0	0
N.S.	1	1.16	0.22	0.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.217	10.057	4.165	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	913	1001	331	313	0	0	0	0	0
N.S.	1	1.10	0.36	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.433	10.278	4.184	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	976	1071	429	371	0	0	0	0	0
N.S.	1	1.10	0.44	0.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.644	10.818	4.164	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	426	422	477	539	0	0	0	0	0
N.S.	1	0.99	1.12	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	10.818	7.936	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	365	356	396	411	0	0	0	0	0
N.S.	1	0.98	1.08	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	10.584	7.161	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	309	299	342	328	0	0	0	0	0
N.S.	1	0.97	1.11	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	10.332	4.033	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	276	270	233	293	0	0	0	0	0
N.S.	1	0.98	0.84	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	10.166	4.351	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	286	386	321	0	0	0	0	0
N.S.	1	0.92	1.25	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	10.281	4.477	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	362	347	374	374	0	0	0	0	0
N.S.	1	0.96	1.03	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	10.520	4.751	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	0
N.S.	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.873	10.879	4.427	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	101	81	89	0	437	0	0	0
N.S.	1	0.98	0.79	0.86	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.211	0.553	5.254	0.000	0.578	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	88	141	0	482	0	0	0
N.S.	1	1.00	0.76	1.22	0.00	4.16	0.00	0.00	0.00
time (sec)	N/A	0.199	0.519	6.369	0.000	0.576	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	220	288	401	0	1962	0	0	0
N.S.	1	1.04	1.36	1.90	0.00	9.30	0.00	0.00	0.00
time (sec)	N/A	0.417	1.796	8.934	0.000	1.706	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	181	255	338	0	733	0	0	0
N.S.	1	1.05	1.47	1.95	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.317	1.199	5.585	0.000	0.405	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	178	244	0	0	0	0	0
N.S.	1	1.00	1.70	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.879	4.252	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	146	229	325	0	0	0	0	0
N.S.	1	1.09	1.71	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	1.788	4.146	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	206	621	329	0	0	0	0	0
N.S.	1	1.14	3.45	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	12.117	4.390	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	279	1172	381	0	0	0	0	0
N.S.	1	1.20	5.03	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	14.498	4.440	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	316	287	294	0	0	0	0	0	0
N.S.	1	0.91	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.625	10.647	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	239	346	0	0	0	0	0	0
N.S.	1	0.87	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	10.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	166	141	160	0	0	0	0	0	0
N.S.	1	0.85	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	10.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	259	225	161	0	0	0	0	0	0
N.S.	1	0.87	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	10.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	304	285	332	0	0	0	0	0	0
N.S.	1	0.94	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	10.273	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	357	358	430	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	10.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	286	349	507	0	2764	0	0	0
N.S.	1	1.02	1.25	1.81	0.00	9.87	0.00	0.00	0.00
time (sec)	N/A	0.553	3.364	5.364	0.000	10.518	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	244	341	408	0	1440	0	0	0
N.S.	1	1.06	1.48	1.77	0.00	6.26	0.00	0.00	0.00
time (sec)	N/A	0.409	2.076	4.509	0.000	0.859	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	142	238	293	0	0	0	0	0
N.S.	1	1.05	1.76	2.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	1.464	4.429	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	171	251	343	0	0	0	0	0
N.S.	1	1.06	1.55	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	2.396	4.401	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	220	285	355	0	0	0	0	0
N.S.	1	1.07	1.39	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	4.372	4.476	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	295	357	445	0	0	0	0	0
N.S.	1	1.11	1.34	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	7.694	4.661	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	353	310	392	0	0	0	0	0	0
N.S.	1	0.88	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	10.481	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	298	261	341	0	0	0	0	0	0
N.S.	1	0.88	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	10.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	308	268	233	0	0	0	0	0	0
N.S.	1	0.87	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	10.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	330	300	337	0	0	0	0	0	0
N.S.	1	0.91	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	10.370	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	390	374	387	0	0	0	0	0	0
N.S.	1	0.96	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	10.539	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	61	0	270	0	0	0
N.S.	1	1.00	0.83	1.15	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.178	0.302	4.669	0.000	6.297	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	65	0	0	0	0	0
N.S.	1	1.00	0.84	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.565	4.867	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	175	106	0	0	0	119	0	0
N.S.	1	0.99	0.60	0.00	0.00	0.00	0.68	0.00	0.00
time (sec)	N/A	0.344	0.282	0.000	0.000	0.000	77.687	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.219	0.172	0.000	0.000	0.000	27.993	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.408	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.510	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	503	48	409	0	0	0	0	0
N.S.	1	0.92	0.09	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.913	10.031	10.584	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	154	118	160	164	306	461	0	173
N.S.	1	1.08	0.83	1.12	1.15	2.14	3.22	0.00	1.21
time (sec)	N/A	0.301	0.329	0.101	0.295	0.322	14.651	0.000	6.787

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	84	115	126	208	129	0	99
N.S.	1	1.00	0.85	1.16	1.27	2.10	1.30	0.00	1.00
time (sec)	N/A	0.235	0.277	0.085	0.291	0.260	9.907	0.000	6.066

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	52	84	106	128	95	0	92
N.S.	1	0.97	0.70	1.14	1.43	1.73	1.28	0.00	1.24
time (sec)	N/A	0.204	0.168	0.081	0.272	0.249	11.491	0.000	6.128

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	72	50	99	42	62	58
N.S.	1	1.00	1.00	1.85	1.28	2.54	1.08	1.59	1.49
time (sec)	N/A	0.166	0.019	0.052	0.280	0.263	0.979	0.289	5.469

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	100	232	0	482	0	0	149
N.S.	1	1.09	0.96	2.23	0.00	4.63	0.00	0.00	1.43
time (sec)	N/A	0.271	0.322	0.222	0.000	0.280	0.000	0.000	5.736

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	162	122	477	0	801	0	401	1195
N.S.	1	1.10	0.83	3.24	0.00	5.45	0.00	2.73	8.13
time (sec)	N/A	0.318	0.533	0.235	0.000	0.305	0.000	0.344	6.434

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	250	189	969	0	1749	0	809	1895
N.S.	1	1.17	0.89	4.55	0.00	8.21	0.00	3.80	8.90
time (sec)	N/A	0.431	1.428	0.261	0.000	0.408	0.000	0.345	7.936

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	166	159	214	190	380	1828	0	327
N.S.	1	1.01	0.97	1.30	1.16	2.32	11.15	0.00	1.99
time (sec)	N/A	0.306	0.399	0.104	0.280	0.280	33.365	0.000	8.027

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	118	115	153	152	268	546	0	197
N.S.	1	0.94	0.91	1.21	1.21	2.13	4.33	0.00	1.56
time (sec)	N/A	0.241	0.367	0.091	0.295	0.259	22.390	0.000	6.706

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	91	73	105	132	164	177	0	81
N.S.	1	0.91	0.73	1.05	1.32	1.64	1.77	0.00	0.81
time (sec)	N/A	0.206	0.258	0.086	0.281	0.275	14.378	0.000	6.705

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	46	78	63	100	92	0	34
N.S.	1	1.07	0.85	1.44	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.178	0.014	0.060	0.280	0.340	1.393	0.000	5.602

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	115	103	247	0	519	0	0	556
N.S.	1	1.08	0.97	2.33	0.00	4.90	0.00	0.00	5.25
time (sec)	N/A	0.273	0.395	0.208	0.000	0.309	0.000	0.000	5.817

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	188	144	502	0	769	0	514	448
N.S.	1	1.21	0.92	3.22	0.00	4.93	0.00	3.29	2.87
time (sec)	N/A	0.386	0.513	0.234	0.000	0.287	0.000	0.337	6.332

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	225	169	1008	0	1765	0	720	1664
N.S.	1	1.08	0.81	4.82	0.00	8.44	0.00	3.44	7.96
time (sec)	N/A	0.411	0.658	0.273	0.000	0.341	0.000	0.376	7.515

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	190	201	273	219	494	5523	0	487
N.S.	1	0.96	1.02	1.38	1.11	2.49	27.89	0.00	2.46
time (sec)	N/A	0.309	0.502	0.154	0.289	0.275	39.992	0.000	10.276

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	136	145	194	181	350	1853	0	271
N.S.	1	0.89	0.95	1.28	1.19	2.30	12.19	0.00	1.78
time (sec)	N/A	0.251	0.405	0.135	0.272	0.277	26.636	0.000	8.108

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	109	94	129	0	222	534	0	99
N.S.	1	0.87	0.75	1.03	0.00	1.78	4.27	0.00	0.79
time (sec)	N/A	0.224	0.305	0.106	0.000	0.283	17.071	0.000	7.754

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	76	64	94	78	139	99	0	34
N.S.	1	1.07	0.90	1.32	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.202	0.060	0.069	0.311	0.278	2.148	0.000	5.878

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	146	117	279	0	659	0	0	1427
N.S.	1	1.09	0.87	2.08	0.00	4.92	0.00	0.00	10.65
time (sec)	N/A	0.347	0.376	0.249	0.000	0.376	0.000	0.000	6.260

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	186	146	517	0	1001	0	667	1153
N.S.	1	1.12	0.88	3.11	0.00	6.03	0.00	4.02	6.95
time (sec)	N/A	0.381	0.482	0.284	0.000	0.334	0.000	0.341	6.586

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	274	205	1035	0	1445	0	947	1476
N.S.	1	1.16	0.86	4.37	0.00	6.10	0.00	4.00	6.23
time (sec)	N/A	0.476	0.653	0.339	0.000	0.488	0.000	0.346	7.775

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	131	95	130	166	233	391	0	107
N.S.	1	1.04	0.75	1.03	1.32	1.85	3.10	0.00	0.85
time (sec)	N/A	0.261	0.324	0.146	0.340	0.265	13.729	0.000	5.926

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	82	66	105	129	158	119	0	63
N.S.	1	1.12	0.90	1.44	1.77	2.16	1.63	0.00	0.86
time (sec)	N/A	0.221	0.250	0.148	0.347	0.249	8.879	0.000	5.801

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	84	109	115	85	87	88
N.S.	1	1.00	1.00	1.65	2.14	2.25	1.67	1.71	1.73
time (sec)	N/A	0.192	0.150	0.134	0.319	0.268	9.371	0.293	6.242

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	98	44	69	66
N.S.	1	1.00	1.00	1.65	1.56	2.28	1.02	1.60	1.53
time (sec)	N/A	0.173	0.021	0.110	0.335	0.265	1.163	0.300	5.742

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	120	104	228	0	542	0	0	1183
N.S.	1	1.11	0.96	2.11	0.00	5.02	0.00	0.00	10.95
time (sec)	N/A	0.263	0.365	0.261	0.000	0.298	0.000	0.000	6.234

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	205	149	469	0	1163	0	493	3813
N.S.	1	1.19	0.87	2.73	0.00	6.76	0.00	2.87	22.17
time (sec)	N/A	0.357	0.776	0.309	0.000	0.356	0.000	0.324	7.824

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	294	215	953	0	2307	0	890	2890
N.S.	1	1.18	0.86	3.81	0.00	9.23	0.00	3.56	11.56
time (sec)	N/A	0.476	1.790	0.328	0.000	0.600	0.000	0.384	9.552

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	141	113	226	200	336	0	0	172
N.S.	1	1.07	0.86	1.71	1.52	2.55	0.00	0.00	1.30
time (sec)	N/A	0.269	0.319	0.185	0.344	0.294	0.000	0.000	6.033

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	100	89	182	164	272	0	187	120
N.S.	1	1.06	0.95	1.94	1.74	2.89	0.00	1.99	1.28
time (sec)	N/A	0.244	0.300	0.171	0.338	0.345	0.000	0.331	5.896

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	159	144	210	224	149	71
N.S.	1	1.00	0.92	2.09	1.89	2.76	2.95	1.96	0.93
time (sec)	N/A	0.210	0.218	0.166	0.354	0.258	14.550	0.324	6.286

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	57	116	85	156	71	114	34
N.S.	1	1.12	0.95	1.93	1.42	2.60	1.18	1.90	0.57
time (sec)	N/A	0.193	0.063	0.133	0.334	0.267	1.725	0.307	5.632

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	182	144	297	0	1075	0	0	3000
N.S.	1	1.24	0.98	2.02	0.00	7.31	0.00	0.00	20.41
time (sec)	N/A	0.344	0.684	0.291	0.000	0.355	0.000	0.000	6.745

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	275	215	533	0	2321	0	0	4274
N.S.	1	1.23	0.96	2.38	0.00	10.36	0.00	0.00	19.08
time (sec)	N/A	0.459	1.157	0.379	0.000	0.618	0.000	0.000	9.869

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	376	298	983	0	4093	0	1089	8936
N.S.	1	1.18	0.93	3.07	0.00	12.79	0.00	3.40	27.92
time (sec)	N/A	0.550	1.879	0.354	0.000	1.453	0.000	0.397	13.527

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	165	134	302	228	483	0	429	194
N.S.	1	1.15	0.94	2.11	1.59	3.38	0.00	3.00	1.36
time (sec)	N/A	0.305	0.350	0.199	0.333	0.271	0.000	0.319	6.097

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	112	287	190	407	0	363	144
N.S.	1	1.01	0.92	2.35	1.56	3.34	0.00	2.98	1.18
time (sec)	N/A	0.252	0.281	0.193	0.343	0.272	0.000	0.313	6.291

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	99	91	254	170	331	1479	259	87
N.S.	1	0.96	0.88	2.47	1.65	3.21	14.36	2.51	0.84
time (sec)	N/A	0.217	0.243	0.182	0.335	0.267	33.930	0.308	7.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	90	72	157	101	225	774	171	34
N.S.	1	1.14	0.91	1.99	1.28	2.85	9.80	2.16	0.43
time (sec)	N/A	0.205	0.043	0.155	0.343	0.276	2.715	0.298	5.872

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	250	200	403	0	1990	0	0	5387
N.S.	1	1.24	1.00	2.00	0.00	9.90	0.00	0.00	26.80
time (sec)	N/A	0.447	0.773	0.334	0.000	1.524	0.000	0.000	8.883

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	349	305	665	0	3887	0	924	5789
N.S.	1	1.22	1.06	2.32	0.00	13.54	0.00	3.22	20.17
time (sec)	N/A	0.548	1.602	0.367	0.000	1.659	0.000	0.396	12.726

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	477	404	1138	0	6171	0	1336	4284
N.S.	1	1.17	0.99	2.78	0.00	15.09	0.00	3.27	10.47
time (sec)	N/A	0.687	2.323	0.464	0.000	4.570	0.000	0.486	11.877

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	129	176	218	0	890	0	0	4674
N.S.	1	1.05	1.43	1.77	0.00	7.24	0.00	0.00	38.00
time (sec)	N/A	0.254	0.516	0.097	0.000	0.601	0.000	0.000	26.273

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	113	155	0	247	0	0	478
N.S.	1	1.00	1.40	1.91	0.00	3.05	0.00	0.00	5.90
time (sec)	N/A	0.207	0.363	0.126	0.000	0.414	0.000	0.000	10.767

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	126	280	0	319	0	0	0
N.S.	1	1.01	1.03	2.30	0.00	2.61	0.00	0.00	0.00
time (sec)	N/A	0.236	0.473	0.156	0.000	0.387	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.169	0.030	0.070	0.352	0.308	0.145	0.279	0.069

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	246	205	277	0	0	0	0	0
N.S.	1	1.06	0.88	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	2.471	2.353	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	246	86	94	0	116	0	0	0
N.S.	1	1.06	0.37	0.41	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.362	2.116	1.137	0.000	0.084	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	281	191	185	0	164	0	0	0
N.S.	1	1.07	0.73	0.71	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.443	3.409	3.468	0.000	0.114	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	390	71	133	123
N.S.	1	0.90	0.89	0.29	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.316	0.085	0.090	0.328	0.262	0.227	0.282	0.273

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	51	48	47	48	82	49	49
N.S.	1	1.08	1.04	0.98	0.96	0.98	1.67	1.00	1.00
time (sec)	N/A	0.209	0.050	3.916	0.235	0.248	0.177	0.298	0.076

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	32	26	21	20	20	24	20	20
N.S.	1	1.23	1.00	0.81	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.178	0.022	3.921	0.261	0.350	0.070	0.276	0.033

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	24	17	24	23	23	27	24	13
N.S.	1	1.41	1.00	1.41	1.35	1.35	1.59	1.41	0.76
time (sec)	N/A	0.164	0.024	3.966	0.258	0.242	0.081	0.276	0.051

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	110	104	66	71	71	102	71	90
N.S.	1	1.06	1.00	0.63	0.68	0.68	0.98	0.68	0.87
time (sec)	N/A	0.295	0.192	56.369	0.330	0.256	1.875	0.302	5.582

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	34	28	23	22	22	26	23	22
N.S.	1	1.13	0.93	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.185	0.006	3.932	0.258	0.303	0.066	0.275	0.039

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	128	186	527	2744	740	131
N.S.	1	1.00	0.83	0.97	1.41	3.99	20.79	5.61	0.99
time (sec)	N/A	0.321	0.682	3.993	0.283	0.286	0.733	0.288	5.691

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	90	96	140	319	1540	450	99
N.S.	1	1.00	0.91	0.97	1.41	3.22	15.56	4.55	1.00
time (sec)	N/A	0.269	0.226	4.093	0.288	0.253	0.490	0.291	5.651

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.226	0.151	3.931	0.282	0.253	0.359	0.281	5.610

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	39	48	69	236	83	38
N.S.	1	1.00	0.92	0.98	1.20	1.72	5.90	2.08	0.95
time (sec)	N/A	0.183	0.115	0.051	0.286	0.250	0.257	0.285	5.580

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	110	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	2.56	0.00	0.00
time (sec)	N/A	0.173	0.135	0.000	0.000	0.000	1.046	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	741	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	10.15	0.00	0.00
time (sec)	N/A	0.196	0.083	0.000	0.000	0.000	3.385	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	149	154	242	667	3376	962	157
N.S.	1	1.00	0.94	0.97	1.53	4.22	21.37	6.09	0.99
time (sec)	N/A	0.343	0.773	4.243	0.285	0.262	7.409	0.298	5.914

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	105	109	168	370	1760	539	108
N.S.	1	1.00	0.94	0.97	1.50	3.30	15.71	4.81	0.96
time (sec)	N/A	0.272	0.763	4.000	0.266	0.251	2.956	0.298	5.792

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.230	0.157	4.096	0.303	0.262	0.355	0.292	5.694

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	93	82	0	0	0	235	0	0
N.S.	1	1.11	0.98	0.00	0.00	0.00	2.80	0.00	0.00
time (sec)	N/A	0.282	0.216	0.000	0.000	0.000	1.762	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	117	95	0	0	0	0	0	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	169	133	0	0	0	0	0	0
N.S.	1	1.06	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	321	133	0	0	0	488	0	0
N.S.	1	1.04	0.43	0.00	0.00	0.00	1.57	0.00	0.00
time (sec)	N/A	0.767	3.528	0.000	0.000	0.000	3.214	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	185	104	0	0	0	360	0	0
N.S.	1	1.07	0.60	0.00	0.00	0.00	2.08	0.00	0.00
time (sec)	N/A	0.488	1.488	0.000	0.000	0.000	2.385	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	93	75	0	0	0	235	0	0
N.S.	1	1.11	0.89	0.00	0.00	0.00	2.80	0.00	0.00
time (sec)	N/A	0.279	0.486	0.000	0.000	0.000	1.810	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	110	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.62	0.00	0.00
time (sec)	N/A	0.179	0.128	0.000	0.000	0.000	1.047	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	141	121	0	0	0	0	0	0
N.S.	1	1.15	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	248	210	0	0	0	0	0	0
N.S.	1	1.18	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	341	344	217	0	0	0	0	0	0
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	6.394	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	200	204	2050	0	0	0	0	0	0
N.S.	1	1.02	10.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	4.826	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	117	666	0	0	0	0	0	0
N.S.	1	1.02	5.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	1.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	741	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	10.29	0.00	0.00
time (sec)	N/A	0.199	0.088	0.000	0.000	0.000	3.216	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	142	108	0	0	0	0	0	0
N.S.	1	1.16	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	224	147	0	0	0	0	0	0
N.S.	1	1.16	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.286	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	343	233	0	0	0	0	0	0
N.S.	1	1.15	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.691	0.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.447	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	385	168	0	0	0	243	0	0
N.S.	1	0.96	0.42	0.00	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.821	5.309	0.000	0.000	0.000	42.878	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	188	140	0	0	0	175	0	0
N.S.	1	0.93	0.69	0.00	0.00	0.00	0.87	0.00	0.00
time (sec)	N/A	0.451	5.198	0.000	0.000	0.000	17.299	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	107	0	0
N.S.	1	0.91	0.96	0.00	0.00	0.00	1.09	0.00	0.00
time (sec)	N/A	0.233	0.081	0.000	0.000	0.000	2.688	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	46	0	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	1.02
time (sec)	N/A	0.171	0.004	0.000	0.000	0.000	0.944	0.000	6.462

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	0.467	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	160	218	1288	0	478	2822	0	0
N.S.	1	0.90	1.22	7.24	0.00	2.69	15.85	0.00	0.00
time (sec)	N/A	0.295	0.212	5.015	0.000	0.274	29.868	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	109	113	588	0	231	1035	0	0
N.S.	1	0.94	0.97	5.07	0.00	1.99	8.92	0.00	0.00
time (sec)	N/A	0.234	0.140	4.506	0.000	0.248	12.920	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	82	199	0	85	311	0	0
N.S.	1	1.00	1.41	3.43	0.00	1.47	5.36	0.00	0.00
time (sec)	N/A	0.181	0.167	4.212	0.000	0.252	2.214	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	47	0	31	42	0	75
N.S.	1	1.00	1.00	2.61	0.00	1.72	2.33	0.00	4.17
time (sec)	N/A	0.142	0.055	4.077	0.000	0.248	0.678	0.000	5.883

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	193	181	0	0	0	0	0	0	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	334	0	108	0	0	0
N.S.	1	1.00	0.96	5.86	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.184	0.571	8.574	0.000	0.264	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	231	136	1059	0	400	2746	0	0
N.S.	1	0.71	0.42	3.24	0.00	1.22	8.40	0.00	0.00
time (sec)	N/A	0.382	0.442	4.525	0.000	0.267	13.315	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	113	94	434	0	173	959	0	0
N.S.	1	0.89	0.74	3.42	0.00	1.36	7.55	0.00	0.00
time (sec)	N/A	0.246	0.207	4.398	0.000	0.264	2.479	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	250	0	64
N.S.	1	1.00	1.10	0.00	0.00	1.36	5.00	0.00	1.28
time (sec)	N/A	0.173	0.050	0.000	0.000	0.254	0.768	0.000	5.934

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	6.530	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	212	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	8.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	333	0	0	0	0	0	0
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	8.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	6405	0	0	0	0	0	0
N.S.	1	1.00	48.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	36.956	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	151	97	92	178	114	0	621	152
N.S.	1	0.99	0.64	0.61	1.17	0.75	0.00	4.09	1.00
time (sec)	N/A	0.274	0.300	4.043	0.315	0.301	0.000	0.449	5.921

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	110	75	68	124	90	0	495	118
N.S.	1	1.01	0.69	0.62	1.14	0.83	0.00	4.54	1.08
time (sec)	N/A	0.241	0.215	4.186	0.306	0.267	0.000	0.377	5.791

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	49	44	70	66	0	361	83
N.S.	1	1.07	0.73	0.66	1.04	0.99	0.00	5.39	1.24
time (sec)	N/A	0.196	0.163	4.146	0.290	0.355	0.000	0.353	5.754

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	75	174	52	80	0	78	248
N.S.	1	1.01	0.94	2.18	0.65	1.00	0.00	0.98	3.10
time (sec)	N/A	0.234	0.223	4.021	0.340	0.260	0.000	0.328	7.769

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	99	74	138	98	85	0	157	584
N.S.	1	1.03	0.77	1.44	1.02	0.89	0.00	1.64	6.08
time (sec)	N/A	0.252	0.287	4.064	0.389	0.258	0.000	0.349	11.295

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	142	99	150	162	100	0	324	1004
N.S.	1	1.17	0.82	1.24	1.34	0.83	0.00	2.68	8.30
time (sec)	N/A	0.266	0.281	4.201	0.372	0.270	0.000	0.362	22.149

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	185	142	184	246	138	0	558	2314
N.S.	1	0.89	0.68	0.88	1.18	0.66	0.00	2.68	11.12
time (sec)	N/A	0.297	0.456	4.186	0.257	0.292	0.000	0.405	49.014

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	143	116	159	192	112	0	432	1681
N.S.	1	0.90	0.73	1.00	1.21	0.70	0.00	2.72	10.57
time (sec)	N/A	0.269	0.363	4.196	0.278	0.268	0.000	0.378	55.455

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	105	92	135	137	88	0	288	734
N.S.	1	0.92	0.81	1.18	1.20	0.77	0.00	2.53	6.44
time (sec)	N/A	0.205	0.263	4.202	0.294	0.254	0.000	0.335	23.405

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	97	74	117	105	83	0	110	243
N.S.	1	0.93	0.71	1.12	1.01	0.80	0.00	1.06	2.34
time (sec)	N/A	0.239	0.296	4.199	0.344	0.259	0.000	0.346	7.592

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	81	124	75	100	0	171	236
N.S.	1	1.01	0.96	1.48	0.89	1.19	0.00	2.04	2.81
time (sec)	N/A	0.231	0.269	4.204	0.340	0.273	0.000	0.336	7.534

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	118	96	129	153	96	0	172	1154
N.S.	1	0.94	0.77	1.03	1.22	0.77	0.00	1.38	9.23
time (sec)	N/A	0.248	0.314	4.223	0.270	0.292	0.000	0.299	45.444

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	98	61	57	95	55	216	108	108
N.S.	1	0.95	0.59	0.55	0.92	0.53	2.10	1.05	1.05
time (sec)	N/A	0.230	0.151	4.317	0.272	0.261	5.567	0.288	6.643

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	82	77	111	113	77	0	121	720
N.S.	1	0.94	0.89	1.28	1.30	0.89	0.00	1.39	8.28
time (sec)	N/A	0.214	0.253	4.283	0.280	0.259	0.000	0.284	29.408

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	38	54	37	202	59	66
N.S.	1	1.00	0.66	0.58	0.83	0.57	3.11	0.91	1.02
time (sec)	N/A	0.193	0.119	4.070	0.283	0.249	4.077	0.276	6.574

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	58	91	74	55	0	69	293
N.S.	1	1.00	1.23	1.94	1.57	1.17	0.00	1.47	6.23
time (sec)	N/A	0.168	0.166	4.029	0.282	0.304	0.000	0.282	17.424

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	62	29	48	162	45	77
N.S.	1	1.00	0.98	1.35	0.63	1.04	3.52	0.98	1.67
time (sec)	N/A	0.204	0.124	4.012	0.359	0.261	15.947	0.280	7.661

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	48	77	44	56	148	58	61
N.S.	1	1.00	1.45	2.33	1.33	1.70	4.48	1.76	1.85
time (sec)	N/A	0.183	0.137	4.030	0.352	0.272	15.319	0.276	6.848

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	71	45	57	0	114	297
N.S.	1	1.00	0.92	1.18	0.75	0.95	0.00	1.90	4.95
time (sec)	N/A	0.202	0.157	4.192	0.404	0.248	0.000	0.274	13.843

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	42	37	54	52	146	116	53
N.S.	1	1.00	0.68	0.60	0.87	0.84	2.35	1.87	0.85
time (sec)	N/A	0.196	0.141	4.018	0.363	0.251	10.918	0.285	6.725

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	99	94	78	94	85	78	0	268	650
N.S.	1	0.95	0.79	0.95	0.86	0.79	0.00	2.71	6.57
time (sec)	N/A	0.232	0.290	4.050	0.366	0.269	0.000	0.288	31.048

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	148	119	160	196	115	0	203	1682
N.S.	1	0.90	0.73	0.98	1.20	0.70	0.00	1.24	10.26
time (sec)	N/A	0.278	0.377	4.076	0.274	0.271	0.000	0.308	58.965

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	110	74	68	124	66	240	124	130
N.S.	1	0.93	0.63	0.58	1.05	0.56	2.03	1.05	1.10
time (sec)	N/A	0.239	0.191	4.191	0.296	0.267	6.166	0.283	7.120

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	107	92	136	142	90	0	140	1048
N.S.	1	0.91	0.78	1.15	1.20	0.76	0.00	1.19	8.88
time (sec)	N/A	0.249	0.286	4.210	0.242	0.273	0.000	0.291	33.909

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	43	69	42	223	65	76
N.S.	1	1.00	0.67	0.60	0.96	0.58	3.10	0.90	1.06
time (sec)	N/A	0.201	0.142	4.194	0.249	0.263	4.396	0.281	7.142

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	111	89	63	0	79	417
N.S.	1	1.00	1.00	1.63	1.31	0.93	0.00	1.16	6.13
time (sec)	N/A	0.189	0.178	4.167	0.251	0.251	0.000	0.282	15.566

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	108	37	61	178	55	108
N.S.	1	1.00	0.96	1.93	0.66	1.09	3.18	0.98	1.93
time (sec)	N/A	0.215	0.147	4.272	0.345	0.261	16.137	0.289	8.310

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	97	55	68	165	66	77
N.S.	1	1.00	1.00	1.70	0.96	1.19	2.89	1.16	1.35
time (sec)	N/A	0.209	0.166	4.217	0.345	0.265	15.379	0.280	7.168

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	123	60	73	0	141	457
N.S.	1	1.00	0.92	1.62	0.79	0.96	0.00	1.86	6.01
time (sec)	N/A	0.223	0.192	4.226	0.352	0.267	0.000	0.307	12.131

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	49	75	67	170	137	79
N.S.	1	1.00	0.72	0.65	1.00	0.89	2.27	1.83	1.05
time (sec)	N/A	0.221	0.168	4.205	0.355	0.253	15.094	0.290	7.002

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	115	101	151	114	100	0	325	1005
N.S.	1	0.93	0.82	1.23	0.93	0.81	0.00	2.64	8.17
time (sec)	N/A	0.253	0.287	4.068	0.348	0.257	0.000	0.302	27.648

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	141	137	275	196	190	0	214	0
N.S.	1	0.88	0.85	1.71	1.22	1.18	0.00	1.33	0.00
time (sec)	N/A	0.279	0.389	4.240	0.270	0.258	0.000	0.325	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	103	72	68	123	80	0	200	90
N.S.	1	0.90	0.63	0.59	1.07	0.70	0.00	1.74	0.78
time (sec)	N/A	0.246	0.200	4.074	0.251	0.260	0.000	0.305	7.252

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	133	108	251	138	159	0	147	0
N.S.	1	0.88	0.71	1.65	0.91	1.05	0.00	0.97	0.00
time (sec)	N/A	0.271	0.288	4.256	0.271	0.266	0.000	0.322	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	73	45	43	69	56	0	152	67
N.S.	1	0.96	0.59	0.57	0.91	0.74	0.00	2.00	0.88
time (sec)	N/A	0.213	0.166	4.245	0.282	0.257	0.000	0.315	6.930

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	71	166	76	129	0	113	0
N.S.	1	1.00	1.13	2.63	1.21	2.05	0.00	1.79	0.00
time (sec)	N/A	0.189	0.286	4.067	0.273	0.258	0.000	0.298	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	70	194	58	101	0	115	0
N.S.	1	1.00	1.08	2.98	0.89	1.55	0.00	1.77	0.00
time (sec)	N/A	0.220	0.238	4.206	0.390	0.247	0.000	0.336	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	66	51	48	71	103	0	219	73
N.S.	1	0.99	0.76	0.72	1.06	1.54	0.00	3.27	1.09
time (sec)	N/A	0.216	0.175	4.244	0.363	0.263	0.000	0.344	7.109

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	105	93	239	104	138	0	211	0
N.S.	1	0.90	0.79	2.04	0.89	1.18	0.00	1.80	0.00
time (sec)	N/A	0.257	0.329	4.253	0.298	0.258	0.000	0.355	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	106	77	73	125	132	0	242	104
N.S.	1	0.89	0.65	0.61	1.05	1.11	0.00	2.03	0.87
time (sec)	N/A	0.248	0.222	4.596	0.283	0.265	0.000	0.396	7.366

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	146	122	267	162	165	0	402	0
N.S.	1	0.88	0.73	1.61	0.98	0.99	0.00	2.42	0.00
time (sec)	N/A	0.294	0.443	4.263	0.275	0.367	0.000	0.410	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	53	23	39	148	40	72
N.S.	1	1.00	1.00	1.32	0.58	0.98	3.70	1.00	1.80
time (sec)	N/A	0.197	0.108	4.025	0.303	0.241	15.828	0.271	8.068

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	244	66	79	65	1867	0	96
N.S.	1	1.00	4.60	1.25	1.49	1.23	35.23	0.00	1.81
time (sec)	N/A	0.225	10.294	4.058	0.297	0.278	36.677	0.000	7.451

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	44	0	0	69	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	0.198	9.814	0.000	0.000	0.247	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	50	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.268	10.038	0.000	0.000	0.262	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.478	0.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	103	103	0	0	180	0	0	0
N.S.	1	1.07	1.07	0.00	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.413	1.417	0.000	0.000	0.299	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [39] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	17	0.118
2	A	2	2	1.00	17	0.118
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	15	0.133
5	A	10	9	0.91	17	0.529
6	A	10	9	0.93	17	0.529
7	A	11	10	0.93	17	0.588
8	A	2	2	1.00	19	0.105
9	A	2	2	1.00	19	0.105
10	A	2	2	1.00	17	0.118
11	A	2	2	1.00	19	0.105
12	A	2	2	1.00	19	0.105
13	A	11	10	0.86	19	0.526
14	A	2	2	1.00	19	0.105
15	A	2	2	1.00	19	0.105
16	A	2	2	1.00	19	0.105
17	A	10	9	0.90	17	0.529
18	A	10	9	0.86	19	0.474
19	A	11	10	0.89	19	0.526
20	A	2	2	1.00	19	0.105
21	A	2	2	1.00	19	0.105
22	A	2	2	1.00	19	0.105

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	19	0.105
24	A	10	9	0.93	17	0.529
25	A	12	11	0.88	19	0.579
26	A	14	13	0.89	19	0.684
27	A	3	3	1.02	20	0.150
28	A	2	2	1.01	20	0.100
29	A	2	2	1.00	20	0.100
30	A	2	2	1.00	20	0.100
31	A	3	3	1.09	20	0.150
32	A	4	4	1.18	20	0.200
33	A	5	5	1.23	20	0.250
34	A	22	21	1.16	22	0.955
35	A	20	19	1.16	22	0.864
36	A	11	10	1.03	22	0.455
37	A	14	13	1.03	22	0.591
38	A	20	19	1.14	22	0.864
39	A	23	22	1.16	22	1.000
40	A	5	5	1.04	22	0.227
41	A	4	4	1.02	22	0.182
42	A	5	5	1.01	22	0.227
43	A	4	4	1.05	22	0.182
44	A	3	3	1.07	22	0.136
45	A	4	4	1.12	22	0.182
46	A	6	6	1.18	22	0.273
47	A	7	7	1.22	22	0.318
48	A	5	5	1.05	22	0.227
49	A	5	5	0.99	22	0.227
50	A	5	5	0.99	22	0.227
51	A	4	4	1.00	22	0.182
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	22	0.182
54	A	5	5	1.05	22	0.227
55	A	5	5	1.05	22	0.227
56	A	4	4	0.87	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	0.91	19	0.158
58	A	2	2	0.96	19	0.105
59	A	2	2	1.04	19	0.105
60	A	2	2	1.00	19	0.105
61	A	3	3	0.97	19	0.158
62	A	4	4	0.95	19	0.211
63	A	5	5	0.94	19	0.263
64	A	3	3	1.00	19	0.158
65	A	3	3	1.00	19	0.158
66	A	3	3	1.00	19	0.158
67	A	3	3	1.00	19	0.158
68	A	3	3	1.00	19	0.158
69	A	3	3	1.00	19	0.158
70	A	5	5	0.79	21	0.238
71	A	4	4	0.85	21	0.190
72	A	3	3	0.94	21	0.143
73	A	4	4	1.00	21	0.190
74	A	3	3	1.08	21	0.143
75	A	3	3	1.08	21	0.143
76	A	4	4	0.86	21	0.190
77	A	5	5	0.88	21	0.238
78	A	6	6	0.82	21	0.286
79	A	4	4	1.06	21	0.190
80	A	4	4	1.06	21	0.190
81	A	4	4	1.06	21	0.190
82	A	4	4	1.02	21	0.190
83	A	4	4	1.00	21	0.190
84	A	4	4	1.01	21	0.190
85	A	4	4	1.11	21	0.190
86	A	7	7	0.99	21	0.333
87	A	5	5	1.01	21	0.238
88	A	3	3	1.02	21	0.143
89	A	1	1	1.00	21	0.048
90	A	2	2	1.06	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	5	1.10	21	0.238
92	A	7	7	1.15	21	0.333
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	A	6	6	0.97	21	0.286
99	A	4	4	1.00	21	0.190
100	A	2	2	1.02	21	0.095
101	A	2	2	0.99	21	0.095
102	A	4	4	1.01	21	0.190
103	A	6	6	1.04	21	0.286
104	A	2	2	1.00	21	0.095
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	21	0.095
109	A	10	10	0.93	21	0.476
110	A	8	8	0.94	21	0.381
111	A	6	6	0.96	21	0.286
112	A	3	3	1.02	21	0.143
113	A	3	3	0.94	21	0.143
114	A	4	4	0.96	21	0.190
115	A	6	6	0.98	21	0.286
116	A	9	9	1.01	21	0.429
117	A	2	2	1.00	21	0.095
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	21	0.095
120	A	2	2	1.00	21	0.095
121	A	2	2	1.00	21	0.095
122	A	3	3	1.04	23	0.130
123	A	3	3	1.04	23	0.130
124	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.00	23	0.087
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	2	2	1.00	23	0.087
130	A	2	2	1.00	23	0.087
131	A	3	3	1.05	23	0.130
132	A	3	3	1.05	23	0.130
133	A	3	3	1.00	19	0.158
134	A	5	5	1.05	19	0.263
135	A	3	3	1.01	17	0.176
136	A	2	2	1.00	19	0.105
137	A	2	2	1.00	19	0.105
138	A	7	7	0.97	19	0.368
139	A	5	5	0.99	19	0.263
140	A	3	3	0.91	17	0.176
141	A	2	2	1.00	9	0.222
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	2	2	1.00	19	0.105
145	A	1	1	1.00	50	0.020
146	A	2	2	1.00	17	0.118
147	A	2	2	1.00	17	0.118
148	A	2	2	1.00	17	0.118
149	A	2	2	1.00	15	0.133
150	A	10	9	0.99	17	0.529
151	A	10	9	1.01	17	0.529
152	A	11	10	1.00	17	0.588
153	A	2	2	1.00	19	0.105
154	A	2	2	1.00	19	0.105
155	A	2	2	1.00	19	0.105
156	A	2	2	1.00	17	0.118
157	A	2	2	1.00	19	0.105
158	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	11	10	0.89	19	0.526
160	A	2	2	1.00	19	0.105
161	A	2	2	1.00	19	0.105
162	A	2	2	1.00	19	0.105
163	A	10	9	0.98	17	0.529
164	A	10	9	0.95	19	0.474
165	A	11	10	0.95	19	0.526
166	A	2	2	1.00	19	0.105
167	A	2	2	1.00	19	0.105
168	A	2	2	1.00	19	0.105
169	A	2	2	1.00	19	0.105
170	A	10	9	1.01	17	0.529
171	A	12	11	0.94	19	0.579
172	A	14	13	0.92	19	0.684
173	A	9	9	1.01	23	0.391
174	A	8	8	0.98	23	0.348
175	A	7	7	0.97	23	0.304
176	A	4	4	1.00	23	0.174
177	A	8	8	0.96	23	0.348
178	A	9	9	1.02	23	0.391
179	A	10	10	1.08	21	0.476
180	A	8	8	1.10	21	0.381
181	A	6	6	1.16	21	0.286
182	A	10	10	1.10	21	0.476
183	A	12	12	1.10	21	0.571
184	A	11	11	0.99	23	0.478
185	A	10	10	0.98	23	0.435
186	A	9	9	0.97	23	0.391
187	A	9	9	0.98	23	0.391
188	A	9	9	0.92	23	0.391
189	A	11	11	0.96	23	0.478
190	A	13	13	1.00	23	0.565
191	A	5	4	0.98	25	0.160
192	A	1	1	1.00	25	0.040

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	12	11	1.04	21	0.524
194	A	10	9	1.05	21	0.429
195	A	5	4	1.00	21	0.190
196	A	6	5	1.09	21	0.238
197	A	9	8	1.14	21	0.381
198	A	11	10	1.20	21	0.476
199	A	14	13	0.91	21	0.619
200	A	11	10	0.87	21	0.476
201	A	5	4	0.85	21	0.190
202	A	10	9	0.87	21	0.429
203	A	12	11	0.94	21	0.524
204	A	14	13	1.00	21	0.619
205	A	13	12	1.02	21	0.571
206	A	11	10	1.06	21	0.476
207	A	6	5	1.05	21	0.238
208	A	6	5	1.06	21	0.238
209	A	8	7	1.07	21	0.333
210	A	10	9	1.11	21	0.429
211	A	13	12	0.88	21	0.571
212	A	11	10	0.88	21	0.476
213	A	12	11	0.87	21	0.524
214	A	11	10	0.91	21	0.476
215	A	13	12	0.96	21	0.571
216	A	5	4	1.00	17	0.235
217	A	5	4	1.00	26	0.154
218	A	3	3	1.00	19	0.158
219	A	5	5	0.99	19	0.263
220	A	3	3	0.91	17	0.176
221	A	2	2	1.00	19	0.105
222	A	2	2	1.00	19	0.105
223	A	10	9	0.92	21	0.429
224	A	9	8	1.08	21	0.381
225	A	8	7	1.00	21	0.333
226	A	6	5	0.97	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	5	4	1.00	11	0.364
228	A	8	7	1.09	21	0.333
229	A	11	10	1.10	21	0.476
230	A	13	12	1.17	21	0.571
231	A	10	9	1.01	21	0.429
232	A	9	8	0.94	21	0.381
233	A	7	6	0.91	19	0.316
234	A	6	5	1.07	11	0.455
235	A	8	7	1.08	21	0.333
236	A	10	9	1.21	21	0.429
237	A	12	11	1.08	21	0.524
238	A	11	10	0.96	21	0.476
239	A	10	9	0.89	21	0.429
240	A	8	7	0.87	19	0.368
241	A	7	6	1.07	11	0.545
242	A	10	9	1.09	21	0.429
243	A	10	9	1.12	21	0.429
244	A	12	11	1.16	21	0.524
245	A	7	6	1.04	21	0.286
246	A	7	6	1.12	21	0.286
247	A	5	4	1.00	19	0.211
248	A	5	4	1.00	11	0.364
249	A	8	7	1.11	21	0.333
250	A	10	9	1.19	21	0.429
251	A	12	11	1.18	21	0.524
252	A	7	6	1.07	21	0.286
253	A	7	6	1.06	21	0.286
254	A	6	5	1.00	19	0.263
255	A	6	5	1.12	11	0.455
256	A	10	9	1.24	21	0.429
257	A	12	11	1.23	21	0.524
258	A	14	13	1.18	21	0.619
259	A	7	6	1.15	21	0.286
260	A	8	7	1.01	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	7	6	0.96	19	0.316
262	A	7	6	1.14	11	0.545
263	A	12	11	1.24	21	0.524
264	A	14	13	1.22	21	0.619
265	A	16	15	1.17	21	0.714
266	A	8	7	1.05	23	0.304
267	A	5	4	1.00	23	0.174
268	A	6	5	1.01	23	0.217
269	A	4	3	1.00	19	0.158
270	A	3	3	1.00	17	0.176
271	A	8	7	1.06	23	0.304
272	A	8	7	1.06	23	0.304
273	A	11	10	1.07	23	0.435
274	A	5	4	1.00	19	0.211
275	A	11	10	0.90	17	0.588
276	A	4	3	1.08	21	0.143
277	A	5	4	1.23	17	0.235
278	A	6	5	1.41	17	0.294
279	A	12	11	1.06	17	0.647
280	A	5	4	1.13	17	0.235
281	A	3	3	1.00	24	0.125
282	A	3	3	1.00	24	0.125
283	A	3	3	1.00	20	0.150
284	A	2	2	1.00	17	0.118
285	A	2	2	1.00	17	0.118
286	A	2	2	1.00	17	0.118
287	A	2	2	1.00	15	0.133
288	A	2	2	1.00	17	0.118
289	A	2	2	1.00	17	0.118
290	A	2	2	1.00	17	0.118
291	A	2	2	1.00	17	0.118
292	A	2	2	1.00	19	0.105
293	A	2	2	1.00	19	0.105
294	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	4	4	1.11	19	0.211
296	A	3	3	1.02	19	0.158
297	A	3	3	1.06	19	0.158
298	A	8	8	1.04	19	0.421
299	A	6	6	1.07	19	0.316
300	A	4	4	1.11	19	0.211
301	A	2	2	1.00	17	0.118
302	A	2	2	1.00	19	0.105
303	A	3	3	1.15	19	0.158
304	A	4	4	1.18	19	0.211
305	A	5	5	1.01	19	0.263
306	A	4	4	1.02	19	0.211
307	A	3	3	1.02	19	0.158
308	A	2	2	1.00	17	0.118
309	A	3	3	1.16	19	0.158
310	A	4	4	1.16	19	0.211
311	A	5	5	1.15	19	0.263
312	A	3	3	1.00	19	0.158
313	A	7	7	0.96	19	0.368
314	A	5	5	0.93	19	0.263
315	A	3	3	0.91	17	0.176
316	A	2	2	1.00	9	0.222
317	A	2	2	1.00	19	0.105
318	A	2	2	1.00	19	0.105
319	A	2	2	1.00	19	0.105
320	A	1	1	1.00	28	0.036
321	A	4	4	0.90	25	0.160
322	A	3	3	0.94	25	0.120
323	A	2	2	1.00	23	0.087
324	A	1	1	1.00	15	0.067
325	A	1	1	1.00	23	0.043
326	A	1	1	1.00	25	0.040
327	A	1	1	1.00	25	0.040
328	A	2	2	0.94	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	1	1	1.00	69	0.014
330	A	5	5	0.71	25	0.200
331	A	3	3	0.89	23	0.130
332	A	2	2	1.00	15	0.133
333	A	2	2	1.00	25	0.080
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	6	6	0.99	31	0.194
338	A	4	4	1.01	31	0.129
339	A	2	2	1.07	29	0.069
340	A	6	5	1.01	31	0.161
341	A	6	5	1.03	31	0.161
342	A	6	5	1.17	31	0.161
343	A	9	8	0.89	31	0.258
344	A	7	6	0.90	31	0.194
345	A	5	4	0.92	28	0.143
346	A	5	4	0.93	31	0.129
347	A	6	5	1.01	31	0.161
348	A	5	5	0.94	29	0.172
349	A	4	4	0.95	29	0.138
350	A	3	3	0.94	29	0.103
351	A	2	2	1.00	27	0.074
352	A	2	2	1.00	26	0.077
353	A	4	3	1.00	29	0.103
354	A	2	2	1.00	29	0.069
355	A	4	3	1.00	29	0.103
356	A	2	2	1.00	29	0.069
357	A	6	5	0.95	29	0.172
358	A	8	7	0.90	31	0.226
359	A	4	4	0.93	31	0.129
360	A	6	5	0.91	31	0.161
361	A	2	2	1.00	29	0.069
362	A	4	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	4	3	1.00	31	0.097
364	A	4	3	1.00	31	0.097
365	A	4	3	1.00	31	0.097
366	A	2	2	1.00	31	0.065
367	A	6	5	0.93	31	0.161
368	A	8	7	0.88	31	0.226
369	A	4	4	0.90	31	0.129
370	A	7	6	0.88	31	0.194
371	A	2	2	0.96	29	0.069
372	A	4	3	1.00	28	0.107
373	A	4	3	1.00	31	0.097
374	A	2	2	0.99	31	0.065
375	A	6	5	0.90	31	0.161
376	A	4	4	0.89	31	0.129
377	A	8	7	0.88	31	0.226
378	A	4	3	1.00	31	0.097
379	A	1	1	1.00	57	0.018
380	A	3	3	1.00	32	0.094
381	A	4	3	1.00	41	0.073
382	A	4	4	1.00	31	0.129
383	A	4	4	1.00	35	0.114
384	A	3	3	1.00	31	0.097
385	A	2	2	1.07	76	0.026

CHAPTER 3

Listing of integrals

3.1 $\int (a+bx^3)(c+dx^3)^4\,dx$

3.1.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7 + \frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

output
```
a*c^4*x+1/4*c^3*(4*a*d+b*c)*x^4+2/7*c^2*d*(3*a*d+2*b*c)*x^7+1/5*c*d^2*(2*a
*d+3*b*c)*x^10+1/13*d^3*(a*d+4*b*c)*x^13+1/16*b*d^4*x^16
```

3.1.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7 + \frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c
*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/
16
```

3.1.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4 \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(c^3x^3(4ad+bc)+2c^2dx^6(3ad+2bc)+d^3x^{12}(ad+4bc)+2cd^2x^9(2ad+3bc)+ac^4+bd^4x^{15}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c
*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/
16
```

3.1.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
norman	$a\,c^4 x + \left(a\,c^3 d + \frac{1}{4} b\,c^4\right) x^4 + \left(\frac{6}{7} a\,c^2 d^2 + \frac{4}{7} b\,c^3 d\right) x^7 + \left(\frac{2}{5} ac\,d^3 + \frac{3}{5} b\,c^2 d^2\right) x^{10} + \left(\frac{1}{13} a\,d^4 + \frac{4}{13} bc\right.$
default	$\frac{b\,d^4 x^{16}}{16} + \frac{(a\,d^4+4bc\,d^3)x^{13}}{13} + \frac{(4ac\,d^3+6b\,c^2d^2)x^{10}}{10} + \frac{(6a\,c^2d^2+4b\,c^3d)x^7}{7} + \frac{(4a\,c^3d+b\,c^4)x^4}{4} + a\,c^4 x$
gosper	$a\,c^4 x + x^4 a\,c^3 d + \frac{1}{4} x^4 b\,c^4 + \frac{6}{7} x^7 a\,c^2 d^2 + \frac{4}{7} x^7 b\,c^3 d + \frac{2}{5} x^{10} ac\,d^3 + \frac{3}{5} x^{10} b\,c^2 d^2 + \frac{1}{13} x^{13} a\,d^4 + \frac{4}{13}$
risch	$a\,c^4 x + x^4 a\,c^3 d + \frac{1}{4} x^4 b\,c^4 + \frac{6}{7} x^7 a\,c^2 d^2 + \frac{4}{7} x^7 b\,c^3 d + \frac{2}{5} x^{10} ac\,d^3 + \frac{3}{5} x^{10} b\,c^2 d^2 + \frac{1}{13} x^{13} a\,d^4 + \frac{4}{13}$
parallelrisch	$a\,c^4 x + x^4 a\,c^3 d + \frac{1}{4} x^4 b\,c^4 + \frac{6}{7} x^7 a\,c^2 d^2 + \frac{4}{7} x^7 b\,c^3 d + \frac{2}{5} x^{10} ac\,d^3 + \frac{3}{5} x^{10} b\,c^2 d^2 + \frac{1}{13} x^{13} a\,d^4 + \frac{4}{13}$

input
```
int((b*x^3+a)*(d*x^3+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+(a*c^3*d+1/4*b*c^4)*x^4+(6/7*a*c^2*d^2+4/7*b*c^3*d)*x^7+(2/5*a*c*d
^3+3/5*b*c^2*d^2)*x^10+(1/13*a*d^4+4/13*b*c*d^3)*x^13+1/16*b*d^4*x^16
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx = \frac{1}{16}bd^4x^{16} + \frac{1}{13}\left(4\,bcd^3 + ad^4\right)x^{13} + \frac{1}{5}\left(3\,bc^2d^2 + 2\,acd^3\right)x^{10} + \frac{2}{7}\left(2\,bc^3d + 3\,ac^2d^2\right)x^7 + ac^4x + \frac{1}{4}\left(bc^4 + 4\,ac^3d\right)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

output
```
1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*
a*c^3*d)*x^4
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x + \frac{bd^4x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4bcd^3}{13}\right) + x^{10}\cdot\left(\frac{2acd^3}{5} + \frac{3bc^2d^2}{5}\right) + x^7\cdot\left(\frac{6ac^2d^2}{7} + \frac{4bc^3d}{7}\right) + x^4\left(ac^3d + \frac{bc^4}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

output
```
a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*
a*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x*
*4*(a*c**3*d + b*c**4/4)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^3)(c+dx^3)^4\,dx = \frac{1}{16}\,bd^4x^{16} + \frac{1}{13}\,(4\,bcd^3 + ad^4)x^{13} + \frac{1}{5}\,(3\,bc^2d^2 + 2\,acd^3)x^{10} + \frac{2}{7}\,(2\,bc^3d + 3\,ac^2d^2)x^7 + ac^4x + \frac{1}{4}\,(bc^4 + 4\,ac^3d)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

output
```
1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*
a*c^3*d)*x^4
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int (a+bx^3)(c+dx^3)^4\,dx = \frac{1}{16}\,bd^4x^{16} + \frac{4}{13}\,bcd^3x^{13} + \frac{1}{13}\,ad^4x^{13} + \frac{3}{5}\,bc^2d^2x^{10} + \frac{2}{5}\,acd^3x^{10} + \frac{4}{7}\,bc^3dx^7 + \frac{6}{7}\,ac^2d^2x^7 + \frac{1}{4}\,bc^4x^4 + ac^3dx^4 + ac^4x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

output
```
1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
 + 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4
+ a*c^3*d*x^4 + a*c^4*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int (a+bx^3)\left(c+dx^3\right)^4\,dx = x^4\left(\frac{b\,c^4}{4}+a\,d\,c^3\right)+x^{13}\left(\frac{a\,d^4}{13}+\frac{4\,b\,c\,d^3}{13}\right)+\frac{b\,d^4\,x^{16}}{16}+a\,c^4\,x+\frac{2\,c^2\,d\,x^7\,(3\,a\,d+2\,b\,c)}{7}+\frac{c\,d^2\,x^{10}\,(2\,a\,d+3\,b\,c)}{5}$$

input
```
int((a + b*x^3)*(c + d*x^3)^4,x)
```

output
```
x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^
16)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d +
3*b*c))/5
```

3.2 $\int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx$

3.2.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx = a c^3 x+\frac{1}{4} c^2 (b c+3 a d) x^4+\frac{3}{7} c d (b c+a d) x^7 +\frac{1}{10} d^2 (3 b c+a d) x^{10}+\frac{1}{13} b d^3 x^{13}$$

output
```
a*c^3*x+1/4*c^2*(3*a*d+b*c)*x^4+3/7*c*d*(a*d+b*c)*x^7+1/10*d^2*(a*d+3*b*c)
*x^10+1/13*b*d^3*x^13
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx = a c^3 x+\frac{1}{4} c^2 (b c+3 a d) x^4+\frac{3}{7} c d (b c+a d) x^7 +\frac{1}{10} d^2 (3 b c+a d) x^{10}+\frac{1}{13} b d^3 x^{13}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*
b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^3 dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(c^2x^3(3ad+bc)+d^2x^9(ad+3bc)+3cdx^6(ad+bc)+ac^3+bd^3x^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*
b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

3.2.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
norman	$\frac{b\,d^{3} x^{13}}{13}+\left(\frac{1}{10} a\,d^{3}+\frac{3}{10} b c\,d^{2}\right) x^{10}+\left(\frac{3}{7} a c\,d^{2}+\frac{3}{7} b\,c^{2} d\right) x^{7}+\left(\frac{3}{4} a\,c^{2} d+\frac{1}{4} c^{3} b\right) x^{4}+a\,c^{3} x$	72
default	$\frac{b\,d^{3} x^{13}}{13}+\frac{\left(a\,d^{3}+3 b c\,d^{2}\right) x^{10}}{10}+\frac{\left(3 a c\,d^{2}+3 b\,c^{2} d\right) x^{7}}{7}+\frac{\left(3 a\,c^{2} d+c^{3} b\right) x^{4}}{4}+a\,c^{3} x$	73
gosper	$\frac{1}{13} b\,d^{3} x^{13}+\frac{1}{10} x^{10} a\,d^{3}+\frac{3}{10} x^{10} b c\,d^{2}+\frac{3}{7} x^{7} a c\,d^{2}+\frac{3}{7} x^{7} b\,c^{2} d+\frac{3}{4} x^{4} a\,c^{2} d+\frac{1}{4} x^{4} c^{3} b+a\,c^{3} x$	75
risch	$\frac{1}{13} b\,d^{3} x^{13}+\frac{1}{10} x^{10} a\,d^{3}+\frac{3}{10} x^{10} b c\,d^{2}+\frac{3}{7} x^{7} a c\,d^{2}+\frac{3}{7} x^{7} b\,c^{2} d+\frac{3}{4} x^{4} a\,c^{2} d+\frac{1}{4} x^{4} c^{3} b+a\,c^{3} x$	75
parallelrisch	$\frac{1}{13} b\,d^{3} x^{13}+\frac{1}{10} x^{10} a\,d^{3}+\frac{3}{10} x^{10} b c\,d^{2}+\frac{3}{7} x^{7} a c\,d^{2}+\frac{3}{7} x^{7} b\,c^{2} d+\frac{3}{4} x^{4} a\,c^{2} d+\frac{1}{4} x^{4} c^{3} b+a\,c^{3} x$	75

input
```
int((b*x^3+a)*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^3*x^13+(1/10*a*d^3+3/10*b*c*d^2)*x^10+(3/7*a*c*d^2+3/7*b*c^2*d)*x
^7+(3/4*a*c^2*d+1/4*c^3*b)*x^4+a*c^3*x
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^3 \, dx = \frac{1}{13} bd^3x^{13} + \frac{1}{10} (3\,bcd^2 + ad^3)x^{10} + \frac{3}{7} (bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4} (bc^3 + 3\,ac^2d)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*
x^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int (a+bx^3)(c+dx^3)^3\,dx = ac^3x+\frac{bd^3x^{13}}{13}+x^{10}\left(\frac{ad^3}{10}+\frac{3bcd^2}{10}\right)+x^7\cdot\left(\frac{3acd^2}{7}+\frac{3bc^2d}{7}\right)+x^4\cdot\left(\frac{3ac^2d}{4}+\frac{bc^3}{4}\right)$$

input

```
integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

output

```
a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a
*c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)^3\,dx = \frac{1}{13}bd^3x^{13}+\frac{1}{10}(3bcd^2+ad^3)x^{10}+\frac{3}{7}(bc^2d+acd^2)x^7+ac^3x+\frac{1}{4}(bc^3+3ac^2d)x^4$$

input

```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

output

```
1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*
x^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^3)(c+dx^3)^3\,dx = \frac{1}{13}bd^3x^{13}+\frac{3}{10}bcd^2x^{10}+\frac{1}{10}ad^3x^{10}+\frac{3}{7}bc^2dx^7+\frac{3}{7}acd^2x^7+\frac{1}{4}bc^3x^4+\frac{3}{4}ac^2dx^4+ac^3x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 +
3/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^3)\,(c+dx^3)^3\,dx = x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right)+x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right)+\frac{b\,d^3\,x^{13}}{13}+a\,c^3\,x+\frac{3\,c\,d\,x^7\,(a\,d+b\,c)}{7}$$

input
```
int((a + b*x^3)*(c + d*x^3)^3,x)
```

output
```
x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*
d^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

3.3 $\int (a+bx^3) (c+dx^3)^2 \, dx$

3.3.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3) (c+dx^3)^2 \, dx = ac^2x + \frac{1}{4}c(bc+2ad)x^4 + \frac{1}{7}d(2bc+ad)x^7 + \frac{1}{10}bd^2x^{10}$$

output
```
a*c^2*x+1/4*c*(2*a*d+b*c)*x^4+1/7*d*(a*d+2*b*c)*x^7+1/10*b*d^2*x^10
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^2 \, dx = ac^2x + \frac{1}{4}c(bc+2ad)x^4 + \frac{1}{7}d(2bc+ad)x^7 + \frac{1}{10}bd^2x^{10}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)
/10
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right) \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^6(ad+2bc)+cx^3(2ad+bc)+ac^2+bd^2x^9\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)
/10
```

3.3.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^{2} x^{10}}{10}+\frac{\left(a\,d^{2}+2 b c d \right) x^{7}}{7}+\frac{\left(2 a c d +b\,c^{2}\right) x^{4}}{4}+a\,c^{2} x$	49
norman	$\frac{b\,d^{2} x^{10}}{10}+\left(\frac{1}{7} a\,d^{2}+\frac{2}{7} b c d \right) x^{7}+\left(\frac{1}{2} a c d +\frac{1}{4} b\,c^{2}\right) x^{4}+a\,c^{2} x$	49
gosper	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
risch	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
parallelrisch	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51

input
```
int((b*x^3+a)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3)\,(c+dx^3)^2\,dx = \frac{1}{10}\,bd^2x^{10} + \frac{1}{7}\,(2\,bcd + ad^2)x^7 + \frac{1}{4}\,(bc^2 + 2\,acd)x^4 + ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 +
a*c^2*x
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (a+bx^3)\,(c+dx^3)^2\,dx = ac^2x + \frac{bd^2x^{10}}{10} + x^7\left(\frac{ad^2}{7} + \frac{2bcd}{7}\right) + x^4\left(\frac{acd}{2} + \frac{bc^2}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

output
```
a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
 b*c**2/4)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3) (c+dx^3)^2 dx = \frac{1}{10} bd^2x^{10} + \frac{1}{7} (2 bcd + ad^2)x^7 + \frac{1}{4} (bc^2 + 2 acd)x^4 + ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 +
a*c^2*x
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^2 dx = \frac{1}{10} bd^2x^{10} + \frac{2}{7} bcdx^7 + \frac{1}{7} ad^2x^7 + \frac{1}{4} bc^2x^4 + \frac{1}{2} acdx^4 + ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*
d*x^4 + a*c^2*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3) (c+dx^3)^2 dx = x^4 \left(\frac{b c^2}{4} + \frac{a d c}{2}\right) + x^7 \left(\frac{a d^2}{7} + \frac{2 b c d}{7}\right) + \frac{b d^2 x^{10}}{10} + a c^2 x$$

input
```
int((a + b*x^3)*(c + d*x^3)^2,x)
```

output
```
x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)
/10 + a*c^2*x
```

3.4 $\int (a+bx^3)(c+dx^3)\,dx$

3.4.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^3)(c+dx^3)\,dx = acx + \frac{1}{4}(bc+ad)x^4 + \frac{1}{7}bdx^7$$

output
```
a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)\,dx = acx + \frac{1}{4}(bc+ad)x^4 + \frac{1}{7}bdx^7$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3),x]
```

output
```
a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

3.4.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^3(ad+bc)+ac+bdx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

input
```
Int[(a + b*x^3)*(c + d*x^3),x]
```

output
```
a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

3.4.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^4}{4}+\frac{bd\,x^7}{7}$	25
norman	$\frac{bd\,x^7}{7}+\left(\frac{ad}{4}+\frac{bc}{4}\right)x^4+acx$	26
gosper	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27
risch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27
parallelrisch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27

input
```
int((b*x^3+a)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3)(c+dx^3)\,dx = \frac{1}{7}bdx^7+\frac{1}{4}(bc+ad)x^4+acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3)(c+dx^3)\,dx = acx+\frac{bdx^7}{7}+x^4\left(\frac{ad}{4}+\frac{bc}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c),x)
```

output
```
a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a + bx^3) (c + dx^3) \, dx = \frac{1}{7} bdx^7 + \frac{1}{4} (bc + ad)x^4 + acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a + bx^3) (c + dx^3) \, dx = \frac{1}{7} bdx^7 + \frac{1}{4} bcx^4 + \frac{1}{4} adx^4 + acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
```

output
```
1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a + bx^3) (c + dx^3) \, dx = \frac{b\,d\,x^7}{7} + \left(\frac{a\,d}{4} + \frac{b\,c}{4}\right) x^4 + a\,c\,x$$

input
```
int((a + b*x^3)*(c + d*x^3),x)
```

output
```
x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

3.5 $\int \frac{a+bx^3}{c+dx^3} \, dx$

3.5.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{a+bx^3}{c+dx^3} \, dx = \frac{bx}{d} + \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}$$

output
```
b*x/d-1/3*(-a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+1/6*(-a*d+b*c)*
ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+1/3*(-a*d+b*c)*a
rctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(4/3)*3^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3} \, dx$$

$$= \frac{6bc^{2/3}\sqrt[3]{d}x + 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) + (bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\right.}{6c^{2/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3),x]
```

output
```
(6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Lo
g[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

3.5.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{c+dx^3}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^3+c}dx}{d}$$

$$\Big\downarrow \text{750}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{1142}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{217}$$

3.5. $\int \frac{a+bx^3}{c+dx^3}\,dx$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$\downarrow$ 1103

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

input
```
Int[(a + b*x^3)/(c + d*x^3),x]
```

output
```
(b*x)/d - ((b*c - a*d)*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/d
```

3.5.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.5. $\int \frac{a+bx^3}{c+dx^3}\,dx$

3.5.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\_Z^{3}+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3d^{2}}$	42
default	$\frac{bx}{d}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)(ad-bc)}{d}$	110

input
```
int((b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+1/3/d^2*sum((a*d-b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.56

$$\int\frac{a+bx^{3}}{c+dx^{3}}\,dx = \left[\frac{6\,bc^{2}dx-3\,\sqrt{\frac{1}{3}}(bc^{2}d-acd^{2})\sqrt{-\frac{(c^{2}d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^{3}-3\,(c^{2}d)^{\frac{1}{3}}cx-c^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+(c^{2}d)^{\frac{2}{3}}x-(c^{2}d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^{2}d)^{\frac{1}{3}}}{d}}}{dx^{3}+c}\right)+}{6\,c^{2}d^{2}}\right.$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^
2*d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) -
2*(c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*
b*c^2*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(s
qrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2)
+ (c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*
c) - 2*(c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t\log\left(\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c),x)
```

output
```
b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**
2*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} - \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/
d)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c
/d)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(
c/d)^(2/3))
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} + \frac{bx}{d} + \frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/
3))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/
3))/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/
d)^(1/3)))/(c*d)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{b\,x}{d} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}$$

input
```
int((a + b*x^3)/(c + d*x^3),x)
```

output
```
(b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (lo
g(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
- b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))
```

3.5. $\int \frac{a+bx^3}{c+dx^3}\,dx$

3.6 $\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$

3.6.1 Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x}{3cd\,(c+dx^3)} - \frac{(bc+2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

output
```
-1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)+1/9*(2*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5
/3)/d^(4/3)-1/18*(2*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(
5/3)/d^(4/3)-1/9*(2*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1
/2))/c^(5/3)/d^(4/3)*3^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$

$$=\frac{-\frac{6c^{2/3}\sqrt[3]{d}(bc-ad)x}{c+dx^3}-2\sqrt{3}(bc+2ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+2(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-(bc+2ad)\log\left(\right.}{18c^{5/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

output
```
((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*
ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3)
+ d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/(18*c^(5/3)*d^(4/3))
```

3.6.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2ad+bc)\int \frac{1}{dx^3+c}dx}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{(2ad+bc)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 16$$

$$\frac{(2ad+bc)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 25$$

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 1082$$

3.6. $\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$

$$\frac{(2ad+bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 217$$

$$\frac{(2ad+bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(2ad+bc)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

input
```
Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

output
```
-1/3*((b*c - a*d)*x)/(c*d*(c + d*x^3)) + ((b*c + 2*a*d)*(Log[c^(1/3) + d^(
1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*
d^(1/3)))/(3*c^(2/3))))/(3*c*d)
```

3.6. $\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.6.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(ad-bc)x}{3dc(d\,x^3+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\,\_Z^3+c\right)}\frac{(2ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{9c\,d^2}$	65
default	$\frac{(ad-bc)x}{3dc(d\,x^3+c)}+\frac{(2ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3cd}$	134

input
```
int((b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(a*d-b*c)/c*x/(d*x^3+c)+1/9/c/d^2*sum((2*a*d+b*c)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*d+c))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(bc^3d+2\,ac^2d^2+(bc^2d^2+2\,acd^3)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)}{dx^3+c}\right.}{}\right.$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*s
qrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(
1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d
))/(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*
log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^
3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d
 - a*c^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a
*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(
1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((
b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d
)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)
*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^
3*d^3*x^3 + c^4*d^2)]
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{x(ad-bc)}{3c^2d+3cd^2x^3} + \operatorname{RootSum}\left(729t^3c^5d^4-8a^3d^3-12a^2bcd^2-6ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

output
```
x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8
*a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t
*log(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x}{3\,(cd^2x^3+c^2d)} + \frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
-1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(
1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b
*c + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1
/9*(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d} - \frac{bcx-adx}{3\,(dx^3+c)cd}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(
x - (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{x\,\left(a\,d-b\,c\right)}{3\,c\,d\,\left(d\,x^3+c\right)}$$

input
```
int((a + b*x^3)/(c + d*x^3)^2,x)
```

output
```
(log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2
)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c)
)/(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))
/(3*c*d*(c + d*x^3))
```

3.7 $\int \frac{a+bx^3}{(c+dx^3)^3} \, dx$

3.7.1 Optimal result

Integrand size = 17, antiderivative size = 197

$$\int \frac{a+bx^3}{(c+dx^3)^3} \, dx = -\frac{(bc-ad)x}{6cd(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d(c+dx^3)} - \frac{(bc+5ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}$$

output
```
-1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)+1/27*
(5*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(4/3)-1/54*(5*a*d+b*c)*ln(c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(4/3)-1/27*(5*a*d+b*c)*arctan
(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(4/3)*3^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{(c+dx^3)^3} \, dx = \frac{-\frac{9c^{5/3}\sqrt[3]{d}(bc-ad)x}{(c+dx^3)^2} + \frac{3c^{2/3}\sqrt[3]{d}(bc+5ad)x}{c+dx^3} - 2\sqrt{3}(bc+5ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{54c^{8/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

output
```
((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*
c + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)
*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c +
5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3)
)
```

3.7.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(5ad+bc)\int \frac{1}{\left(dx^3+c\right)^2}dx}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(5ad+bc)\left(\frac{2\int \frac{1}{dx^3+c}dx}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 25$$

3.7. $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 27

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1082

3.7. $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 217

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1103

3.7. $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x)/(c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*(x/(3*c*(c + d*x
^3)) + (2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(3*c)))/(6*c*d)
```

3.7.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.7. $\int \frac{a+bx^3}{(c+dx^3)^3} \, dx$

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(dx^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(5ad+bc)\ln(x-\_R)}{\_R^2}}{27c^2d^2}$	84
default	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(dx^3+c)^2}+\frac{(5ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{9c^2d}$	153

input
```
int((b*x^3+a)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+1/27/c^2/d^2*
sum((5*a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(156) = 312.

Time = 0.32 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx$$

$$= \left[\frac{3\,(bc^3d^2+5\,ac^2d^3)x^4+3\,\sqrt{\frac{1}{3}}((bc^2d^3+5\,acd^4)x^6+bc^4d+5\,ac^3d^2+2\,(bc^3d^2+5\,ac^2d^3)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log}{}\right.$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
[1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d
^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(
c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(
2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*
x^3 + c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*
a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*
c) + 2*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d
^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c
^2*d^3)*x^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3
*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(
1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((
b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*
(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d
^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*
d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^
4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2)]
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5ad^2+bcd)+x(8acd-2bc^2)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6} + \mathrm{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3, \left(t\mapsto t\log\left(\frac{27tc^3d}{5ad+bc}+x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

output
```
(x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d*
*2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d*
*3 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27
*_t*c**3*d/(5*a*d + b*c) + x)))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{(bcd+5\,ad^2)x^4-2\,(bc^2-4\,acd)x}{18\,(c^2d^3x^6+2\,c^3d^2x^3+c^4d)} + \frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+5\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+5\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*
d^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c
/d)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)
*log(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2} - \frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2} - \frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d} + \frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\,(dx^3+c)^2c^2d}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3)
 + (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*lo
g(abs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2
*x + 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{\frac{x^4\,(5\,a\,d+b\,c)}{18\,c^2}+\frac{x\,(4\,a\,d-b\,c)}{9\,c\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}$$

input
```
int((a + b*x^3)/(c + d*x^3)^3,x)
```

output
```
((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6
+ 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3
)) - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/
3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

3.8 $\int (a+bx^3)^2 (c+dx^3)^3 \, dx$

3.8.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{1}{4}ac^2(2bc+3ad)x^4 + \frac{1}{7}c(b^2c^2+6abcd+3a^2d^2)\,x^7 + \frac{1}{10}d(3b^2c^2+6abcd+a^2d^2)\,x^{10} + \frac{1}{13}bd^2(3bc+2ad)x^{13} + \frac{1}{16}b^2d^3x^{16}$$

output
```
a^2*c^3*x+1/4*a*c^2*(3*a*d+2*b*c)*x^4+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^7+1/10*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^10+1/13*b*d^2*(2*a*d+3*b*c)*x^1
3+1/16*b^2*d^3*x^16
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{1}{4}ac^2(2bc+3ad)x^4 + \frac{1}{7}c(b^2c^2+6abcd+3a^2d^2)\,x^7 + \frac{1}{10}d(3b^2c^2+6abcd+a^2d^2)\,x^{10} + \frac{1}{13}bd^2(3bc+2ad)x^{13} + \frac{1}{16}b^2d^3x^{16}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*
b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

3.8.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(dx^9 \left(a^2d^2+6abcd+3b^2c^2\right)+cx^6 \left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^3(3ad+2bc)+bd^2x^{12}(2ad+3bc)+b^2\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+ \frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*
b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

3.8.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x+\left(\frac{3}{4}a^2c^2d+\frac{1}{2}ab\,c^3\right)x^4+\left(\frac{3}{7}c\,a^2d^2+\frac{6}{7}ab\,c^2d+\frac{1}{7}b^2c^3\right)x^7+\left(\frac{1}{10}a^2d^3+\frac{3}{5}abc\,d^2+\frac{3}{10}b^2c^2$
default	$\frac{b^2d^3x^{16}}{16}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{13}}{13}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{10}}{10}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^7}{7}+\frac{(3a^2c^2d+2ab\,c^3)x^4}{4}+$
gosper	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$
risch	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$
parallelrisch	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$

input
```
int((b*x^3+a)^2*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x+(3/4*a^2*c^2*d+1/2*a*b*c^3)*x^4+(3/7*c*a^2*d^2+6/7*a*b*c^2*d+1/7
*b^2*c^3)*x^7+(1/10*a^2*d^3+3/5*a*b*c*d^2+3/10*b^2*c^2*d)*x^10+(2/13*a*b*d
^3+3/13*b^2*c*d^2)*x^13+1/16*b^2*d^3*x^16
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^3\,dx &= \frac{1}{16}\,b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}\\&\quad+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}\\&\quad+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7\\&\quad+a^2c^3x+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4\end{aligned}$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{16}}{16} + x^{13} \cdot \left(\frac{2abd^3}{13} + \frac{3b^2cd^2}{13}\right) + x^{10}\left(\frac{a^2d^3}{10} + \frac{3abcd^2}{5} + \frac{3b^2c^2d}{10}\right) + x^7 \cdot \left(\frac{3a^2cd^2}{7} + \frac{6abc^2d}{7} + \frac{b^2c^3}{7}\right) + x^4 \cdot \left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

output
```
a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13
) + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**
2*c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c
**3/2)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{1}{16}\, b^2d^3x^{16} + \frac{1}{13}\left(3\, b^2cd^2 + 2\, abd^3\right)x^{13} + \frac{1}{10}\left(3\, b^2c^2d + 6\, abcd^2 + a^2d^3\right)x^{10} + \frac{1}{7}\left(b^2c^3 + 6\, abc^2d + 3\, a^2cd^2\right)x^7 + a^2c^3x + \frac{1}{4}\left(2\, abc^3 + 3\, a^2c^2d\right)x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{1}{16}\,b^2d^3x^{16} + \frac{3}{13}\,b^2cd^2x^{13} + \frac{2}{13}\,abd^3x^{13} + \frac{3}{10}\,b^2c^2dx^{10} + \frac{3}{5}\,abcd^2x^{10} + \frac{1}{10}\,a^2d^3x^{10} + \frac{1}{7}\,b^2c^3x^7 + \frac{6}{7}\,abc^2dx^7 + \frac{3}{7}\,a^2cd^2x^7 + \frac{1}{2}\,abc^3x^4 + \frac{3}{4}\,a^2c^2dx^4 + a^2c^3x$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2
*d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a
*b*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a
^2*c^3*x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = x^7 \left(\frac{3\,a^2\,c\,d^2}{7} + \frac{6\,a\,b\,c^2\,d}{7} + \frac{b^2\,c^3}{7}\right) + x^{10} \left(\frac{a^2\,d^3}{10} + \frac{3\,a\,b\,c\,d^2}{5} + \frac{3\,b^2\,c^2\,d}{10}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^{16}}{16} + \frac{a\,c^2\,x^4\,(3\,a\,d+2\,b\,c)}{4} + \frac{b\,d^2\,x^{13}\,(2\,a\,d+3\,b\,c)}{13}$$

input
```
int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

output
```
x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
 + (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (
a*c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

3.8. $\int (a+bx^3)^2 (c+dx^3)^3 \, dx$

3.9 $\int (a+bx^3)^2 (c+dx^3)^2 \, dx$

3.9.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}(b^2c^2+4abcd+a^2d^2)\,x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

output
```
a^2*c^2*x+1/2*a*c*(a*d+b*c)*x^4+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+1/5*b*
d*(a*d+b*c)*x^10+1/13*b^2*d^2*x^13
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}(b^2c^2+4abcd+a^2d^2)\,x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

output
```
a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

3.9.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(x^6\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^9(ad+bc)+2acx^3(ad+bc)+b^2d^2x^{12}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

output
```
a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

3.9.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^2 d^2 x^{13}}{13}+\left(\frac{1}{5}ab\,d^2+\frac{1}{5}b^2 cd\right)x^{10}+\left(\frac{1}{7}a^2 d^2+\frac{4}{7}abcd+\frac{1}{7}b^2 c^2\right)x^7+\left(\frac{1}{2}a^2 cd+\frac{1}{2}b\,c^2 a\right)x^4+a^2 c^2 x$
default	$\frac{b^2 d^2 x^{13}}{13}+\frac{(2ab\,d^2+2b^2 cd)x^{10}}{10}+\frac{(a^2 d^2+4abcd+b^2 c^2)x^7}{7}+\frac{(2a^2 cd+2b\,c^2 a)x^4}{4}+a^2 c^2 x$
gosper	$\frac{1}{13}b^2 d^2 x^{13}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2 cd+\frac{1}{7}x^7 a^2 d^2+\frac{4}{7}x^7 abcd+\frac{1}{7}x^7 b^2 c^2+\frac{1}{2}x^4 a^2 cd+\frac{1}{2}x^4 b\,c^2 a+c$
risch	$\frac{1}{13}b^2 d^2 x^{13}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2 cd+\frac{1}{7}x^7 a^2 d^2+\frac{4}{7}x^7 abcd+\frac{1}{7}x^7 b^2 c^2+\frac{1}{2}x^4 a^2 cd+\frac{1}{2}x^4 b\,c^2 a+c$
parallelrisch	$\frac{1}{13}b^2 d^2 x^{13}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2 cd+\frac{1}{7}x^7 a^2 d^2+\frac{4}{7}x^7 abcd+\frac{1}{7}x^7 b^2 c^2+\frac{1}{2}x^4 a^2 cd+\frac{1}{2}x^4 b\,c^2 a+c$

input
```
int((b*x^3+a)^2*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d^2*x^13+(1/5*a*b*d^2+1/5*b^2*c*d)*x^10+(1/7*a^2*d^2+4/7*a*b*c*d+
1/7*b^2*c^2)*x^7+(1/2*a^2*c*d+1/2*b*c^2*a)*x^4+a^2*c^2*x
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2\,dx=\frac{1}{13}b^2d^2x^{13}+\frac{1}{5}\left(b^2cd+abd^2\right)x^{10}+\frac{1}{7}\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+a^2c^2x+\frac{1}{2}\left(abc^2+a^2cd\right)x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^{13}}{13} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^4\left(\frac{a^2cd}{2} + \frac{abc^2}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*
(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\,\left(b^2cd + abd^2\right)x^{10} + \frac{1}{7}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + a^2c^2x + \frac{1}{2}\,\left(abc^2 + a^2cd\right)x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\,b^2cdx^{10} + \frac{1}{5}\,abd^2x^{10} + \frac{1}{7}\,b^2c^2x^7 + \frac{4}{7}\,abcdx^7 + \frac{1}{7}\,a^2d^2x^7 + \frac{1}{2}\,abc^2x^4 + \frac{1}{2}\,a^2cdx^4 + a^2c^2x$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7
+ 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 +
a^2*c^2*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = x^7 \left(\frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7}\right) + a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + \frac{a c x^4 (a d + b c)}{2} + \frac{b d x^{10} (a d + b c)}{5}$$

input
```
int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

output
```
x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^1
3)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

3.10 $\int (a+bx^3)^2 (c+dx^3) \, dx$

3.10.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)^2 (c+dx^3) \, dx = a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10}$$

output
```
a^2*c*x+1/4*a*(a*d+2*b*c)*x^4+1/7*b*(2*a*d+b*c)*x^7+1/10*b^2*d*x^10
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3) \, dx = a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)
/10
```

3.10.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2c+bx^6(2ad+bc)+ax^3(ad+2bc)+b^2dx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)
/10
```

3.10.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d x^{10}}{10}+\frac{(2abd+b^2c)x^7}{7}+\frac{(a^2d+2abc)x^4}{4}+a^2cx$	49
norman	$\frac{b^2 d x^{10}}{10}+\left(\frac{2}{7}abd+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{4}a^2d+\frac{1}{2}abc\right)x^4+a^2cx$	49
gosper	$\frac{1}{10}b^2 d x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51
risch	$\frac{1}{10}b^2 d x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51
parallelrisch	$\frac{1}{10}b^2 d x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51

input
```
int((b*x^3+a)^2*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx=\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,(b^2c+2\,abd)x^7+\frac{1}{4}\,(2\,abc+a^2d)x^4+a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 +
a^2*c*x
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx=a^2cx+\frac{b^2dx^{10}}{10}+x^7\cdot\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c),x)
```

output
```
a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4
+ a*b*c/2)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,(b^2c+2\,abd)x^7 + \frac{1}{4}\,(2\,abc+a^2d)x^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 +
a^2*c*x
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,b^2cx^7 + \frac{2}{7}\,abdx^7 + \frac{1}{2}\,abcx^4 + \frac{1}{4}\,a^2dx^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

output
```
1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*
d*x^4 + a^2*c*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx = x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right) + x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right) + \frac{b^2\,d\,x^{10}}{10} + a^2\,c\,x$$

input
```
int((a + b*x^3)^2*(c + d*x^3),x)
```

output
```
x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)
/10 + a^2*c*x
```

3.11 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(a+bx^3)^2}{c+dx^3} \, dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}-\frac{(bc-ad)^2 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}+\frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d+1/3*(-a*d+b*c)^2*ln(c^(1/3)+d^(1/3)*x)
/c^(2/3)/d^(7/3)-1/6*(-a*d+b*c)^2*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3
)*3^(1/2))/c^(2/3)/d^(7/3)*3^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2}{c+dx^3} \, dx = \frac{-12bc^{2/3}\sqrt[3]{d}(bc-2ad)x+3b^2c^{2/3}d^{4/3}x^4+4\sqrt{3}(bc-ad)^2 \arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)+4(bc-ad)^2 \log\left(\sqrt[3]{c}+\right.}{12c^{2/3}d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

output
```
(-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*
(b*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

3.11.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$\Big\downarrow \text{ 915}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^3)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^3}{d}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1
/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c -
 a*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

3.11.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{b^2x^4}{4d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^3}$	78
default	$\frac{b\left(\frac{1}{4}bd\,x^4+2adx-bcx\right)}{d^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)\left(a^2d^2-2abcd+b^2c^2\right)}{d^2}$	140

input
```
int((b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*c*x+1/3/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.92

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$= \left[\frac{3\,b^2c^2d^2x^4 + 6\,\sqrt{\frac{1}{3}}(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3 - 3\,(c^2d)^{\frac{1}{3}}cx - c^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2 + (c^2d)^{\frac{2}{3}}x - (c^2d)^{\frac{1}{3}}c\right.}{dx^3+c}\right.}{}\right.$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3
*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(
1/3)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*lo
g(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*
c^2*d^2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d -
 2*a*b*c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^
(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^
2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2x^4}{4d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right)$$

$$+\text{RootSum}\left(27t^3c^2d^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto\right.\right.$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c),x)
```

output
```
b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
 a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*
d**3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t
*log(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2dx^4 - 4\,(b^2c - 2\,abd)x}{4\,d^2} + \frac{\sqrt{3}(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(
c/d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(x + (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^4} + \frac{b^2d^3x^4-4\,b^2cd^2x+8\,abd^3x}{4\,d^4}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c
/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3
)))/(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

3.11.9 Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2\,x^4}{4\,d} - x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right) + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)^2}{3\,c^{2/3}\,d^{7/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^2}{c^{2/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2}{3\,c^{2/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3),x)
```

output
```
(b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*
(a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2)/(3*c^(2/3)*d^(7/3))
```

3.11. $\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}$$

output
```
b^2*x/d^2+1/3*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)-2/9*(-a*d+b*c)*(a*d+2*b*c)*ln
(c^(1/3)+d^(1/3)*x)/c^(5/3)/d^(7/3)+1/9*(-a*d+b*c)*(a*d+2*b*c)*ln(c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(7/3)+2/9*(-a*d+b*c)*(a*d+2*b*c)*
arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/d^(7/3)*3^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$$

$$= \frac{9b^2\sqrt[3]{d}x+\frac{3\sqrt[3]{d}(bc-ad)^2x}{c(c+dx^3)}+\frac{2\sqrt{3}(2b^2c^2-abcd-a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}-\frac{2(2b^2c^2-abcd-a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{(2b^2c^2-ab}{}}{9d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

output
```
(9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3
]*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[
3]])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x]
)/c^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

3.12.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{ 915}$$

$$\int \left(\frac{b^2}{d^2}-\frac{-a^2d^2+2bdx^3(bc-ad)+b^2c^2}{d^2\,(c+dx^3)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(bc-ad)(ad+2bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}}+\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2\left(c+dx^3\right)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*
b*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c
^(5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(
9*c^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

3.12.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

method	result	siz
risch	$\frac{b^2 x}{d^2}+\frac{(a^2 d^2-2abcd+b^2 c^2) x}{3c\, d^2 (d\, x^3+c)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(a^2 d^2+abcd-2b^2 c^2)\ln(x-\_R)}{\_R^2}\right)}{9d^3 c}$	99
default	$\frac{b^2 x}{d^2}+\frac{\frac{(a^2 d^2-2abcd+b^2 c^2) x}{3c(d\, x^3+c)}+\frac{2(a^2 d^2+abcd-2b^2 c^2)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}} x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}}{d^2}$	16

input
```
int((b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^3+c)+2/9/d^3/c*sum(
(a^2*d^2+a*b*c*d-2*b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.

Time = 0.34 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.80

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\, dx$$

$$= \left[\frac{9\, b^2c^3d^2x^4 - 3\,\sqrt{\frac{1}{3}}(2\, b^2c^4d - abc^3d^2 - a^2c^2d^3 + (2\, b^2c^3d^2 - abc^2d^3 - a^2cd^4)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\, cdx^3 - 3\,(c^2\right.}{}}{}\right.$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2
*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/
d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (
c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (
2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^
3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b
^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*
(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2
+ a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt
(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*
d^3 - a^2*c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2
/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2
*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*lo
g(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d
- a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c
*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(
c^3*d^4*x^3 + c^4*d^3)]
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3c^2d^2 + 3cd^3x^3}$$
$$+\text{RootSum}\Big(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c^6$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

output
```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**
3*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 2
4*a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 -
96*a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*
d**2 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(cd^3x^3+c^2d^2)}+\frac{b^2x}{d^2} - \frac{2\,\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 -
2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2
*d^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*
b^2*c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{2\,\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(dx^3+c)cd^2}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)
*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2
- a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2
/3)*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-
c/d)^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3
 + c)*c*d^2)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2\,x}{d^2}+\frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{3\,c\,(d^3\,x^3+c\,d^2)} + \frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}} + \frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}} - \frac{2\,\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3))
+ (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3
)) + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/
3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2
*b*c))/(9*c^(5/3)*d^(7/3))
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx$

3.13.1 Optimal result

Integrand size = 19, antiderivative size = 258

$$\begin{aligned}\int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx = & -\frac{(bc-ad)x(a+bx^3)}{6cd(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2(c+dx^3)} \\ & - \frac{(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} \\ & + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} \\ & - \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}\end{aligned}$$

output
```
-1/6*(-a*d+b*c)*x*(b*x^3+a)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+4*b*c)*
x/c^2/d^2/(d*x^3+c)+1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(1/3)+d^(1/3
)*x)/c^(8/3)/d^(7/3)-1/54*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(7/3)-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*
c^2)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(7/3)*3^(
1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$$
$$= \frac{-\frac{3c^{2/3}\sqrt[3]{d}(bc-ad)x\left(ad(8c+5dx^3)+bc(4c+7dx^3)\right)}{(c+dx^3)^2} - 2\sqrt{3}(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(2b^2c^2+2a}{54c^{8/3}d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

output
```
((-3*c^(2/3)*d^(1/3)*(b*c - a*d)*x*(a*d*(8*c + 5*d*x^3) + b*c*(4*c + 7*d*x
^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTan
[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d
^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

3.13.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {930, 910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$$
$$\Big\downarrow \ 930$$
$$\frac{\int \frac{2b(2bc+ad)x^3+a(bc+5ad)}{(dx^3+c)^2}dx}{6cd} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$
$$\Big\downarrow \ 910$$
$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\int \frac{1}{dx^3+c}dx - \frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

3.13. $\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+4\cdots\right.}{3(c+dx^3)}}{6cd}$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\cdots\right.}{3(c+dx^3\cdots}}{6cd}$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1103

3.13. $\int\frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3))/(c*d*(c + d*x^3)^2) + (-1/3*((a*b + (4*b^
2*c)/d - (5*a^2*d)/c)*x)/(c + d*x^3) + (2*(2*a*b + (2*b^2*c)/d + (5*a^2*d)
/c)*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1
- (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/3)/(6*c*d)
```

3.13.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.13. $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

3.13.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.89 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{\left(d\,x^3+c\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{27c^2d^3}$
default	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{\left(d\,x^3+c\right)^2}+\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\sqrt{\ \ }\right.}{3d}\right.}{9c^2d^2}$

input
```
int((b*x^3+a)^2/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*b*c*d-b^2
*c^2)/c/d^2*x)/(d*x^3+c)^2+1/27/c^2/d^3*sum((5*a^2*d^2+2*a*b*c*d+2*b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(217) = 434.

Time = 0.32 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.14

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
[-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3
)*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^
2*d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^
4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2
+ 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d
)^(1/3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6
+ 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2
 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)
^(1/3)*c) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^
3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4
*d^2 - 2*a^2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3
*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*
c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5
*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*s
qrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c
)*sqrt((c^2*d)^(1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*
x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2
*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^
2*d)^(1/3)*c) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^...
```

3.13.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5a^2d^3+2abcd^2-7b^2c^2d)+x(8a^2cd^2-4abc^2d-4b^2c^3)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}$$
$$+\mathrm{RootSum}\Big(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210a^4b^2c^2d^4-128a^3b^3c^3d^3-84a^2b^4c^4d^2-24ab^5c^5d$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

output
```
(x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*
a*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**
4*x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**
5 - 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*
d**2 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(
5*a**2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3)x^4+4\,(b^2c^3+abc^2d-2\,a^2cd^2)x}{18\,(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2)} + \frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
-1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*
d - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)
*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2
*d^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*
(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2
/3))
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^2\right)^{\frac{2}{3}}c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^2\right)^{\frac{2}{3}}c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^2} - \frac{7\,b^2c^2dx^4-2\,abcd^2x^4-5\,a^2d^3x^4+4\,b^2c^3x+4\,abc^2dx-8\,a^2cd^2x}{18\left(dx^3+c\right)^2c^2d^2}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x
+ (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2
*a*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(
2/3)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(ab
s(x - (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 -
 5*a^2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)
^2*c^2*d^2)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\frac{2\,x\,\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

output
```
(log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)
*d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2
*d^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (
log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*
a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1
/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*
c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

3.13. $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

3.14.1 Optimal result

Integrand size = 19, antiderivative size = 252

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}-\frac{(bc-ad)^4\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}$$

output
```
d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/4*d^2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)*x^4/b^3+1/7*d^3*(-a*d+4*b*c)*x^7/b^2+1/10*d^4*x^10/b+1/3*(-
a*d+b*c)^4*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/6*(-a*d+b*c)^4*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/3)-1/3*(-a*d+b*c)^4*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$= \frac{420\sqrt[3]{b}d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+105b^{4/3}d^2(6b^2c^2-4abcd+a^2d^2)\,x^4+60b^{7/3}d^3(4bc-ad)x}{}$$

input
```
Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

output
```
(420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 1
05*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b
*c - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(
-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*L
og[a^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

3.14.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{b^3}+\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d}{b^4\left(a+bx^3\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4}{\sqrt{3}a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

input
```
Int[(c + d*x^3)^4/(a + b*x^3),x]
```

output
```
(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^
2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) +
(d^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

3.14.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

method	result
risch	$\frac{d^4x^{10}}{10b}-\frac{d^4x^7a}{7b^2}+\frac{4d^3x^7c}{7b}-\frac{d^3x^4ac}{b^2}+\frac{3d^2x^4c^2}{2b}+\frac{d^4x^4a^2}{4b^3}-\frac{d^4a^3x}{b^4}+\frac{4d^3a^2cx}{b^3}-\frac{6d^2ac^2x}{b^2}+\frac{4dc^3x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}(b}}{}$
default	$-\frac{d\left(-\frac{d^3x^{10}b^3}{10}+\frac{((ad-2bc)b^2d^2-2b^3cd^2)x^7}{7}+\frac{(2(ad-2bc)b^2cd-bd(a^2d^2-2abcd+2b^2c^2))x^4}{4}+(ad-2bc)(a^2d^2-2abcd+2b^2c^2)x\right)}{b^4}+$

input
```
int((d*x^3+c)^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*d^4*x^10/b-1/7*d^4/b^2*x^7*a+4/7*d^3/b*x^7*c-d^3/b^2*x^4*a*c+3/2*d^2/
b*x^4*c^2+1/4*d^4/b^3*x^4*a^2-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*
c^2*x+4*d/b*c^3*x+1/3/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$= \left[\frac{42\,a^2b^4d^4x^{10}+60\,(4\,a^2b^4cd^3-a^3b^3d^4)x^7+105\,(6\,a^2b^4c^2d^2-4\,a^3b^3cd^3+a^4b^2d^4)x^4+210\,\sqrt{\tfrac{1}{3}}(ab^5c^4-4}{}\right.$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105
*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(
a*b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*
d^4)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3
*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^
2*b)^(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^
4*c^3*d - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1
/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(
6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^
4)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/
3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2
/3)*x + (a^2*b)^(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 42
0*(4*a^2*b^4*c^3*d - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(
a^2*b^5)]
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)$$
$$+\mathrm{RootSum}\Big(27t^3a^2b^{13}-a^{12}d^{12}+12a^{11}bcd^{11}-66a^{10}b^2c^2d^{10}+220a^9b^3c^3d^9-495a^8b^4c^4d^8+792a^7b^5c^5d^7$$
$$+\frac{d^4x^{10}}{10b}$$

input
```
integrate((d*x**3+c)**4/(b*x**3+a),x)
```

output
```
x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*
d**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 -
 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d*
*12 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d
**9 - 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c*
*6*d**6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**
9*c**9*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12,
 Lambda(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*
c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= \frac{14\,b^3d^4x^{10} + 20\,(4\,b^3cd^3 - ab^2d^4)x^7 + 35\,(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4)x^4 + 140\,(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a^2b}{140\,b^4}$$
$$+ \frac{\sqrt{3}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2
+ 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b^3}$$
$$- \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b^3}$$
$$- \frac{(b^{10}c^4 - 4\,ab^9c^3d + 6\,a^2b^8c^2d^2 - 4\,a^3b^7cd^3 + a^4b^6d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{10}}$$
$$+ \frac{14\,b^9d^4x^{10} + 80\,b^9cd^3x^7 - 20\,ab^8d^4x^7 + 210\,b^9c^2d^2x^4 - 140\,ab^8cd^3x^4 + 35\,a^2b^7d^4x^4 + 560\,b^9c^3dx - 840\,a}{140\,b^{10}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)
^(2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b
^3) - 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(1
4*b^9*d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4
 - 140*a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*
c^2*d^2*x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

3.14.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^7\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)$$
$$+x^4\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right)+\frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^4}{3\,a^{2/3}\,b^{13/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^4}{a^{2/3}\,b^{13/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^4}{3\,a^{2/3}\,b^{13/3}}$$

input
```
int((c + d*x^3)^4/(a + b*x^3),x)
```

output
```
x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b
) - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*
d^3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x +
 a^(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i +
2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(1
3/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$

3.15.1 Optimal result

Integrand size = 19, antiderivative size = 208

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} - \frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/4*d^2*(-a*d+3*b*c)*x^4/b^2+1/7*d^3
*x^7/b+1/3*(-a*d+b*c)^3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a*d+b
*c)^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/3)-1/3*(-a*d
+b*c)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(10/3)
*3^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$

$$=\frac{84\sqrt[3]{b}d(3b^2c^2-3abcd+a^2d^2)\,x+21b^{4/3}d^2(3bc-ad)x^4+12b^{7/3}d^3x^7+\frac{28\sqrt{3}(bc-ad)^3\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+}{84b^{10/3}}$$

input
```
Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

output
```
(84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c
- a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/
3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1
/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

3.15.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3(a+bx^3)}+\frac{d^2x^3(3bc-ad)}{b^2}+\frac{d^3x^6}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3}{\sqrt{3}a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

input
```
Int[(c + d*x^3)^3/(a + b*x^3),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b
^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) +
b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

3.15.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.63

method	result
risch	$\frac{d^3x^7}{7b}-\frac{d^3a\,x^4}{4b^2}+\frac{3d^2c\,x^4}{4b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^4}$
default	$\frac{d\left(\frac{1}{7}b^2d^2x^7-\frac{1}{4}ab\,d^2x^4+\frac{3}{4}b^2cd\,x^4+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b^3}$

input
```
int((d*x^3+c)^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^3*x^7/b-1/4*d^3/b^2*a*x^4+3/4*d^2/b*c*x^4+d^3/b^3*a^2*x-3*d^2/b^2*a*
c*x+3*d/b*c^2*x+1/3/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)
/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.37

$$\int \frac{(c + dx^3)^3}{a + bx^3}\, dx$$

$$= \left[\frac{12\, a^2 b^3 d^3 x^7 + 21\, (3\, a^2 b^3 c d^2 - a^3 b^2 d^3) x^4 - 42\, \sqrt{\tfrac{1}{3}} (a b^4 c^3 - 3\, a^2 b^3 c^2 d + 3\, a^3 b^2 c d^2 - a^4 b d^3) \sqrt{\tfrac{(-a^2 b)^{\frac{1}{3}}}{b}} \log \left(\right.}{} \right.$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sq
rt(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((
-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)
*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))
/(b*x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a
^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (
-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^
2*b^4), 1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4
+ 84*sqrt(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)
*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(
1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2
*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*
d^2 + a^4*b*d^3)*x)/(a^2*b^4)]
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{10}+a^9d^9-9a^8bcd^8+36a^7b^2c^2d^7-84a^6b^3c^3d^6+126a^5b^4c^4d^5-126a^4b^5c^5d^4+84a^3\right.$$
$$+\frac{d^3x^7}{7b}$$

input
```
integrate((d*x**3+c)**3/(b*x**3+a),x)
```

output
```
x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/
b**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*
d**8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**
4*d**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c
**7*d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a*
*3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**
7/(7*b)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \frac{4\,b^2d^3x^7+7\,(3\,b^2cd^2-abd^3)x^4+28\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{28\,b^3}$$
$$+\frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b
^4*(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(
2/3))
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,b^6d^3x^7+21\,b^6cd^2x^4-7\,ab^5d^3x^4+84\,b^6c^2dx-84\,ab^5cd^2x+28\,a^2b^4d^3x}{28\,b^7}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a
^2*b^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7
) + 1/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*
d*x - 84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

3.15.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)+\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^3}{a^{2/3}\,b^{10/3}}$$

input
```
int((c + d*x^3)^3/(a + b*x^3),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
 - (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b
*c)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(
1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c
)^3)/(a^(2/3)*b^(10/3))
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$

3.16.1 Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} + \frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/4*d^2*x^4/b+1/3*(-a*d+b*c)^2*ln(a^(1/3)+b^(1/3)*x)/
a^(2/3)/b^(7/3)-1/6*(-a*d+b*c)^2*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(2/3)/b^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{-12a^{2/3}\sqrt[3]{b}d(-2bc+ad)x+3a^{2/3}b^{4/3}d^2x^4+4\sqrt{3}(bc-ad)^2\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{a}\right.}{12a^{2/3}b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

output
```
(-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sq
rt[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4
*(b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

3.16.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$$

$$\Big\downarrow \ 915$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^3)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2}{\sqrt{3}a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*
d)^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

3.16.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result	s
risch	$\frac{d^2x^4}{4b} - \frac{d^2ax}{b^2} + \frac{2dcx}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^3}$	7
default	$-\frac{d\left(-\frac{1}{4}bd\,x^4+adx-2bcx\right)}{b^2} + \frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^2d^2-2abcd+b^2c^2\right)}{b^2}$	1

input
```
int((d*x^3+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*c*x+1/3/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.93

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \left[\frac{3\,a^2b^2d^2x^4+6\,\sqrt{\frac{1}{3}}(ab^3c^2-2\,a^2b^2cd+a^3bd^2)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}\right.}{bx^3+a}\right.}{}\right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*
d^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3
*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^
3*b*d^2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 -
 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2
*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^
2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right) + \mathrm{RootSum}\left(27t^3a^2b^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d-b^6c^6,\left(t\mapsto\right.\right. + \frac{d^2x^4}{4b}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a*
*5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b*
*4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/
(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{bd^2x^4+4\,(2\,bcd-ad^2)x}{4\,b^2} + \frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(
a/b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b} - \frac{(b^4c^2-2\,ab^3cd+a^2b^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{b^3d^2x^4+8\,b^3cdx-4\,ab^2d^2x}{4\,b^4}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*
(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{d^2\,x^4}{4\,b} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right) + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^2}{3\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^2}{a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2}{3\,a^{2/3}\,b^{7/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3),x)
```

output
```
(d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*
(a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2)/(3*a^(2/3)*b^(7/3))
```

3.16. $\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$

3.17 $\int \frac{c+dx^3}{a+bx^3}\,dx$

3.17.1 Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} - \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

output
```
d*x/b+1/3*(-a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(-a*d+b*c)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(-a*d+b*c)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{a+bx^3}\,dx$$

$$= \frac{6a^{2/3}\sqrt[3]{b}dx - 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3),x]
```

output
```
(6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

3.17.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{a+bx^3}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{bx^3+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{750}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\downarrow 25$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\downarrow 27$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\downarrow 1082$$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\downarrow 217$$

3.17. $\int \frac{c+dx^3}{a+bx^3}\,dx$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$\Big\downarrow$ 1103

$$\frac{(bc-ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3),x]
```

output
```
(d*x)/b + ((b*c - a*d)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

3.17.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.17. $\int \frac{c+dx^3}{a+bx^3}\,dx$

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$	42
default	$\frac{dx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{b}$	110

input
```
int((d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+1/3/b^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int \frac{c+dx^3}{a+bx^3}\,dx$$

$$= \left[\frac{6\,a^2bdx-3\,\sqrt{\frac{1}{3}}(ab^2c-a^2bd)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b)^{\frac{1}{3}}\,ax-a^2-3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(-a^2b)^{\frac{2}{3}}\,x+(-a^2b)^{\frac{1}{3}}\,a\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right.}{6\,a^{}}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-
a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) -
(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2),
 1/6*(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^
(1/3)/b)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2
/3)))/(a^2*b^2)]
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \text{RootSum}\left(27t^3a^2b^4+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(-\frac{3tab}{ad-bc}+x\right)\right)\right) + \frac{dx}{b}$$

input
```
integrate((d*x**3+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
 - b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3}{a+bx^3}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}} + \frac{dx}{b} - \frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{d\,x}{b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,a^{2/3}\,b^{4/3}}$$

input

```
int((c + d*x^3)/(a + b*x^3),x)
```

output

```
(d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
- b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

3.17. $\int \frac{c+dx^3}{a+bx^3}\,dx$

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

3.18.1 Optimal result

Integrand size = 19, antiderivative size = 288

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

output
```
1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3
)+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/
3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\right.}{}}{-6bc+6ad}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) -
 (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
2/3))/(-6*b*c + 6*a*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {917, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$

$$\Big\downarrow \text{917}$$

$$\frac{b\int \frac{1}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{1}{dx^3+c}dx}{bc-ad}$$

$$\Big\downarrow \text{750}$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}$$

$$\downarrow\ 16$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\downarrow\ 1142$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\downarrow\ 25$$

3.18. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1082$$

3.18. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

3.18. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

$$\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(Log[c^(1/3)
 + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

3.18.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.18. $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

3.18.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\,\mathrm{arcta}\ldots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}d^{3}-3a^{4}bc\,d^{2}+3a^{3}b^{2}c^{2}d-a^{2}b^{3}c^{3}\right)\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-b^{5}c^{5}\right)\_R\ldots\right.\right.\right.}{3}$

input
```
int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2\ldots\right.}{\ldots}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3
) - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d
^2/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2
/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*
(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-
b^2/a^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a
*d)
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b
)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)
^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*
d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/
3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = &-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}\\&+\frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}\\&-\frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}\\&+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)}\\&-\frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}\end{aligned}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)
*a^2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*
(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*
a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d +
b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b
^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3
) + log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 -
18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d
+ b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-
d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3) + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)
*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*
c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3)
)^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c
^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b
^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(
81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*
(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5
))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d
^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(...
```

3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

3.19.1 Optimal result

Integrand size = 19, antiderivative size = 346

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = &-\frac{dx}{3c(bc-ad)(c+dx^3)}-\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}\\&+\frac{d^{2/3}(5bc-2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2}\\&+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2}\\&-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}\\&+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}\end{aligned}$$

output
```
-1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)+1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/
(-a*d+b*c)^2-1/9*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*
d+b*c)^2-1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a
*d+b*c)^2+1/18*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/c^(5/3)/(-a*d+b*c)^2-1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)^2*3^(1/2)+1/9*d^(2/3)*(-2*a*d+5*b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^2*3^(1/2
)
```

3.19.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$

$$= \frac{6a^{2/3}c^{2/3}d(-bc+ad)x - 6\sqrt{3}b^{5/3}c^{5/3}(c+dx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt{3}a^{2/3}d^{2/3}(-5bc+2ad)\left(c+dx^3\right) a}{}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

output
```
(6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x
^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3
)*(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]]
 + 6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2
/3)*(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5
/3)*(c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d
^(2/3)*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

3.19.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {931, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-2bdx^3+3bc-2ad}{(bx^3+a)(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{3b^2c\int\frac{1}{bx^3+a}dx}{bc-ad}-\frac{d(5bc-2ad)\int\frac{1}{dx^3+c}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 750

$$\frac{\frac{3b^2c\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 16

$$\frac{\frac{3b^2c\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1142

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/}\cdots}\cdots\right)}{\cdots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 25

3.19. $\int\frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx$

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\ldots}{3c^{2}\ldots}\right.\ldots}{\ldots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\ldots}{3\ldots}\right.\ldots}{\ldots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1082

$$\frac{\frac{3b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}d\ldots}{\ldots}\right.\ldots}{\ldots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 217

3.19. $\int\frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

$$\frac{\frac{3b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\ldots\right)}{\ldots}}{3c^{2/3}}\ldots\right)}{bc-ad}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1103

$$\frac{\frac{3b^2c\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\ldots\right)}{2\sqrt[3]{\ldots}}}{3c^{2/3}}\ldots\right)}{bc-ad}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b^2*c*(Log[a^(1/3) + b^(1/3)*
x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/
3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(5*b*c - 2*a*d)*(Log[c^(1/3) + d^(1/3)
*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/S
qrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1
/3)))/(3*c^(2/3))))/(b*c - a*d))/(3*c*(b*c - a*d))
```

3.19. $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.19.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.71

method	result
default	$\frac{d\left(\frac{(ad-bc)x}{3c\left(d\,x^{3}+c\right)}+\frac{(2ad-5bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d/(a*d-b*c)^2*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+1/3*(2*a*d-5*b*c)/c*(1/3/d/(c/d
)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/
3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+(1
/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*b^2/(a*d-b*c)^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$
$$= \frac{6\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}((5\,bcd-2\,ad^2)x^3+5\,bc^2-2\,acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\text{arcta}}{}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*
a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 +
 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 -
a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 +
5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b
^2/a^2)^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

output
```
Timed out
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx &= \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
&- \frac{\sqrt{3}(5bc-2ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-2abc^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\
&- \frac{dx}{3\left(bc^3-ac^2d+(bc^2d-acd^2)x^3\right)} \\
&- \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\
&+ \frac{(5bc-2ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-2abc^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} \\
&+ \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\
&- \frac{(5bc-2ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-2abc^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}\end{aligned}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^
2*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3))
- 1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)
^(1/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^
(1/3))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3)
- 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3
) + a^2*c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/
3) - 2*a*b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*lo
g(x + (c/d)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*
d^2*(c/d)^(2/3))
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = -\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c^2-2\,a^2bcd+a^3d^2)} + \frac{(-ab^2)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\,\sqrt{3}a^2bcd+\sqrt{3}a^3d^2} + \frac{(-ab^2)^{\frac{1}{3}}\,b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab^2c^2-2\,a^2bcd+a^3d^2)} + \frac{(5\,bcd-2\,ad^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,(b^2c^4-2\,abc^3d+a^2c^2d^2)} - \frac{\left(5\,(-cd^2)^{\frac{1}{3}}\,bc-2\,(-cd^2)^{\frac{1}{3}}\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(\sqrt{3}b^2c^4-2\,\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2)} - \frac{\left(5\,(-cd^2)^{\frac{1}{3}}\,bc-2\,(-cd^2)^{\frac{1}{3}}\,ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,(b^2c^4-2\,abc^3d+a^2c^2d^2)} - \frac{dx}{3\,(dx^3+c)(bc^2-acd)}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/
6*(-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2
*a^2*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (
-c/d)^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3
)*b*c - 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) -
1/18*(5*(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3
) + (-c/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3
+ c)*(b*c^2 - a*c*d))
```

3.19.9 Mupad [B] (verification not implemented)

Time = 21.57 (sec) , antiderivative size = 2589, normalized size of antiderivative = 7.48

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

output
```
log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
 + (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c
^5*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c
^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c
^5*(a*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*
b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*
c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*
d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 +
 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x
*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*
(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*
d))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3
+ 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d
- b*c)^6))^(1/3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*
d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*
c^6 - 162*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*
a^6*b^2*c^2*d^4 - 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1
/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*
c*d))/c + (27*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*...
```

3.19. $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

3.20 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

3.20.1 Optimal result

Integrand size = 19, antiderivative size = 320

$$\begin{aligned}\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = {} & \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\ & + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc-2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} \\ & + \frac{(bc-ad)^5x}{3ab^5\,(a+bx^3)} - \frac{(bc-ad)^4(2bc+13ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}} \\ & + \frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} \\ & - \frac{(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}\end{aligned}$$

output
```
d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3
*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/1
0*d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)+1/9*(-a*d+b*c)^4*(13*a*d
+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(16/3)-1/18*(-a*d+b*c)^4*(13*a*d+2
*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(16/3)-1/9*(-a*d
+b*c)^4*(13*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(5/3)/b^(16/3)*3^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$$

$$= \frac{1260\sqrt[3]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+315b^{4/3}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4+180b^{7/3}d^4}{}$$

input
```
Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

output
```
(1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(
7/3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c -
 a*d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*A
rcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a
*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)
^4*(2*b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3
))/(1260*b^(16/3))
```

3.20.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^3x^3(3a^2d^2-10abcd+10b^2c^2)}{b^4}+\frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{5bdx^3(bc-ad)^4+(4ad+}{b^5(a+bx^3)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4(13ad+2bc)}{3\sqrt{3}a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}}+\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{3ab^5(a+bx^3)}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{d^5x^{10}}{10b^2}$$

input
```
Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a
*d)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b
*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 1
3*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*
b^(16/3))
```

3.20.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.95

method	result
risch	$\frac{d^5x^{10}}{10b^2}-\frac{2d^5a\,x^7}{7b^3}+\frac{5d^4c\,x^7}{7b^2}+\frac{3d^5a^2x^4}{4b^4}-\frac{5d^4ac\,x^4}{2b^3}+\frac{5d^3c^2x^4}{2b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5}{}$
default	$-\frac{d^2\left(-\frac{1}{10}d^3x^{10}b^3+\frac{2}{7}a\,b^2d^3x^7-\frac{5}{7}b^3c\,d^2x^7-\frac{3}{4}a^2b\,d^3x^4+\frac{5}{2}a\,b^2c\,d^2x^4-\frac{5}{2}b^3c^2d\,x^4+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+$

input
```
int((d*x^3+c)^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*d^5*x^10/b^2-2/7*d^5/b^3*a*x^7+5/7*d^4/b^2*c*x^7+3/4*d^5/b^4*a^2*x^4-
5/2*d^4/b^3*a*c*x^4+5/2*d^3/b^2*c^2*x^4-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x
-20*d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/3*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2
*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^3+a)+1/9/b
^6/a*sum((13*a^5*d^5-50*a^4*b*c*d^4+70*a^3*b^2*c^2*d^3-40*a^2*b^3*c^3*d^2+
5*a*b^4*c^4*d+2*b^5*c^5)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(275) = 550.

Time = 0.32 (sec) , antiderivative size = 1619, normalized size of antiderivative = 5.06

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^1
0 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*
(40*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d
^5)*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*
d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5
+ 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c
*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2
*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(
1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4
*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6
*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^
3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a
^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6
*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^
2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 42
0*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3
 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(12
6*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*
a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*...
```

3.20.6 Sympy [A] (verification not implemented)

Time = 127.22 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = x^7\left(-\frac{2ad^5}{7b^3}+\frac{5cd^4}{7b^2}\right)+x^4\cdot\left(\frac{3a^2d^5}{4b^4}-\frac{5acd^4}{2b^3}+\frac{5c^2d^3}{2b^2}\right)$$
$$+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)$$
$$+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{3a^2b^5+3ab^6x^3}$$
$$+\mathrm{RootSum}\Big(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^2c^2d^{13}+418280a^{12}b^3c^3d^{12}-874635$$
$$+\frac{d^5x^{10}}{10b^2}$$

input
```
integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

output
```
x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
 - 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15
*a**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d*
*5 + 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5
*a*b**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t
**3*a**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b
**2*c**2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**
11 + 1271886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*
a**8*b**7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d*
*6 + 7194*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*
b**12*c**12*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**1
5*c**15, Lambda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4
 + 70*a**3*b**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b
**5*c**5) + x))) + d**5*x**10/(10*b**2)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{3\,(ab^6x^3 + a^2b^5)}$$
$$+ \frac{14\,b^3d^5x^{10} + 20\,(5\,b^3cd^4 - 2\,ab^2d^5)x^7 + 35\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^4 + 140\,(10\,b^3c^3d^2 - 20\,ab^2}{140\,b^5}$$
$$+ \frac{\sqrt{3}(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 +
 20*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4
+ 3*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d
^4 - 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b
^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5
*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*
c*d^4 + 13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2
/3)) + 1/9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^
2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/
3))
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx =$$

$$-\frac{\sqrt{3}(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^4}$$
$$-\frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab^4}$$
$$-\frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^5}$$
$$+\frac{b^5c^5x-5\,ab^4c^4dx+10\,a^2b^3c^3d^2x-10\,a^3b^2c^2d^3x+5\,a^4bcd^4x-a^5d^5x}{3\,(bx^3+a)ab^5}$$
$$+\frac{14\,b^{18}d^5x^{10}+100\,b^{18}cd^4x^7-40\,ab^{17}d^5x^7+350\,b^{18}c^2d^3x^4-350\,ab^{17}cd^4x^4+105\,a^2b^{16}d^5x^4+1400\,b^{18}c^3d}{140\,b^{20}}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*
c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*
d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*
b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4
*b*c*d^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) +
 1/3*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*
d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*
d^5*x^10 + 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 -
 350*a*b^17*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*
a*b^17*c^2*d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$$
$$= x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)$$
$$-x^7\left(\frac{2\,a\,d^5}{7\,b^3}-\frac{5\,c\,d^4}{7\,b^2}\right)+x^4\left(\frac{a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{2\,b}-\frac{a^2\,d^5}{4\,b^4}+\frac{5\,c^2\,d^3}{2\,b^2}\right)+\frac{d^5\,x^{10}}{10\,b^2}$$
$$-\frac{x\,(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5)}{3\,a\,(b^6\,x^3+a\,b^5)}$$
$$+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^4\,(13\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{16/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^4\,(13\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{16/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^4\,(13\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{16/3}}$$

input
```
int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

output
```
x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2
*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b
^2) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3
 - (5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5
*x^10)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*
c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b
^(1/3)*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) -
 (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d
+ 2*b*c))/(9*a^(5/3)*b^(16/3))
```

3.20. $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2} dx$

3.21.1 Optimal result

Integrand size = 19, antiderivative size = 267

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2} dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} - \frac{2(bc-ad)^3(bc+5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}} - \frac{(bc-ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}$$

output
```
d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7
*d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)+2/9*(-a*d+b*c)^3*(5*a*d+b*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(13/3)-1/9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-2/9*(-a*d+b*c)^3*(
5*a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13
/3)*3^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$

$$= \frac{126\sqrt[3]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+63b^{4/3}d^3(2bc-ad)x^4+18b^{7/3}d^4x^7+\frac{42\sqrt[3]{b}(bc-ad)^4x}{a(a+bx^3)}+\frac{28\sqrt{3}(bc-ad)^3(bc+}{}}{126b^{13/}}$$

input
```
Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

output
```
(126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2
*b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a
+ b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Lo
g[a^(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

3.21.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^2(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{4bdx^3(bc-ad)^3+(3ad+bc)(bc-ad)^3}{b^4(a+bx^3)^2}+\frac{2d^3x^3(2bc-ad)}{b^3}+\frac{d^4x^6}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3(5ad+bc)}{3\sqrt{3}a^{5/3}b^{13/3}} - \frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}} + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{3ab^4(a+bx^3)} + \frac{d^3x^4(2bc-ad)}{2b^3} + \frac{d^4x^7}{7b^2}$$

input
```
Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

output
```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/
(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2
*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

3.21.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.93 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.82

method	result
risch	$\frac{d^4x^7}{7b^2} - \frac{d^4a\,x^4}{2b^3} + \frac{d^3c\,x^4}{b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{3a\,b^4(b\,x^3+a)} - \frac{2\left(\sum_{\_R=\text{RootO}}\right.}{2\left(5a^4d^4-14a^3bc\,d^3+\right.}$
default	$\frac{d^2\left(\frac{1}{7}b^2d^2x^7-\frac{1}{2}ab\,d^2x^4+b^2cd\,x^4+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{3a\left(b\,x^3+a\right)}+}{}$

input
```
int((d*x^3+c)^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*d^4*x^7/b^2-1/2*d^4/b^3*a*x^4+d^3/b^2*c*x^4+3*d^4/b^4*a^2*x-8*d^3/b^3*
a*c*x+6*d^2/b^2*c^2*x+1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)/a*x/b^4/(b*x^3+a)-2/9/b^5/a*sum((5*a^4*d^4-14*a^3*b*c*d^3+
12*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d-b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(224) = 448.

Time = 0.31 (sec) , antiderivative size = 1316, normalized size of antiderivative = 4.93

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 6
3*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/
3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3
- 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4
*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3
*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-
a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a
^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4
+ 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3
)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(
a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*
d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5
*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*
c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d
^4)*x)/(a^3*b^6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4
*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 +
5*a^5*b^2*d^4)*x^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*
b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*
d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^
2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*...
```

3.21.6 Sympy [A] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)$$
$$+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{3a^2b^4+3ab^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c^4\right.$$
$$+\frac{d^4x^7}{7b^2}$$

input
```
integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

output
```
x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b
**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + Root
Sum(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720
*a**10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*
d**8 - 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b*
*7*c**7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a*
*2*b**10*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*lo
g(-9*_t*a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.49

$$
\begin{aligned}
\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx
&= \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)x}{3\,(ab^5x^3+a^2b^4)} \\
&\quad + \frac{2\,b^2d^4x^7+7\,(2\,b^2cd^3-abd^4)x^4+14\,(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4)x}{14\,b^4} \\
&\quad + \frac{2\,\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad - \frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad + \frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}
$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4
)*x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*
(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
- 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a
^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(
b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)
*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$
$$= -\frac{2\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3} - \frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3} - \frac{2\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4} + \frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{3\left(bx^3+a\right)ab^4} + \frac{2\,b^{12}d^4x^7+14\,b^{12}cd^3x^4-7\,ab^{11}d^4x^4+84\,b^{12}c^2d^2x-112\,ab^{11}cd^3x+42\,a^2b^{10}d^4x}{14\,b^{14}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^
3 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*
b^2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14
*a^3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^
2)^(2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a
^3*b*c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4)
+ 1/3*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x
 + a^4*d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x
^4 - 7*a*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^
10*d^4*x)/b^14
```

3.21.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = x\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)+\frac{d^4\,x^7}{7\,b^2}+\frac{x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{3\,a\,(b^5\,x^3+a\,b^4)}-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$

input
```
int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

output
```
x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b
^2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*
b^4 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))
/(9*a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
- (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

3.22.1 Optimal result

Integrand size = 19, antiderivative size = 234

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)} - \frac{(bc-ad)^2(2bc+7ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a
)+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(10/3)-1/
18*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(5/3)/b^(10/3)-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}d^2(3bc-2ad)x+9b^{4/3}d^3x^4+\frac{12\sqrt[3]{b}(bc-ad)^3x}{a(a+bx^3)}+\frac{4\sqrt{3}(bc-ad)^2(2bc+7ad)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{4(bc-ad)^2(2bc+7}{}}{36b^{10/3}}$$

input
```
Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

output
```
(36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
 a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2
*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b
*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b
^(10/3))
```

3.22.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^3(bc-ad)^2+(bc-ad)^2(2ad+bc)}{b^3\,(a+bx^3)^2}+\frac{d^3x^3}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2(7ad+2bc)}{3\sqrt{3}a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3(a+bx^3)}+\frac{d^3x^4}{4b^2}$$

input
```
Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b
^3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d
)^2*(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^
(5/3)*b^(10/3))
```

3.22.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
risch	$\frac{d^3x^4}{4b^2} - \frac{2d^3ax}{b^3} + \frac{3d^2cx}{b^2} - \frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x}{3ab^3(bx^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3)\ln(x \cdots}{\_R^2}}{9b^4a} \cdots$
default	$-\frac{d^2\left(-\frac{1}{4}bdx^4+2adx-3bcx\right)}{b^3} + \frac{-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x}{3a(bx^3+a)} + \frac{(7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b} \cdots\right.\right.}{6b \cdots} \cdots\right.}{3a}}{b^3} \cdots$

input
```
int((d*x^3+c)^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^3+a)+1/9/b^4/a*sum((7*a^3*d^3-12*a^2*b*c*
d^2+3*a*b^2*c^2*d+2*b^3*c^3)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(193) = 386.

Time = 0.29 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.39

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sq
rt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3
+ (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*
sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt
(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a
^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b
^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*
a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3
*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*
c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(
2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 +
3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b
*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2
*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b...
```

3.22.6 Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{3a^2b^3+3ab^4x^3}$$
$$+\operatorname{RootSum}\Big(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a^4b$$
$$+\frac{d^3x^4}{4b^2}$$

input
```
integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**
3*a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d*
*7 + 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c*
*5*d**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**
8*d - 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2
*b*c*d**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{3\,(ab^4x^3+a^2b^3)} + \frac{bd^3x^4+4\,(3\,bcd^2-2\,ad^3)x}{4\,b^3} + \frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2
*b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b
^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a
*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^
2 + 7*a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(bx^3+a)ab^3} + \frac{b^6d^3x^4+12\,b^6cd^2x-8\,ab^5d^3x}{4\,b^8}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) -
 1/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*
a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a
^3*d^3*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^
5*d^3*x)/b^8
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{d^3\,x^4}{4\,b^2} - x\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{3\,a\,(b^4\,x^3+a\,b^3)} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}}$$

input
```
int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

output
```
(d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/
3)*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i +
2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c
))/(9*a^(5/3)*b^(10/3))
```

3.22. $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2} \, dx$

3.23.1 Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2} \, dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{3ab^2(a+bx^3)} - \frac{2(bc-ad)(bc+2ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}$$

output
```
d^2*x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)+2/9*(-a*d+b*c)*(2*a*d+b*c)*ln
(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/9*(-a*d+b*c)*(2*a*d+b*c)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-2/9*(-a*d+b*c)*(2*a*d+b*c)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$= \frac{9\sqrt[3]{b}d^2x + \frac{3\sqrt[3]{b}(bc-ad)^2x}{a(a+bx^3)} - \frac{2\sqrt{3}(b^2c^2+abcd-2a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2(b^2c^2+abcd-2a^2d^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{(b^2c^2+abcd}{}}{9b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

output
```
(9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3
]*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x]
)/a^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

3.23.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{-a^2d^2+2bdx^3(bc-ad)+b^2c^2}{b^2(a+bx^3)^2}+\frac{d^2}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)(2ad+bc)}{3\sqrt{3}a^{5/3}b^{7/3}}-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2(a+bx^3)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*
c + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(
9*a^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

3.23.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3a\,b^2(b\,x^3+a)}-\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2a^2d^2-abcd-b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^3a}$
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{3a\left(b\,x^3+a\right)}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}}{b^2}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^3+a)-2/9/b^3/a*sum(
(2*a^2*d^2-a*b*c*d-b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.

Time = 0.30 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{9\,a^3b^2d^2x^4-3\,\sqrt{\tfrac{1}{3}}(a^2b^3c^2+a^3b^2cd-2\,a^4bd^2+(ab^4c^2+a^2b^3cd-2\,a^3b^2d^2)x^3)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a\right.}{}}{}\right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b
*d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 +
(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a))
- (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)
*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
2*(a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)
*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*
b^2*c*d + 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4
+ 6*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*
b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2
*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-
a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2
*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-
a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3)]
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{3a^2b^2+3ab^3x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6\right.$$
$$+\frac{d^2x}{b^2}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + Root
Sum(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c
**2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**
5*d - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*
c*d - 2*b**2*c**2) + x))) + d**2*x/b**2
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(ab^3x^3+a^2b^2)} + \frac{d^2x}{b^2} + \frac{2\,\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 +
2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2
*d^2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^
2*c^2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2x}{b^2} - \frac{2\,\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(bx^3+a)ab^2}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 +
a*b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2
/3)*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3
 + a)*a*b^2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{3\,a\,(b^3\,x^3 + a\,b^2)} - \frac{2\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(a\,d - b\,c)\,(2\,a\,d + b\,c)}{9\,a^{5/3}\,b^{7/3}} - \frac{2\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d - b\,c)\,(2\,a\,d + b\,c)}{9\,a^{5/3}\,b^{7/3}} + \frac{2\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d - b\,c)\,(2\,a\,d + b\,c)}{9\,a^{5/3}\,b^{7/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3))
- (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3
)) - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/
3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d +
 b*c))/(9*a^(5/3)*b^(7/3))
```

3.24 $\int \frac{c+dx^3}{(a+bx^3)^2} dx$

3.24.1 Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{c+dx^3}{(a+bx^3)^2} dx = \frac{(bc-ad)x}{3ab(a+bx^3)} - \frac{(2bc+ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

output
```
1/3*(-a*d+b*c)*x/a/b/(b*x^3+a)+1/9*(a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/
3)/b^(4/3)-1/18*(a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5
/3)/b^(4/3)-1/9*(a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(5/3)/b^(4/3)*3^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6a^{2/3}\sqrt[3]{b}(-bc+ad)x}{a+bx^3} - 2\sqrt{3}(2bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (2bc+ad)\log\left(\right.}{18a^{5/3}b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

output
```
((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/
3) + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(5/3)*b^(4/3))
```

3.24.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+2bc)\int \frac{1}{bx^3+a}dx}{3ab} + \frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 750$$

$$\frac{(ad+2bc)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3ab} + \frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 16$$

$$\frac{(ad+2bc)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

3.24. $\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx$

$$\frac{(ad+2bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{(ad+2bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{(ad+2bc)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

output
```
((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) + ((2*b*c + a*d)*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a*b)
```

3.24. $\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$

3.24.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(ad+2bc)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{(ad+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

input
```
int((d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d-b*c)/b/a*x/(b*x^3+a)+1/9/a/b^2*sum((a*d+2*b*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}(2a^2b^2c+a^3bd+(2ab^3c+a^2b^2d)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)}{bx^3+a}\right.}{}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*s
qrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(
1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b
))/(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^
3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2
*c - a^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c +
 a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b
)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^
3*b^3*x^3 + a^4*b^2)]
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{x(-ad+bc)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4-a^3d^3-6a^2bcd^2-12ab^2c^2d-8b^3c^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 -
a**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _
t*log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,bc+ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,bc+ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*
b*c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/
9*(2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(2\,bc+ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx-adx}{3\,(bx^3+a)ab}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}} - \frac{x\,\left(a\,d-b\,c\right)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

input
```
int((c + d*x^3)/(a + b*x^3)^2,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c)
)/(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))
/(3*a*b*(a + b*x^3))
```

3.25 $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

3.25.1 Optimal result

Integrand size = 19, antiderivative size = 346

$$\begin{aligned}\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^3)}-\frac{b^{2/3}(2bc-5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}\\&-\frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2}\\&+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2}\\&-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}\\&-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}\end{aligned}$$

output
```
1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)+1/9*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(1/3)+b^(1/
3)*x)/a^(5/3)/(-a*d+b*c)^2+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d
+b*c)^2-1/18*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(5/3)/(-a*d+b*c)^2-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*
x^2)/c^(2/3)/(-a*d+b*c)^2-1/9*b^(2/3)*(-5*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^2*3^(1/2)-1/3*d^(5/3)*arct
an(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)^2*3^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$$

$$= \frac{6a^{2/3}bc^{2/3}(bc-ad)x-2\sqrt{3}b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\sqrt{3}a^{5/3}d^{5/3}(a+bx^3)\text{arcta}}{}$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

output
```
(6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*
a*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a
^(5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2
*b^(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*
a^(5/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*
b*c - 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
3*a^(5/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2])/(18*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

3.25.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {931, 25, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{3a(a+bx^3)(bc-ad)}-\frac{\int -\frac{2bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)}dx}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 1020

$$\frac{\frac{3ad^2\int\frac{1}{dx^3+c}dx}{bc-ad}+\frac{b(2bc-5ad)\int\frac{1}{bx^3+a}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 750

$$\frac{\frac{b(2bc-5ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 16

$$\frac{\frac{b(2bc-5ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 1142

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/}\cdots}}{\cdots}\cdots\right.}{\cdots}}{3a(bc-ad)}$$
$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 25

3.25. $\int\frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}dx$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{3c^{2/3}}\cdots\right)}{\cdots}}{3a(bc-ad)}+$$
$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{\cdots}\cdots\right)}{\cdots}}{3a(bc-ad)}+$$
$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 1082

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}d\cdots}{\cdots}\cdots\right)}{\cdots}}{3a(bc-ad)}+$$
$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\Big\downarrow$ 217

3.25. $\displaystyle\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\cdots\right)}{\cdots}}{3c^{2/3}}\cdots\right)}{bc-ad}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$\big\downarrow$ 1103

$$\frac{\frac{b(2bc-5ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\cdots\right)}{2\sqrt[3]{\cdots}}}{3c^{2/3}}\cdots\right)}{bc-ad}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) + ((b*(2*b*c - 5*a*d)*(Log[a^(1/3) + b
^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) + (3*a*d^2*(Log[c^(1/3) + d^(1/3)*x]
/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt
[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)
))/(3*c^(2/3))))/(b*c - a*d))/(3*a*(b*c - a*d))
```

3.25. $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.25.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{2}}{(ad-bc)^{2}}-\frac{b\left(\frac{(ad-bc)x}{3a\left(b\,x^{3}+a\right)}+\frac{(5ad-2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\right.}{}\right.}{}\right.}{(ad-}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d^2/(a*d-b*c)^2-1/(a*d-b*c)^2*b*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+1/3*(
5*a*d-2*b*c)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}((2b^2c-5abd)x^3+2abc-5a^2d)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}(abdx^3+a^2d)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}}{}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(
3)*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)
^(2/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b
^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3))
 + 3*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/
3) + c^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d
)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(
d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^
2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^
3)
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

output
```
Timed out
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \frac{\sqrt{3}(2\,bc-5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{bx}{3\left(a^2bc-a^3d+(ab^2c-a^2bd)x^3\right)} - \frac{(2\,bc-5\,ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{(2\,bc-5\,ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1
/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^
(1/3))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3)
- 1/18*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(
a/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2
/3) + a^2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b
^2*c^2*(a/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*
d*log(x + (c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*
d^2*(c/d)^(2/3))
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx =& -\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^3-2\,abc^2d+a^2cd^2)} \\ &+ \frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\,\sqrt{3}abc^2d+\sqrt{3}a^2cd^2} \\ &+ \frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^3-2\,abc^2d+a^2cd^2)} \\ &- \frac{\left(2\,b^2c-5\,abd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)} \\ &+ \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^2c^2-2\,\sqrt{3}a^3bcd+\sqrt{3}a^4d^2\right)} \\ &+ \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)} \\ &+ \frac{bx}{3\,(bx^3+a)(abc-a^2d)}\end{aligned}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/
6*(-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3
)*b*c - 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) +
1/18*(2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3
+ a)*(a*b*c - a^2*d))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 20.56 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.20

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

output
```
log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5
*(a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/
3))/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^
2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^
(1/3))/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b
^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*
c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*
d^4 - 4374*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a
^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c
)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 -
(b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^
4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^
3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4
))*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6
*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3)
 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^
3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^...
```

3.25. $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

3.26 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

3.26.1 Optimal result

Integrand size = 19, antiderivative size = 419

$$
\begin{aligned}
\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx =& \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} \\
&- \frac{2b^{5/3}(bc-4ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} \\
&- \frac{2d^{5/3}(4bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} \\
&+ \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} \\
&+ \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} \\
&- \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} \\
&- \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}
\end{aligned}
$$

output
```
1/3*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+1/3*b*x/a/(-a*d+b*c)/(b*x^3+a
)/(d*x^3+c)+2/9*b^(5/3)*(-4*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b
*c)^3+2/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^3-
1/9*b^(5/3)*(-4*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
/(-a*d+b*c)^3-1/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/c^(5/3)/(-a*d+b*c)^3-2/9*b^(5/3)*(-4*a*d+b*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^3*3^(1/2)-2/9*d^(5/3)*(-a*
d+4*b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b
*c)^3*3^(1/2)
```

3.26. $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

3.26.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \frac{1}{9}\left(\frac{3b^2x}{a(bc-ad)^2(a+bx^3)} + \frac{3d^2x}{c(bc-ad)^2(c+dx^3)} + \frac{2\sqrt{3}b^{5/3}(bc-4ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(-bc+ad)^3} + \frac{2\sqrt{3}d^{5/3}(-4bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(-bc+4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(-bc+ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}(bc-ad)^3} + \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(-bc+ad)^3} + \frac{d^{5/3}(-4bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

output
```
((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
 d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) +
 (2*b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) +
a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c
 - a*d)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9
```

3.26.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {931, 25, 1024, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}-\frac{\int -\frac{5bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)^2}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)^2}dx}{3a(bc-ad)}+\frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{\int \frac{6\left(bd(bc+ad)x^3+b^2c^2+a^2d^2-3abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{3c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2-3abcd}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 1020}$$

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\int\frac{1}{bx^3+a}dx}{bc-ad}+\frac{ad^2(4bc-ad)\int\frac{1}{dx^3+c}dx}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\big\downarrow$ 750

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\big\downarrow$ 16

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\big\downarrow$ 1142

3.26. $\int\frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}dx$

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}\cdots}\cdots\right)}{\cdots}\right)}{c(bc-ad)}}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x\cdots}\cdots\right)}{\cdots}\right)}{c(bc-ad)}}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\Big\downarrow$ 27

3.26. $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}\cdots}\cdots\right.}{\cdots}\cdots\right.}{c(bc-ad)}\cdots}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x\cdots}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{c(bc-ad)}\cdots}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{ 217}$$

3.26. $\displaystyle\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\,\mathrm{a}\cdots}{\cdots}}{3c^{2/3}}\cdots\right)}{bc-ad}\cdots\right)}{c(bc-ad)}\cdots}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}\cdots\right)}{\cdots}}{3c^{2/3}}\cdots\right)}{bc-ad}\cdots\right)}{c(bc-ad)}\cdots}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

3.26. $\int\frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^3)) + (2*((b^2*c*(b*c - 4*a*d)*(Log[a^(1/3) + b^(1/3)*x
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(2/3))))/(b*c - a*d) + (a*d^2*(4*b*c - a*d)*(Log[c^(1/3) + d^(1/3
)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/
Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(
1/3)))/(3*c^(2/3))))/(b*c - a*d)))/(c*(b*c - a*d)))/(3*a*(b*c - a*d))
```

3.26.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.26. $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

3.26.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.68

method	result
default	$\frac{d^2\left(\frac{(ad-bc)x}{3c(d\,x^3+c)}+\frac{2(ad-4bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^3}+\frac{b^2\left(\frac{(ad-bc)x}{3a(b\,x^3+a)}+\frac{2(4ad-}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(a*d-b*c)^3*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+2/3*(a*d-4*b*c)/c*(1/3/d/(c/d
)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/
3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+b^
2/(a*d-b*c)^3*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+2/3*(4*a*d-b*c)/a*(1/3/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(341) = 682$.

Time = 52.87 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.14

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$$

$$= \frac{3\left(b^3c^2d - a^2bd^3\right)x^4 + 2\sqrt{3}\left(\left(b^3c^2d - 4\,ab^2cd^2\right)x^6 + ab^2c^3 - 4\,a^2bc^2d + \left(b^3c^3 - 3\,ab^2c^2d - 4\,a^2bcd^2\right)x^3\right)\left(\frac{b^2}{a^2}\right.}{}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/9*(3*(b^3*c^2*d - a^2*b*d^3)*x^4 + 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2
)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d
^2)*x^3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(
3)*b)/b) + 2*sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^
3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - ((b^3*c^2*d - 4
*a*b^2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d -
 4*a^2*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) +
 a^2*(b^2/a^2)^(2/3)) - ((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d -
 a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3
)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*((b^3*c^2
*d - 4*a*b^2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c
^2*d - 4*a^2*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) +
2*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2
)^(1/3)) + 3*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c
^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^
2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^3)
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

output
```
Timed out
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(341) = 682.

Time = 0.30 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.87

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2}\,dx$$
$$= \frac{2\sqrt{3}(b^2c-4\,abd)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{2\sqrt{3}(4\,bcd-ad^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$
$$-\frac{(b^2c-4\,abd)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$
$$-\frac{(4\,bcd-ad^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$
$$+\frac{2\,(b^2c-4\,abd)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$
$$+\frac{2\,(4\,bcd-ad^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$
$$+\frac{(b^2cd+abd^2)x^4+(b^2c^2+a^2d^2)x}{3\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^6+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*
d^2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d
 - a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c
/d)^(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*
c*d^3*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3)
+ 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)
*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3
*d*(c/d)^(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/
9*(b^2*c - 4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^
2*c^2*d*(a/b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2
/9*(4*b*c*d - a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c
^3*d*(c/d)^(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) +
1/3*((b^2*c*d + a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3
*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x
^6 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

3.26. $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

3.26.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2}\,dx = -\frac{2\,(b^3c-4\,ab^2d)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)} - \frac{2\,(4\,bcd^2-ad^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)} + \frac{2\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^3c^3-3\,\sqrt{3}a^3b^2c^2d+3\,\sqrt{3}a^4bcd^2-\sqrt{3}a^5d^3\right)} + \frac{2\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^3c^5-3\,\sqrt{3}ab^2c^4d+3\,\sqrt{3}a^2bc^3d^2-\sqrt{3}a^3c^2d^3\right)} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)} + \frac{\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)} + \frac{b^2cdx^4+abd^2x^4+b^2c^2x+a^2d^2x}{3\,(bdx^6+bcx^3+adx^3+ac)(ab^2c^3-2\,a^2bc^2d+a^3cd^2)}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*
c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*
b*c^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*
b*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^
3*c^3 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3)
 + 2/3*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*
d + 3*sqrt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b
^2*c - 4*(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^
(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3
))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*
d*x^4 + a*b*d^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3
 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 28.74 (sec) , antiderivative size = 3637, normalized size of antiderivative = 8.68

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
4*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d +
b*c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c
^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a
*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*
d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6
 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5
*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3
)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 2
68*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*
c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^
3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^
9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3
 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6
 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^
3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(
c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/
3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c
^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*...
```

3.26. $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

3.27 $\int (a-bx^3) (a+bx^3)^{2/3} \, dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (a-bx^3) (a+bx^3)^{2/3} \, dx = \frac{7}{18} ax (a+bx^3)^{2/3} - \frac{1}{6} x (a+bx^3)^{5/3} + \frac{7a^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{7a^2 \log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

output
```
7/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(b*x^3+a)^(5/3)-7/18*a^2*ln(-b^(1/3)*x+(b*x
^3+a)^(1/3))/b^(1/3)+7/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/b^(1/3)*3^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int (a-bx^3) (a+bx^3)^{2/3} \, dx = \frac{3\sqrt[3]{b}(a+bx^3)^{2/3}(4ax-3bx^4) + 14\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 14a^2 \log\left(-\sqrt[3]{b}x+\sqrt[3]{a\right.}}{54\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(1/3)*(a + b*x^3)^(2/3)*(4*a*x - 3*b*x^4) + 14*Sqrt[3]*a^2*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 14*a^2*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] + 7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

3.27.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{7}{6}a\int \left(bx^3+a\right)^{2/3}dx-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{748}$$

$$\frac{7}{6}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{7}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

input
```
Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

output
```
-1/6*(x*(a + b*x^3)^(5/3)) + (7*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b
^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6
```

3.27.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.27.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{-9x^{4}\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}}+12ax\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}-14a^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-14a^{2}\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+7a^{2}\ln}{54b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-9*x^4*(b*x^3+a)^(2/3)*b^(4/3)+12*a*x*(b*x^3+a)^(2/3)*b^(1/3)-14*a^2
*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-14*a^
2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+7*a^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(1/3)
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.56

$$\int (a-bx^3)\,(a+bx^3)^{2/3}\,dx = \left[\frac{21\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+\right.\right.}{}\right.$$

$$-\frac{42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+14\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-7\,a^2}{54\,b}$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(
1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x
^2) - 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^
2*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1
/3))*sqrt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3
 + a)^(1/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 +
 a)^(2/3))/b]
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx=\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

output
```
a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.88

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx=$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3\right.}{}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*ar
ctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*
log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3)
 + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3
)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 +
(b*x^3 + a)^2*b/x^6))*b
```

3.27.8 Giac [F]

$$\int \left(a-bx^3\right) \left(a+bx^3\right)^{2/3} \, dx = \int -\left(bx^3+a\right)^{\frac{2}{3}} \left(bx^3-a\right) \, dx$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right) \left(a+bx^3\right)^{2/3} \, dx = \int \left(b\,x^3+a\right)^{2/3} \left(a-b\,x^3\right) \, dx$$

input
```
int((a + b*x^3)^(2/3)*(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

output
```
-1/3*x*(b*x^3+a)^(2/3)-2/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+4/9*a*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{-3\sqrt[3]{b}x(a+bx^3)^{2/3}+4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-4a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+2a\log\left(b^{2/3}x^2+\right.}{9\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + 2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)])/(9*b^(1/3))
```

3.28.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{4}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{1}{3}x\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{4}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{1}{3}x\left(a+bx^3\right)^{2/3}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
-1/3*(x*(a + b*x^3)^(2/3)) + (4*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(
2*b^(1/3))))/3
```

3.28.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.28.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{4\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a+\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}}x\,b^{\frac{1}{3}}}{4}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}\right.}{x^{2}}\right.}{2}\right.}{9b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*
a+3/4*(b*x^3+a)^(2/3)*x*b^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-1/2*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(1/3)
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.99

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \left[\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3 - (bx^3+a)^{\frac{1}{3}}bx^2 + 2\,(bx^3+a)^{\frac{2}{3}}(-\right.\right.}{} \right.$$
$$-\frac{12\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) + 3\,(bx^3+a)^{\frac{2}{3}}bx + 4\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x}{}\right.}{9\,b}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)
*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2
+ 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 +
 a)^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) +
 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*
x^3 + a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan
(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) +
3*(b*x^3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(
1/3))/x) - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2))/b]
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

output
```
a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.68

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

3.28.8 Giac [F]

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{a-b\,x^3}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((a - b*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

3.29 $\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
2*x/(b*x^3+a)^(1/3)+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)-1/3*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(2*x)/(a + b*x^3)^(1/3) - ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b
*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3
*b^(1/3)) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2
/3)]/(6*b^(1/3))
```

3.29.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{2x}{\sqrt[3]{a+bx^3}}-\int \frac{1}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{ 769}$$

$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)
```

3.29.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 3.93 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)}{x^{2}}\right.}{2}}{3 b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1
/3)/x)*(b*x^3+a)^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*(b*x^3+a)^(1/3)-
1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*(b*x^3
+a)^(1/3)+6*b^(1/3)*x)/(b*x^3+a)^(1/3)
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.38

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(b^2x^3+ab)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2\right.\right.}{}\right.$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3
+ a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^
2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a
)^(2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2
*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2) + 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x
^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

3.29.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

output
```
x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(
4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{1}{6}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{\left(bx^3+a\right)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+\right.}{x^2}\right.}{b^{\frac{4}{3}}}\right.$$

$$+\frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)
*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)
^(1/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

3.29.8 Giac [F]

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx = \int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{a-b\,x^3}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((a - b*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

3.30 $\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}$$

output
```
1/4*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+3/4*x/a/(b*x^3+a)^(1/3)
```

3.30.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{2ax+bx^4}{2a\,(a+bx^3)^{4/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(2*a*x + b*x^4)/(2*a*(a + b*x^3)^(4/3))
```

3.30.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}$$

$$\Big\downarrow \; 746$$

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

3.30.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.30.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25
trager	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25
pseudoelliptic	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25

input
```
int((-b*x^3+a)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{(bx^4+2\,ax)(bx^3+a)^{\frac{2}{3}}}{2\,(ab^2x^6+2\,a^2bx^3+a^3)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(37) = 74$.

Time = 19.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx = a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) - \frac{bx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

output
```
a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)
*gamma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx = -\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a} - \frac{bx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
 + a)^(4/3)*a)
```

3.30.8 Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{x\,(b\,x^3+a)+a\,x}{2\,a\,(b\,x^3+a)^{4/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(7/3),x)
```

output
```
(x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

3.31 $\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx$

3.31.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx=\frac{2x}{7(a+bx^3)^{7/3}}+\frac{5x}{28a(a+bx^3)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

output
```
2/7*x/(b*x^3+a)^(7/3)+5/28*x/a/(b*x^3+a)^(4/3)+15/28*x/a^2/(b*x^3+a)^(1/3)
```

3.31.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx=\frac{28a^2x+35abx^4+15b^2x^7}{28a^2(a+bx^3)^{7/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(28*a^2*x + 35*a*b*x^4 + 15*b^2*x^7)/(28*a^2*(a + b*x^3)^(7/3))
```

3.31.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{5}{7}\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{5}{7}\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{5}{7}\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(2*x)/(7*(a + b*x^3)^(7/3)) + (5*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2
*(a + b*x^3)^(1/3))))/7
```

3.31.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.31.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37
trager	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37
pseudoelliptic	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37

input
```
int((-b*x^3+a)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(15\,b^2x^7+35\,abx^4+28\,a^2x)(bx^3+a)^{\frac{2}{3}}}{28\,(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
 3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(49) = 98$.

Time = 100.31 (sec) , antiderivative size = 709, normalized size of antiderivative = 12.89

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = a\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+\cdots}\right.$$

$$+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\cdots\right.}$$

$$+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\cdots\right.}$$

$$+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\cdots\right.}$$

$$-b\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right.$$

$$\left.+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

output
```
a*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) - b*(7*a*x**4*gamma(4/3)/(9*
a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gam
ma(10/3)) + 3*b*x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(1
0/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*
b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,\left(bx^3+a\right)}{x^3}\right)bx^7}{28\,\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,\left(bx^3+a\right)b}{x^3}+\frac{14\,\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\,\left(bx^3+a\right)^{\frac{7}{3}}a^2}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

3.31.8 Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{15\,x\,(b\,x^3+a)^2+8\,a^2\,x+5\,a\,x\,(b\,x^3+a)}{28\,a^2\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(10/3),x)
```

output
```
(15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/
3))
```

3.32 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

3.32.1 Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

output
```
1/5*x/(b*x^3+a)^(10/3)+4/35*x/a/(b*x^3+a)^(7/3)+6/35*x/a^2/(b*x^3+a)^(4/3)
+18/35*x/a^3/(b*x^3+a)^(1/3)
```

3.32.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{35a^3x+70a^2bx^4+60ab^2x^7+18b^3x^{10}}{35a^3\left(a+bx^3\right)^{10/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
(35*a^3*x + 70*a^2*b*x^4 + 60*a*b^2*x^7 + 18*b^3*x^10)/(35*a^3*(a + b*x^3)
^(10/3))
```

3.32.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{4}{5}\int \frac{1}{(bx^3+a)^{10/3}}dx+\frac{x}{5(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{4}{5}\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{x}{5(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{4}{5}\left(\frac{6\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{x}{5(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{4}{5}\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{x}{5(a+bx^3)^{10/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
x/(5*(a + b*x^3)^(10/3)) + (4*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a +
 b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/5
```

3.32.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.32.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3)}{35(b\,x^3+a)^{\frac{10}{3}}a^3}$	48
trager	$\frac{x(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3)}{35(b\,x^3+a)^{\frac{10}{3}}a^3}$	48
pseudoelliptic	$\frac{x(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3)}{35(b\,x^3+a)^{\frac{10}{3}}a^3}$	48

input
```
int((-b*x^3+a)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x)(bx^3+a)^{\frac{2}{3}}}{35\,(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2
/3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x
^3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3
+ a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

3.32.8 Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x}{5\,(b\,x^3+a)^{10/3}} + \frac{18\,x}{35\,a^3\,(b\,x^3+a)^{1/3}} + \frac{6\,x}{35\,a^2\,(b\,x^3+a)^{4/3}} + \frac{4\,x}{35\,a\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

output
```
x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a
^2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

3.33 $\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx=\frac{2x}{13(a+bx^3)^{13/3}}+\frac{11x}{130a(a+bx^3)^{10/3}}+\frac{99x}{910a^2(a+bx^3)^{7/3}}+\frac{297x}{1820a^3(a+bx^3)^{4/3}}+\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}$$

output
```
2/13*x/(b*x^3+a)^(13/3)+11/130*x/a/(b*x^3+a)^(10/3)+99/910*x/a^2/(b*x^3+a)
^(7/3)+297/1820*x/a^3/(b*x^3+a)^(4/3)+891/1820*x/a^4/(b*x^3+a)^(1/3)
```

3.33.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx=\frac{x(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12})}{1820a^4(a+bx^3)^{13/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

output
```
(x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^
4*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

3.33.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{11}{13}\int \frac{1}{\left(bx^3+a\right)^{13/3}}dx+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{11}{13}\left(\frac{9\int \frac{1}{\left(bx^3+a\right)^{10/3}}dx}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{11}{13}\left(\frac{9\left(\frac{6\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{11}{13}\left(\frac{9\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{11}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

output
```
(2*x)/(13*(a + b*x^3)^(13/3)) + (11*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(
7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a +
 b*x^3)^(1/3))))/(7*a)))/(10*a)))/13
```

3.33.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.33.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^4}$	59
trager	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^4}$	59
pseudoelliptic	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^4}$	59

input
```
int((-b*x^3+a)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820
*a^4)/(b*x^3+a)^(13/3)/a^4
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x)(bx^3+a)^{\frac{2}{3}}}{1820\,(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
 + 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b
^3*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(73) = 146.

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.65

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^4}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(
b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 +
 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

3.33.8 Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2\,x}{13\,{\left(b\,x^3+a\right)}^{13/3}}+\frac{891\,x}{1820\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}+\frac{297\,x}{1820\,a^3\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{99\,x}{910\,a^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{11\,x}{130\,a\,{\left(b\,x^3+a\right)}^{10/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

output
```
(2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (29
7*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (
11*x)/(130*a*(a + b*x^3)^(10/3))
```

3.34 $\int \frac{(a+bx^3)^{7/3}}{a-bx^3} dx$

3.34.1 Optimal result

Integrand size = 22, antiderivative size = 483

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\frac{7}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{4\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}} - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{2\sqrt[3]{2}a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2}a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

output
```
-7/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(b*x^3+a)^(4/3)-7/5*a^2*x*(1+b*x^3/a)^(2/3)
*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2/3*2^(1/3)*a^(5/3)*
ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+2/3*2^(1/3)*a^(5/
3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)-4/3*2^(1/3)*a^(5/3)*ln(1+2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(5/3)*ln(2*2^(1/3)+(a^(
1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))/b^(1/3)-4/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)-2/3*2^(1/3)*a^(5/3)*arctan(1/
3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.34.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \frac{27abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+4x\left(-8a^2-9abx^3-b^2x^6+\frac{}{(a-b}\right.}{20\,(a+}$$

input
```
Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

output
```
(27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)
/a])))))/(20*(a + b*x^3)^(2/3))
```

3.34.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.16, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {933, 27, 1025, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$$

$$\Big\downarrow \; 933$$

$$-\frac{\int -\frac{2ab\sqrt[3]{bx^3+a}(7bx^3+3a)}{a-bx^3}dx}{5b}-\frac{1}{5}x(a+bx^3)^{4/3}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}a\int \frac{\sqrt[3]{bx^3+a}(7bx^3+3a)}{a-bx^3}dx-\frac{1}{5}x(a+bx^3)^{4/3}$$

$$\Big\downarrow \; 1025$$

$$\frac{2}{5}a\left(-\frac{\int -\frac{ab(27bx^3+13a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 25

$$\frac{2}{5}a\left(\frac{\int \frac{ab(27bx^3+13a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 27

$$\frac{2}{5}a\left(\frac{1}{2}a\int \frac{27bx^3+13a}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 1026

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx-27\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 779

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx-\frac{27\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 778

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\frac{7}{2}x\sqrt[3]{a+b\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 928

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\frac{7}{2}\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\big\downarrow$ 779

3.34. $\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3}\right.}{\left(a+bx^3\right)^{2/3}}\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hyper}}{(a}\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{927}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeo}}{2a\,(a}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{982}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{821}$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)\right)}{2a^2}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow 16$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\cdots\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow 1142$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+\cdots}}{\cdots}\right.\right.}{\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(bx^3+a\right)^{4/3}$$

$\downarrow$ 25

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+}\right.\right.}{}\right.\right.\right.$$

$$\frac{1}{5}x\left(bx^3+a\right)^{4/3}$$

$$\downarrow 27$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 1082

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{3\int -\frac{\frac{1}{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}}{\frac{}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \; 217$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log \cdots}{\cdots}\right.\right.}{\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\downarrow 1103$$

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\sqrt[3]{a+bx^3}}\cdots\right.}{3\ 2^{2/3}a^{2/3}}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

input
```
Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

output
```
-1/5*(x*(a + b*x^3)^(4/3)) + (2*a*((-7*x*(a + b*x^3)^(1/3))/2 + (a*((-27*x
*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a
+ b*x^3)^(2/3) + 40*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3
, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan
[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1
/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3
) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))
)/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3
)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(
a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3)
 + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b
^(1/3)))))/2))/5
```

3.34. $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

3.34.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

3.34. $\int \frac{(a+bx^3)^{7/3}}{a-bx^3} \, dx$

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.34. $\int \frac{(a+bx^3)^{7/3}}{a-bx^3} dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.34.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{7}{3}}}{-b\,x^{3}+a}dx$$

input
```
int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

3.34.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.34.6 Sympy [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\int \frac{a^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{b^2x^6\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{2abx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
```

output
```
-Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*
(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**
(1/3)/(-a + b*x**3), x)
```

3.34.7 Maxima [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

3.34.8 Giac [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{7/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(7/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(7/3)/(a - b*x^3), x)
```

3.35 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} dx$

3.35.1 Optimal result

Integrand size = 22, antiderivative size = 464

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{2\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$
$$-\frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$
$$-\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$
$$-\frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$+\frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$-\frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$+\frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{b}}$$

output
```
-1/2*x*(b*x^3+a)^(1/3)-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3
],-b*x^3/a)/(b*x^3+a)^(2/3)-1/3*2^(1/3)*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/
3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b
^(1/3)-2/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3
))/b^(1/3)+1/6*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+
2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(1/3)-2/3*2^(1/3)*a
^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2
))/b^(1/3)*3^(1/2)-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.35.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \frac{x\left(-4(a+bx^3)+5bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{\cdots}{(a-bx^3)\left(4a\,\text{AppellF1}\cdots\right.}\right.}{8\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

output
```
(x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7
/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3
)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/
a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(8*(a + b*
x^3)^(2/3))
```

3.35.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {933, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.35. $\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx$

$$\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}dx$$

$\big\downarrow$ 933

$$-\frac{\int -\frac{ab\left(5bx^3+3a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b}-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 25

$$\frac{\int \frac{ab\left(5bx^3+3a\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{2b}-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 27

$$\frac{1}{2}a\int \frac{5bx^3+3a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 1026

$$\frac{1}{2}a\left(8a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-5\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 779

$$\frac{1}{2}a\left(8a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{5\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 778

$$\frac{1}{2}a\left(8a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 928

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\big\downarrow$ 779

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}dx$

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{778}$$

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeomet}}{\left(a+bx^3\right)^{2}}\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{927}$$

$$\frac{1}{2}a\left(8a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric}}{2a\left(a+bx^3\right)^{2}}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{982}$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+}{\sqrt[3]{a}\sqrt[3]{bx}}\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{821}$$

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\cdots}{2a^{2/3}\sqrt[3]{b}}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow 16$$

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \cdots}{\cdots}\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \ 1142$$

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\Bigg(8a\Bigg(\frac{9\Bigg(\frac{2}{9}\Bigg(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{}}{\sqrt[3]{a}}\cdots}{2\sqrt[3]{2}\sqrt[3]{a}}\cdots}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots}{\cdots}\cdots$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \; 25$$

3.35. $\int\frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{\cdots}}{\sqrt[3]{a}\sqrt[3]{\cdots}}\cdots}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 27

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{}{\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)\right)}{}\right)\right)$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 1082

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}} \cdots$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \; 217$$

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

$$\frac{1}{2}a\left(8a\frac{\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\cdots\right.}{\sqrt[3]{\cdots}}\right.}{3\cdots}\right.\right.\right.}{\cdots}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 1103

3.35. $\int\frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}dx$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right.\right.}{2a}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

input
```
Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

output
```
-1/2*(x*(a + b*x^3)^(1/3)) + (a*((-5*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometri
c2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 8*a*((x*(1 + (b*x^3
)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3
)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x
))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(
1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-
1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3
)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/
3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3
)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/
(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/2
```

3.35. $\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

3.35. $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} \, dx$

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.35. $\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx$

3.35.4 Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{-b\,x^3+a}dx$$

input
```
int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

3.35.5 Fricas [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
integral(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

3.35.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx=-\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
```

output
```
-Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b
*x**3)**(1/3)/(-a + b*x**3), x)
```

3.35.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

3.35.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)/(a - b*x^3), x)
```

3.36 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

3.36.1 Optimal result

Integrand size = 22, antiderivative size = 398

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

output
```
-1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1
/3)+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*ln(1+2^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)+1/12*ln(2*2^(1/3
)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)*3^(1/2)-1/6*arc
tan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a
^(1/3)/b^(1/3)*3^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$$
$$= \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

output
```
(4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1
/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3
)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Lo
g[2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/
3)*a^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/
3) + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a +
b*x^3)^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]
+ 2*Log[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a
+ b*x^3)^(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3))
```

3.36.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$$

$$\Big\downarrow \text{927}$$

$$\frac{9\sqrt[3]{a}\int \frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^3}{bx^3+a}\right)\left(\frac{2\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^3}{bx^3+a}+1\right)}d\frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{982}$$

$$\frac{9\sqrt[3]{a}\left(\frac{1}{9}\int \frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^3}{bx^3+a}\right)}d\frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^3}{bx^3+a}+1\right)}d\frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right.}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{821}$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \frac{}{2^{2/3}\ldots}\right.\right.}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}\cdots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}\cdots}\cdots}{\cdots}\right.\right.\cdots}{\sqrt[3]{b}}$$

$\downarrow$ 1142

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{\cdots}}{\sqrt[3]{a}\sqrt[3]{bx^3\cdots}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.$$

$\downarrow$ 25

3.36. $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3 \cdots}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} \cdots\right.\right.$$

$$\Big\downarrow \; 27$$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^{\cdots}}}}{3\sqrt[3]{2}\sqrt[3]{a}} \cdots\right.\right.$$

$$\Big\downarrow \; 1082$$

3.36. $\displaystyle\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3\int\frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 217

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\sqrt[3]{a+bx\cdots}}\right.}{3\ 2^{2/3}a^{2/\cdots}}\cdots\right.\right.$$

$\downarrow$ 1103

3.36. $\int\frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\right.}{\sqrt[3]{b}}$$

input
```
Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

output
```
(9*a^(1/3)*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3
) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a +
 b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3)))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3
*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3))
- ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(
2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*
2^(2/3)*a^(1/3)))/9))/b^(1/3)
```

3.36.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.36. $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

3.36.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{-b\,x^{3}+a}dx$$

input
```
int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(284) = 568$.

Time = 16.54 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx =$$

$$-\frac{1}{18}\sqrt{3}2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}(ab^6x^{16}+33\,a^2b^5x^{13}+110\,a^3b^4x^{10}+110\,a^4b^3x^7+33\,a^5b^2x^4+a^6bx}{}\right.$$

$$-\frac{1}{36}$$

$$\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(ab^3x^8+4\,a^2b^2x^5+a^3bx^2)(bx^3+a)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(b^4x^{12}+32\,ab^3x^9+78\,a^2b^2x^6}{b^4x^{12}-4\,ab^3x^9+6\,a^2b^2x^6}\right.$$

$$+\frac{1}{18}$$

$$\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(-\frac{12\,(bx^3+a)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(b^2x^6-2\,abx^3+a^2)\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}(bx^4+ax)(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{ab}\right.}{b^2x^6-2\,abx^3+a^2}\right.$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^
6*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2
*x^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a
*b^5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b
*x^3 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417
*a^2*b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(
b^6*x^18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b
^2*x^6 + 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(
2/3)*(a*b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^
(2/3) - 2^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 +
 a^4)*(-1/(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x
)*(b*x^3 + a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3
 + a^4)) + 1/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 +
2^(2/3)*(b^2*x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 +
a*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

3.36.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

3.36.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

3.36.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

3.37 $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

3.37.1 Optimal result

Integrand size = 22, antiderivative size = 452

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{4/3}\sqrt[3]{b}}$$

output
```
1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(
2/3)-1/12*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)
/b^(1/3)+1/12*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*ln(1+2^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/24*ln(2*
2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/b^(1/3)*3^(1/2)-1
/12*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^
(1/3)/a^(4/3)/b^(1/3)*3^(1/2)
```

3.37.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \frac{4ax\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b}{}\right.}{(a-bx^3)\,(a+bx^3)^{2/3}\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\text{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

output
```
(4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(
a + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a]
+ b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

3.37.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{928}$$

$$\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}$$

$$\Big\downarrow \text{779}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{927}$$

$$\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{982}$$

$$\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{821}$$

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

$$\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \frac{\cdots}{2^{2/3}-\cdots}\cdots}{\cdots}\cdots\right)\cdots\right)}{2a^{2/3}\sqrt[3]{b}\cdots}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 16

$$\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{\cdots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\cdots}\cdots}{\cdots}\cdots\right)\cdots\right)}{2a^{2/3}\sqrt[3]{b}\cdots}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 1142

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

$$9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 25

$$9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 27

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

$$9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\ldots}{\ldots}\right.}{\ldots}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{217}$$

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

$$9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\right.\right.}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

input
```
Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 +
 (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) -
 Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(
2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(
2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x
)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3))
```

3.37.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.37. $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

3.37.4 Maple [F]

$$\int \frac{1}{(-b\,x^{3}+a)\,(b\,x^{3}+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

3.37.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.37.6 Sympy [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = -\int \frac{1}{-a\,(a+bx^3)^{\frac{2}{3}} + bx^3\,(a+bx^3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
```

output
```
-Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

3.37.8 Giac [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(a - b*x^3)), x)
```

3.38 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

3.38.1 Optimal result

Integrand size = 22, antiderivative size = 473

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{x}{4a^2(a+bx^3)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{7/3}\sqrt[3]{b}}$$

output
```
1/4*x/a^2/(b*x^3+a)^(2/3)+1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/
3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)-1/24*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x
^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)+1/24*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)
^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^
(7/3)/b^(1/3)-1/12*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/
3)/a^(7/3)/b^(1/3)+1/48*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)
+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)-1/12
*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(
1/3)/a^(7/3)/b^(1/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(7/3)/b^(1/3)*3^(1/2)
```

3.38.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{x\left(\frac{4}{a^2}-\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{\cdots}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)\cdots\right.}\right.}{16\,(a+bx^3)^{2/3}}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

output
```
(x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x
^3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3
)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/
3))
```

3.38.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.14, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {931, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}dx$$

$$\Big\downarrow \ 931$$

$$\frac{x}{4a^2(a+bx^3)^{2/3}}-\frac{\int -\frac{b(3a-bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{b(3a-bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3a-bx^3}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 1026$$

$$\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx+2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 779$$

$$\frac{2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 778$$

$$\frac{2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 928$$

$$\frac{2a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 779$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}dx$

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\Big\downarrow$ 778

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\Big\downarrow$ 927

$$\frac{2a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right.}{2a(a+bx^3)^{2/3}}\right.}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\Big\downarrow$ 982

$$\frac{2a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}+\right.}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$\Big\downarrow \; 821$$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int \ldots\right.\right.}{2a^{2/3}\sqrt[3]{b}}\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 16$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{\sqrt[3]{b}\cdots}{\sqrt[3]{b}\cdots}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\frac{2^{2/3}\left(\cdots\right.}{\sqrt[3]{\cdots}}}\cdots}{\cdots}\cdots\right.\right)}{2a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 1142$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$\frac{x}{4a^2\left(bx^3+a\right)^{2/3}}+$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(bx^3+a)^{2/3}}+2a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}}{\left(bx^3+a\right)^{2/3}}\right.\right.}{}\right.}{}$$

$\downarrow$ 25

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$\frac{x}{4a^2\left(bx^3+a\right)^{2/3}}+$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(bx^3+a)^{2/3}}+2a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{\cdots}{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\cdots}}{\left(bx^3+a\right)^{2/3}\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 27

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$2a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{3\sqrt[3]{2}\sqrt[3]{a}}$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 1082$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 217$$

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$2a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\right.\right.\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 1103

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+$$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\log \cdots}{\cdots}\right.\cdots\right)}{2a^{2/3}\sqrt[3]{b}}\cdots\right.$$

input
```
Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

output
```
x/(4*a^2*(a + b*x^3)^(2/3)) + ((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 2*a*((x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/
3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3)
+ b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(
a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*L
og[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) -
((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/S
qrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/
3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^
(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3))))/(4*a^2)
```

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

3.38.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.38. $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}} \, dx$

3.38.4 Maple [F]

$$\int \frac{1}{(-b\,x^{3}+a)\,(b\,x^{3}+a)^{\frac{5}{3}}}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

3.38.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.38.6 Sympy [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{5/3}}\,dx = -\int \frac{1}{-a^2\,(a+bx^3)^{\frac{2}{3}}+b^2x^6\,(a+bx^3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
```

output
```
-Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x
)
```

3.38.7 Maxima [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

3.38.8 Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)
```

3.39 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

3.39.1 Optimal result

Integrand size = 22, antiderivative size = 492

$$\begin{aligned}\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx &= \frac{x}{10a^2(a+bx^3)^{5/3}}+\frac{13x}{40a^3(a+bx^3)^{2/3}} \\ &- \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} \\ &+ \frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{20a^3(a+bx^3)^{2/3}} \\ &- \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}} \\ &- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{48\ 2^{2/3}a^{10/3}\sqrt[3]{b}}\end{aligned}$$

output
```
1/10*x/a^2/(b*x^3+a)^(5/3)+13/40*x/a^3/(b*x^3+a)^(2/3)+9/20*x*(1+b*x^3/a)^
(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)-1/48*ln(2^(
2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/48*l
n(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*ln(1+2^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/96*ln(2*2^(1/3)+(a^(
1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1/3)*3^(1/2)-1/48*arcta
n(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
10/3)/b^(1/3)*3^(1/2)
```

3.39.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{x\left(16a^2+52a(a+bx^3)-13bx^3(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\right.\right.}{}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

output
```
(x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)
*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3
)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*A
ppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3
, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

3.39.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.16, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {931, 25, 27, 1024, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{x}{10a^2(a+bx^3)^{5/3}}-\frac{\int -\frac{b(9a-4bx^3)}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b(9a-4bx^3)}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2b}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{9a-4bx^3}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{13x}{4a(a+bx^3)^{2/3}}-\frac{\int -\frac{ab(23a-13bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{ab(23a-13bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{23a-13bx^3}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{\frac{13\int \frac{1}{(bx^3+a)^{2/3}}dx+10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 779}$$

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}dx$

$$\frac{\frac{10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{13\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{10a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{928}$$

$$\frac{\frac{10a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{927}$$

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$\frac{10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)+}{\frac{4a}{10a^2}}$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 982

$$\frac{10a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}+\frac{x(}{}\right.}{\frac{4a}{10a^2}}$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 821

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{\sqrt[3]{b}x\cdots}{\sqrt[3]{bx\cdots}}}\cdots}{\cdots}\right.\right.\cdots}{2a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow\ 16$$

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{\sqrt[3]{b}x+\cdots}{\sqrt[3]{bx^3\cdots}}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{\cdots}\right.}{\sqrt[3]{b\cdots}}}\cdots}{3\cdots}\right.\cdots\right)}{2a^{2/3}\sqrt[3]{b}\cdots}\cdots\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\downarrow 1142$$

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$\frac{x}{10a^2\left(bx^3+a\right)^{5/3}}+$$

$$\frac{13x}{4a(bx^3+a)^{2/3}}+\frac{\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(bx^3+a\right)^{2/3}}+10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{2^{2/3}\left(\sqrt[3]{b}\ldots\right.}}{\left(bx^3+\ldots\right.}\right.\right.}{}\right.}{}$$

$\downarrow$ 25

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$\frac{x}{10a^2\left(bx^3+a\right)^{5/3}}+$$

$$\frac{\frac{13x}{4a(bx^3+a)^{2/3}}+\frac{\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(bx^3+a\right)^{2/3}}+10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{2^{2/3}\left(\sqrt[3]{b}\ldots\right.}}{\left(bx^3+\ldots\right.}\right.\right.}{}\right.}{}}{}$$

$\downarrow$ 27

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3 \cdots}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right. \cdots$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 1082

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\ldots\right.}{\ldots}\right.\right.\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 217

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$10a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\right.\right.\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \text{ 1103}$$

3.39. $\int\frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+$$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)}{2a^{2/3}\sqrt[3]{b}}+\frac{1}{9}\left(-\frac{\log\left(2\ldots\right.}{\ldots}\right.\right.\right.$$

input
```
Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

output
```
x/(10*a^2*(a + b*x^3)^(5/3)) + ((13*x)/(4*a*(a + b*x^3)^(2/3)) + ((13*x*(1
 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b
*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4
/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1
 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)
*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) -
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3)))/(
3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(
1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/
(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) +
(a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1
/3))))/(4*a))/(10*a^2)
```

3.39.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

3.39. $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.39.4 Maple [F]

$$\int \frac{1}{\left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{8}{3}}}d x$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

3.39.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.39.6 Sympy [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = -\int \frac{1}{-a^3(a+bx^3)^{\frac{2}{3}}-a^2bx^3(a+bx^3)^{\frac{2}{3}}+ab^2x^6(a+bx^3)^{\frac{2}{3}}+b^3x^9(a+bx^3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
```

output
```
-Integral(1/(-a**3*(a + b*x**3)**(2/3) - a**2*b*x**3*(a + b*x**3)**(2/3) +
 a*b**2*x**6*(a + b*x**3)**(2/3) + b**3*x**9*(a + b*x**3)**(2/3)), x)
```

3.39.7 Maxima [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx=\int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

3.39.8 Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)
```

3.40 $\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx$

3.40.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{8}{27}ax(a+bx^3)^{5/3} - \frac{1}{9}x(a-bx^3)(a+bx^3)^{5/3} + \frac{76a^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{38a^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81\sqrt[3]{b}}$$

output
```
38/81*a^2*x*(b*x^3+a)^(2/3)-8/27*a*x*(b*x^3+a)^(5/3)-1/9*x*(-b*x^3+a)*(b*x
^3+a)^(5/3)-38/81*a^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+76/243*a^3*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.20

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \frac{3\sqrt[3]{b}(a+bx^3)^{2/3}(5a^2x-24abx^4+9b^2x^7)+76\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-76a^3\log\left(\cdots\right.}{243\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(1/3)*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + 76*Sqrt[
3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 76*
a^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 38*a^3*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(1/3))
```

3.40.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {933, 27, 913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int 2ab\left(5a-8bx^3\right)\left(bx^3+a\right)^{2/3}dx}{9b}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}a\int \left(5a-8bx^3\right)\left(bx^3+a\right)^{2/3}dx-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\int \left(bx^3+a\right)^{2/3}dx-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

output
```
-1/9*(x*(a - b*x^3)*(a + b*x^3)^(5/3)) + (2*a*((-4*x*(a + b*x^3)^(5/3))/3
+ (19*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1
/3)]/(2*b^(1/3))))/3))/3))/9
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.40.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{27b^{\frac{7}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}x^{7}-72a\,b^{\frac{4}{3}}x^{4}\left(b\,x^{3}+a\right)^{\frac{2}{3}}+15a^{2}x\,b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}-76\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a^{3}-76\ln\left(\frac{-b^{\frac{1}{3}}x+}{}\right.}{243b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/243*(27*b^(7/3)*(b*x^3+a)^(2/3)*x^7-72*a*b^(4/3)*x^4*(b*x^3+a)^(2/3)+15*
a^2*x*b^(1/3)*(b*x^3+a)^(2/3)-76*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(
b*x^3+a)^(1/3))/b^(1/3)/x)*a^3-76*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^3+3
8*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^3)/b^(
1/3)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.03

$$\int (a-bx^3)^2 (a+bx^3)^{2/3}\,dx = \left[\frac{114\sqrt{\frac{1}{3}}a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+\right.\right.}{}\right.$$

$$-\frac{228\sqrt{\frac{1}{3}}a^3b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+76\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-38}{243\,b}$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)
^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*
b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a
^3*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3
)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1
/243*(228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3
)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(
((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*
x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*
x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

3.40.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx = \frac{a^{\frac{8}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{5}{3}} b x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

output
```
a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*h
yper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.97

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{27}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

3.40. $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx$

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*
arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^
2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/
3) + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2
/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3
+ (b*x^3 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(
1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^
3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/
3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 1
1*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x
^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

3.40.8 Giac [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} \, dx=\int \left(bx^3+a\right)^{\frac{2}{3}} \left(bx^3-a\right)^2 \, dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} \, dx=\int \left(b\,x^3+a\right)^{2/3} \left(a-b\,x^3\right)^2 \, dx$$

input
```
int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

3.41 $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

3.41.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{17a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

output
```
-13/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(-b*x^3+a)*(b*x^3+a)^(2/3)-17/18*a^2*ln(-
b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+17/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*
x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(a+bx^3\right)^{2/3}\left(-16ax+3bx^4\right)+\frac{17a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27\sqrt[3]{b}} + \frac{17a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-16*a*x + 3*b*x^4))/18 + (17*a^2*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (17*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (17*a^2*Log[b^(2/3)*x^
2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

3.41.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {933, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \frac{ab(7a-13bx^3)}{\sqrt[3]{bx^3+a}}dx}{6b} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{7a-13bx^3}{\sqrt[3]{bx^3+a}}dx - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{913}$$

$$\frac{1}{6}a\left(\frac{34}{3}a\int\frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{13}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{6}a\left(\frac{34}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{13}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
-1/6*(x*(a - b*x^3)*(a + b*x^3)^(2/3)) + (a*((-13*x*(a + b*x^3)^(2/3))/3 +
 (34*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(
1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6
```

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.41. $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.41.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{9x^4\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{4}{3}}-48ax\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}-34a^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-34a^2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)+17a^2\ln}{54b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*x^4*(b*x^3+a)^(2/3)*b^(4/3)-48*a*x*(b*x^3+a)^(2/3)*b^(1/3)-34*a^2*
3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-34*a^2
*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+17*a^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(1/3)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.32

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}\right.\right.}{} \right.$$

$$-\frac{102\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+34\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-}{54\,b}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(
1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*
log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/
x^2) + 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)
*a^2*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*
x^3 + a)^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*
x^3 + a)^(2/3))/b]
```

3.41.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

output
```
a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(94) = 188.

Time = 0.27 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.63

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

3.41. $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*
(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a
^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)
^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x
^3 + (b*x^3 + a)^2*b^2/x^6))*b^2
```

3.41.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.42 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$

3.42.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{10a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

output
```
2*x*(-b*x^3+a)/(b*x^3+a)^(1/3)+7/3*x*(b*x^3+a)^(2/3)+5/3*a*ln(-b^(1/3)*x+(
b*x^3+a)^(1/3))/b^(1/3)-10/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))/b^(1/3)*3^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.36

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\left(\frac{3(13ax+bx^4)}{\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{10a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}} - \frac{5a\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}\right)$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((3*(13*a*x + b*x^4))/(a + b*x^3)^(1/3) - (10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(1/3) + (10*a*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)]])/b^(1/3) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/b^(1/3))/9
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {930, 25, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int -\frac{ab(a-7bx^3)}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 25$$

$$\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} - \frac{\int \frac{ab(a-7bx^3)}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}-\int \frac{a-7bx^3}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{913}$$

$$-\frac{10}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{769}$$

$$-\frac{10}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
(2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*(A
rcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.42.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{3b^{\frac{4}{3}}x^4+10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a\left(b\,x^3+a\right)^{\frac{1}{3}}+39xa\,b^{\frac{1}{3}}+10\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)a\left(b\,x^3+a\right)^{\frac{1}{3}}-5\ln\left(\frac{b^{\frac{2}{3}}x \cdots}{\cdots}\right.}{9b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*(3*b^(4/3)*x^4+10*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1
/3))/b^(1/3)/x)*a*(b*x^3+a)^(1/3)+39*x*a*b^(1/3)+10*ln((-b^(1/3)*x+(b*x^3+
a)^(1/3))/x)*a*(b*x^3+a)^(1/3)-5*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2)*a*(b*x^3+a)^(1/3))/b^(1/3)/(b*x^3+a)^(1/3)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.

Time = 0.36 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.65

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left[\frac{15\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}\right.\right.}{}\right.$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b
*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)
*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*
x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 +
a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^
(2/3))/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1
/9*(10*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5
*(a*b*x^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3
)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13
*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

3.42.6 Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}b^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3ab-\frac{4(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)$$
$$+\frac{1}{3}ab\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)$$
$$+\frac{ax}{(bx^3+a)^{\frac{1}{3}}}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/
x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3)
+ (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^
(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

3.42.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.43 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx$

3.43.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx = \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
1/2*x*(-b*x^3+a)/(b*x^3+a)^(4/3)-1/2*x/(b*x^3+a)^(1/3)-1/2*ln(-b^(1/3)*x+(
b*x^3+a)^(1/3))/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(1/3)*3^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx = \frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}} + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + \log\left(b^{2/3}\right.}{6\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)
/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(6*b^(1/3))
```

3.43.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {930, 27, 910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{2ab(2bx^3+a)}{(bx^3+a)^{4/3}}dx}{4ab}+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2bx^3+a}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{x}{\sqrt[3]{a+bx^3}}\right)+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{2}\left(2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{x}{\sqrt[3]{a+bx^3}}\right)+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
(x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) + (-(x/(a + b*x^3)^(1/3)) + 2*(ArcTa
n[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-
(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/2
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.43.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-6 b^{\frac{4}{3}} x^{4}+\left(b \,x^{3}+a \right)^{\frac{4}{3}} \left(-2 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(b^{\frac{1}{3}} x +2 \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln \left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2}}\right)-2 \ln \left(\frac{-b^{\frac{1}{3}} x +\left(b \right.}{x}\right.}{6 b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{4}{3}}}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/6/b^(1/3)/(b*x^3+a)^(4/3)*(-6*b^(4/3)*x^4+(b*x^3+a)^(4/3)*(-2*3^(1/2)*ar
ctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+
b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)
^(1/3))/x)))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(87) = 174$.

Time = 0.35 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left[-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4 - 3\,\sqrt{\frac{1}{3}}(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-\right.}{}\right.$$

$$-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4 + 6\,\sqrt{\frac{1}{3}}(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(b}{6\,(b^3x^6}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
[-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2
 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)
^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 +
a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a
*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3
)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b
*x^3 + a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqr
t(-(-b)^(1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(
1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)]
```

3.43.6 Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4(bx^3+a)}{x^3}\right)x^4}{4(bx^3+a)^{\frac{4}{3}}} - \frac{bx^4}{2(bx^3+a)^{\frac{4}{3}}} - \frac{1}{12}\left(\frac{3\left(b+\frac{4(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4}{}\right.$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)
^(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*s
qrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/
3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2
```

3.43.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.44 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} \, dx$

3.44.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} \, dx = \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}} + \frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}} + \frac{9x}{14a\sqrt[3]{a+bx^3}}$$

output
```
1/7*x*(-b*x^3+a)^2/a/(b*x^3+a)^(7/3)+3/14*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+9
/14*x/a/(b*x^3+a)^(1/3)
```

3.44.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} \, dx = \frac{7a^2x + 7abx^4 + 4b^2x^7}{7a\,(a+bx^3)^{7/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(7*a^2*x + 7*a*b*x^4 + 4*b^2*x^7)/(7*a*(a + b*x^3)^(7/3))
```

3.44.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx$$

$$\Big\downarrow \text{903}$$

$$\frac{6}{7}\int \frac{a-bx^3}{(bx^3+a)^{7/3}}dx+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{6}{7}\left(\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{6}{7}\left(\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*((x*(a - b*x^3))/(4*a*(a +
b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))))/7
```

3.44.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.44.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37
trager	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37
pseudoelliptic	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{(4\,b^2x^7+7\,abx^4+7\,a^2x)(bx^3+a)^{\frac{2}{3}}}{7\,(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2
*b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

3.44.6 Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(10/3), x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{14\,(bx^3+a)^{\frac{7}{3}}a} + \frac{b^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/(
(b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^
2/x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

3.44.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.58

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{4\,x\,(b\,x^3+a)^2+4\,a^2\,x-a\,x\,(b\,x^3+a)}{7\,a\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
```

output
```
(4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

3.45 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx$

3.45.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

output
```
1/20*x*(-b*x^3+a)^3/a^2/(b*x^3+a)^(10/3)+19/140*x*(-b*x^3+a)^2/a^2/(b*x^3+
a)^(7/3)+57/280*x*(-b*x^3+a)/a^2/(b*x^3+a)^(4/3)+171/280*x/a^2/(b*x^3+a)^(
1/3)
```

3.45.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{140a^3x+245a^2bx^4+230ab^2x^7+69b^3x^{10}}{140a^2(a+bx^3)^{10/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(140*a^3*x + 245*a^2*b*x^4 + 230*a*b^2*x^7 + 69*b^3*x^10)/(140*a^2*(a + b*
x^3)^(10/3))
```

3.45.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {907, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{19\int \frac{(a-bx^3)^2}{(bx^3+a)^{10/3}}dx}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{19\left(\frac{6}{7}\int \frac{a-bx^3}{(bx^3+a)^{7/3}}dx+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{19\left(\frac{6}{7}\left(\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19\left(\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{6}{7}\left(\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)\right)}{20a}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*((x*(a - b*x^3)^2)/(7*
a*(a + b*x^3)^(7/3)) + (6*((x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)
/(4*a*(a + b*x^3)^(1/3))))/7))/(20*a)
```

3.45.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.45.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x\left(69 b^{3} x^{9}+230 a \,b^{2} x^{6}+245 a^{2} b \,x^{3}+140 a^{3}\right)}{140 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48
trager	$\frac{x\left(69 b^{3} x^{9}+230 a \,b^{2} x^{6}+245 a^{2} b \,x^{3}+140 a^{3}\right)}{140 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48
pseudoelliptic	$\frac{x\left(69 b^{3} x^{9}+230 a \,b^{2} x^{6}+245 a^{2} b \,x^{3}+140 a^{3}\right)}{140 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/
a^2
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x)(bx^3+a)^{\frac{2}{3}}}{140\,(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a
)^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.48

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)b^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^2}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*
(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)
^(10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*
b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

3.45.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{69\,x}{140\,a^2\,(b\,x^3+a)^{1/3}} - \frac{2\,x}{35\,(b\,x^3+a)^{7/3}} + \frac{23\,x}{140\,a\,(b\,x^3+a)^{4/3}} + \frac{2\,a\,x}{5\,(b\,x^3+a)^{10/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
```

output
```
(69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)
/(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

3.46 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx$

3.46.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{2x(a-bx^3)}{13\,(a+bx^3)^{13/3}}+\frac{8x}{65\,(a+bx^3)^{10/3}} +\frac{47x}{455a\,(a+bx^3)^{7/3}}+\frac{141x}{910a^2\,(a+bx^3)^{4/3}}+\frac{423x}{910a^3\sqrt[3]{a+bx^3}}$$

output
```
2/13*x*(-b*x^3+a)/(b*x^3+a)^(13/3)+8/65*x/(b*x^3+a)^(10/3)+47/455*x/a/(b*x
^3+a)^(7/3)+141/910*x/a^2/(b*x^3+a)^(4/3)+423/910*x/a^3/(b*x^3+a)^(1/3)
```

3.46.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{910a^4x+2275a^3bx^4+3055a^2b^2x^7+1833ab^3x^{10}+423b^4x^{13}}{910a^3\,(a+bx^3)^{13/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(910*a^4*x + 2275*a^3*b*x^4 + 3055*a^2*b^2*x^7 + 1833*a*b^3*x^10 + 423*b^4
*x^13)/(910*a^3*(a + b*x^3)^(13/3))
```

3.46.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {930, 27, 910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{ab(11a-5bx^3)}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{13}\int \frac{11a-5bx^3}{(bx^3+a)^{13/3}}dx+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{ 910}$$

$$\frac{1}{13}\left(\frac{47}{5}\int \frac{1}{(bx^3+a)^{10/3}}dx+\frac{8x}{5(a+bx^3)^{10/3}}\right)+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{8x}{5(a+bx^3)^{10/3}}\right)+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{8x}{5(a+bx^3)^{10/3}}\right)+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{8x}{5\left(a+bx^3\right)^{10/3}}\right)+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + ((8*x)/(5*(a + b*x^3)^(10/3))
+ (47*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(
4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/5)/13
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.46.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x\left(423b^{4}x^{12}+1833a\,b^{3}x^{9}+3055a^{2}b^{2}x^{6}+2275a^{3}b\,x^{3}+910a^{4}\right)}{910\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{3}}$	59
trager	$\frac{x\left(423b^{4}x^{12}+1833a\,b^{3}x^{9}+3055a^{2}b^{2}x^{6}+2275a^{3}b\,x^{3}+910a^{4}\right)}{910\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{3}}$	59
pseudoelliptic	$\frac{x\left(423b^{4}x^{12}+1833a\,b^{3}x^{9}+3055a^{2}b^{2}x^{6}+2275a^{3}b\,x^{3}+910a^{4}\right)}{910\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{3}}$	59

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a
^4)/(b*x^3+a)^(13/3)/a^3
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(423\,b^4x^{13}+1833\,ab^3x^{10}+3055\,a^2b^2x^7+2275\,a^3bx^4+910\,a^4x)(bx^3+a)^{\frac{2}{3}}}{910\,(a^3b^5x^{15}+5\,a^4b^4x^{12}+10\,a^5b^3x^9+10\,a^6b^2x^6+5\,a^7bx^3+a^8)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4
+ 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3
*x^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.10

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{\left(35\,b^2 - \frac{91\,(bx^3+a)b}{x^3} + \frac{65\,(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} + \frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^3} + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*
x^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x
^3 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) +
 1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455
*(b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^
3)
```

3.46.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx=\frac{423\,x}{910\,a^3\,(b\,x^3+a)^{1/3}}-\frac{2\,x}{65\,(b\,x^3+a)^{10/3}}+\frac{141\,x}{910\,a^2\,(b\,x^3+a)^{4/3}}+\frac{47\,x}{455\,a\,(b\,x^3+a)^{7/3}}+\frac{4\,a\,x}{13\,(b\,x^3+a)^{13/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
```

output
```
(423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141
*x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*
x)/(13*(a + b*x^3)^(13/3))
```

3.47 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx$

3.47.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}+\frac{11x}{104(a+bx^3)^{13/3}}+\frac{x}{13a(a+bx^3)^{10/3}}+\frac{9x}{91a^2(a+bx^3)^{7/3}}+\frac{27x}{182a^3(a+bx^3)^{4/3}}+\frac{81x}{182a^4\sqrt[3]{a+bx^3}}$$

output
```
1/8*x*(-b*x^3+a)/(b*x^3+a)^(16/3)+11/104*x/(b*x^3+a)^(13/3)+1/13*x/a/(b*x^
3+a)^(10/3)+9/91*x/a^2/(b*x^3+a)^(7/3)+27/182*x/a^3/(b*x^3+a)^(4/3)+81/182
*x/a^4/(b*x^3+a)^(1/3)
```

3.47.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{364a^5x+1183a^4bx^4+2080a^3b^2x^7+1872a^2b^3x^{10}+864ab^4x^{13}+162b^5x^{16}}{364a^4(a+bx^3)^{16/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(364*a^5*x + 1183*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1872*a^2*b^3*x^10 + 864*a
*b^4*x^13 + 162*b^5*x^16)/(364*a^4*(a + b*x^3)^(16/3))
```

3.47.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {930, 27, 910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{2ab(7a-4bx^3)}{(bx^3+a)^{16/3}}dx}{16ab}+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\int \frac{7a-4bx^3}{(bx^3+a)^{16/3}}dx+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \; 910$$

$$\frac{1}{8}\left(\frac{80}{13}\int \frac{1}{(bx^3+a)^{13/3}}dx+\frac{11x}{13(a+bx^3)^{13/3}}\right)+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\int \frac{1}{(bx^3+a)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{11x}{13(a+bx^3)^{13/3}}\right)+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{11x}{13(a+bx^3)^{13/3}}\right)+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\left(\frac{3\int\frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{11x}{13\left(a+bx^3\right)^{13/3}}\right)+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

$\downarrow$ 746

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{11x}{13\left(a+bx^3\right)^{13/3}}\right)+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + ((11*x)/(13*(a + b*x^3)^(13/3)) +
 (80*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/
(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(10*a)
))/13)/8
```

3.47. $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.47.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x \left(162 b^{5} x^{15}+864 a \,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b \,x^{3}+364 a^{5}\right)}{364 \left(b \,x^{3}+a \right)^{\frac{16}{3}} a^{4}}$	70
trager	$\frac{x \left(162 b^{5} x^{15}+864 a \,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b \,x^{3}+364 a^{5}\right)}{364 \left(b \,x^{3}+a \right)^{\frac{16}{3}} a^{4}}$	70
pseudoelliptic	$\frac{x \left(162 b^{5} x^{15}+864 a \,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b \,x^{3}+364 a^{5}\right)}{364 \left(b \,x^{3}+a \right)^{\frac{16}{3}} a^{4}}$	70

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+118
3*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(162\,b^5x^{16}+864\,ab^4x^{13}+1872\,a^2b^3x^{10}+2080\,a^3b^2x^7+1183\,a^4bx^4+364\,a^5x)(bx^3+a}{364\,(a^4b^6x^{18}+6\,a^5b^5x^{15}+15\,a^6b^4x^{12}+20\,a^7b^3x^9+15\,a^8b^2x^6+6\,a^9bx^3+a^{10})}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

output
```
1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^
7 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^
5*x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 +
 a^10)
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

output
```
Timed out
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(94) = 188$.

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.20

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = -\frac{\left(455\,b^3 - \frac{1680\,(bx^3+a)b^2}{x^3} + \frac{2184\,(bx^3+a)^2b}{x^6} - \frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4} - \frac{\left(455\,b^4 - \frac{2240\,(bx^3+a)b^3}{x^3} + \frac{4368\,(bx^3+a)^2b^2}{x^6} - \frac{4160\,(bx^3+a)^3b}{x^9} + \frac{1820\,(bx^3+a)^4}{x^{12}}\right)bx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^4} - \frac{\left(91\,b^5 - \frac{560\,(bx^3+a)b^4}{x^3} + \frac{1456\,(bx^3+a)^2b^3}{x^6} - \frac{2080\,(bx^3+a)^3b^2}{x^9} + \frac{1820\,(bx^3+a)^4b}{x^{12}} - \frac{1456\,(bx^3+a)^5}{x^{15}}\right)x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^4}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

output
```
-1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 1
040*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4
 - 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a
)^3*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1
456*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*
(b*x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^1
5)*x^16/((b*x^3 + a)^(16/3)*a^4)
```

3.47.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{81\,x}{182\,a^4\,(b\,x^3+a)^{1/3}} - \frac{x}{52\,(b\,x^3+a)^{13/3}} + \frac{27\,x}{182\,a^3\,(b\,x^3+a)^{4/3}} + \frac{9\,x}{91\,a^2\,(b\,x^3+a)^{7/3}} + \frac{x}{13\,a\,(b\,x^3+a)^{10/3}} + \frac{a\,x}{4\,(b\,x^3+a)^{16/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
```

output
```
(81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(1
82*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a
+ b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

3.48 $\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx$

3.48.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx = -\frac{9}{44} a x (a+bx^3)^{7/3} - \frac{1}{11} x (a-bx^3) (a+bx^3)^{7/3} + \frac{57 a^3 x \sqrt[3]{a+bx^3} \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44 \sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-9/44*a*x*(b*x^3+a)^(7/3)-1/11*x*(-b*x^3+a)*(b*x^3+a)^(7/3)+57/44*a^3*x*(b
*x^3+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

3.48.2 Mathematica [A] (verified)

Time = 8.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int (a-bx^3)^2 (a +bx^3)^{4/3} \, dx = \frac{x\left(106a^4+53a^3bx^3-78a^2b^2x^6-5ab^3x^9+20b^4x^{12}+114a^4\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\right.}{220(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

output
```
(x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 +
114*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)]))/(220*(a + b*x^3)^(2/3))
```

3.48.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int 6ab\left(2a-3bx^3\right)\left(bx^3+a\right)^{4/3}dx}{11b}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11}a\int \left(2a-3bx^3\right)\left(bx^3+a\right)^{4/3}dx-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{913}$$

$$\frac{6}{11}a\left(\frac{19}{8}a\int \left(bx^3+a\right)^{4/3}dx-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{6}{11}a\left(\frac{19a^2\sqrt[3]{a+bx^3}\int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{8\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{6}{11}a\left(\frac{19a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

output
```
-1/11*(x*(a - b*x^3)*(a + b*x^3)^(7/3)) + (6*a*((-3*x*(a + b*x^3)^(7/3))/8
 + (19*a^2*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)
/a)])/(8*(1 + (b*x^3)/a)^(1/3))))/11
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.48.4 Maple [F]

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}d x$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

output
```
int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

3.48.5 Fricas [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3}\,dx=\int \left(bx^{3}+a\right)^{\frac{4}{3}}\left(bx^{3}-a\right)^{2}\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

3.48.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.79

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3}\,dx=\frac{a^{\frac{10}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{7}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{a^{\frac{4}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}b^3x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
```

output
```
a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hy
per((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(
1/3)*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(
I*pi)/a)/(3*gamma(13/3))
```

3.48.7 Maxima [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(bx^3-a\right)^2 \, dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

3.48.8 Giac [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(bx^3-a\right)^2 \, dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3}\,dx=\int \left(b\,x^3+a\right)^{4/3}\,\left(a-b\,x^3\right)^2\,dx$$

input
```
int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)
```

3.49 $\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx$

3.49.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = -\frac{3}{8}ax(a+bx^3)^{4/3}-\frac{1}{8}x(a-bx^3)(a+bx^3)^{4/3} + \frac{3a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-3/8*a*x*(b*x^3+a)^(4/3)-1/8*x*(-b*x^3+a)*(b*x^3+a)^(4/3)+3/2*a^2*x*(b*x^3
+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

3.49.2 Mathematica [A] (verified)

Time = 6.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \frac{x\left(2a^3-a^2bx^3-2ab^2x^6+b^3x^9+6a^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{8(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

output
```
(x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

3.49.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int 3ab\left(3a-5bx^3\right)\sqrt[3]{bx^3+a}dx}{8b}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}a\int \left(3a-5bx^3\right)\sqrt[3]{bx^3+a}dx-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{913}$$

$$\frac{3}{8}a\left(4a\int \sqrt[3]{bx^3+a}dx-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{3}{8}a\left(\frac{4a\sqrt[3]{a+bx^3}\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{3}{8}a\left(\frac{4ax\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

output
```
-1/8*(x*(a - b*x^3)*(a + b*x^3)^(4/3)) + (3*a*(-(x*(a + b*x^3)^(4/3)) + (4
*a*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(1
 + (b*x^3)/a)^(1/3)))/8
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.49.4 Maple [F]

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}dx$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

output
```
int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

3.49.5 Fricas [F]

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(bx^3-a\right)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

3.49.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{4}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
```

output
```
a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*h
yper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.49.7 Maxima [F]

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

3.49.8 Giac [F]

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \int \left(b\,x^3+a\right)^{1/3}\left(a-b\,x^3\right)^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(a - b*x^3)^2, x)
```

3.50 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx$

3.50.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = -\frac{6}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3} + \frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}$$

output
```
-6/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(-b*x^3+a)*(b*x^3+a)^(1/3)+12/5*a^2*x*(1+b*
x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

3.50.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{-7a^2x-6abx^4+b^2x^7+12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

3.50.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \frac{6ab(a-2bx^3)}{(bx^3+a)^{2/3}}dx}{5b}-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}a\int \frac{a-2bx^3}{(bx^3+a)^{2/3}}dx-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{913}$$

$$\frac{6}{5}a\left(2a\int \frac{1}{(bx^3+a)^{2/3}}dx-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{779}$$

$$\frac{6}{5}a\left(\frac{2a\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{778}$$

$$\frac{6}{5}a\left(\frac{2ax\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
-1/5*(x*(a - b*x^3)*(a + b*x^3)^(1/3)) + (6*a*(-(x*(a + b*x^3)^(1/3)) + (2
*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])
/(a + b*x^3)^(2/3)))/5
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.50. $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx$

3.50.4 Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

3.50.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt[3]{a}bx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

output
```
a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

3.50.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

3.50.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.51 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$

3.51.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}$$

output
```
x*(-b*x^3+a)/(b*x^3+a)^(2/3)+3/4*b*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4
/3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

3.51.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\frac{5ax+bx^4-3ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4
/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

3.51.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {930, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{6ab^2x^3}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$3b\int \frac{x^3}{(bx^3+a)^{2/3}}dx+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{889}$$

$$\frac{3b\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.51.4 Maple [F]

$$\int \frac{(-b\,x^{3}+a)^{2}}{(b\,x^{3}+a)^{\frac{5}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

3.51.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}} \, dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^
3 + a^2), x)
```

3.51.6 Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{5}{3}}} \, dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

3.51.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}} \, dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

3.51.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.52 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx$

3.52.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}$$

output
```
2/5*x*(-b*x^3+a)/(b*x^3+a)^(5/3)+3/5*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2
/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

3.52.2 Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\frac{2x(a-bx^3)+3x(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{5/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

3.52.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {930, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{3ab}{(bx^3+a)^{2/3}}dx}{5ab}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{5}\int \frac{1}{(bx^3+a)^{2/3}}dx+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{5(a+bx^3)^{2/3}}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.52.4 Maple [F]

$$\int \frac{(-b\,x^{3}+a)^{2}}{(b\,x^{3}+a)^{\frac{8}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

3.52.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*
x^6 + 3*a^2*b*x^3 + a^3), x)
```

3.52.6 Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(8/3), x)
```

3.52.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

3.52.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.53 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx$

3.53.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \frac{x(a-bx^3)}{4(a+bx^3)^{8/3}} + \frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}$$

output
```
1/4*x*(-b*x^3+a)/(b*x^3+a)^(8/3)+3/4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 8
/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

3.53.2 Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \frac{7a^2x+5abx^4+3b^2x^7+3x(a+bx^3)^2\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10a(a+bx^3)^{8/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

output
```
(7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

3.53.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {930, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{6a^2b}{(bx^3+a)^{8/3}}dx}{8ab}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{4}a\int \frac{1}{(bx^3+a)^{8/3}}dx+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \ 779$$

$$\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}}dx}{4a(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \ 778$$

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

output
```
(x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.53.4 Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{11}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

3.53.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3
*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
```

output
```
Timed out
```

3.53.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

3.53.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{11/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)
```

3.54 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx$

3.54.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}+\frac{3x}{22(a+bx^3)^{8/3}} + \frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{22a^2(a+bx^3)^{2/3}}$$

output
```
2/11*x*(-b*x^3+a)/(b*x^3+a)^(11/3)+3/22*x/(b*x^3+a)^(8/3)+15/22*x*(1+b*x^3
/a)^(2/3)*hypergeom([1/3, 8/3],[4/3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)
```

3.54.2 Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \frac{x\left(16a^3+23a^2bx^3+21ab^2x^6+6b^3x^9+6(a+bx^3)^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\right.}{22a^2(a+bx^3)^{11/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

output
```
(x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1
+ (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^
2*(a + b*x^3)^(11/3))
```

3.54.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {930, 27, 910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{3ab(3a-bx^3)}{(bx^3+a)^{11/3}}dx}{11ab}+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{11}\int \frac{3a-bx^3}{(bx^3+a)^{11/3}}dx+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{3}{11}\left(\frac{5}{2}\int \frac{1}{(bx^3+a)^{8/3}}dx+\frac{x}{2(a+bx^3)^{8/3}}\right)+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{3}{11}\left(\frac{5\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}}dx}{2a^2(a+bx^3)^{2/3}}+\frac{x}{2(a+bx^3)^{8/3}}\right)+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{3}{11}\left(\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2(a+bx^3)^{2/3}}+\frac{x}{2(a+bx^3)^{8/3}}\right)+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

output
```
(2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*(x/(2*(a + b*x^3)^(8/3)) +
(5*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(2*a^2*(a + b*x^3)^(2/3))))/11
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.54.4 Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{14}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

3.54.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4
*x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
```

output
```
Timed out
```

3.54.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

3.54.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{14/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(14/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(14/3), x)
```

3.55 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx$

3.55.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx=\frac{x(a-bx^3)}{7\,(a+bx^3)^{14/3}}+\frac{9x}{77\,(a+bx^3)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77a^3\,(a+bx^3)^{2/3}}$$

output
```
1/7*x*(-b*x^3+a)/(b*x^3+a)^(14/3)+9/77*x/(b*x^3+a)^(11/3)+57/77*x*(1+b*x^3
/a)^(2/3)*hypergeom([1/3, 11/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)
```

3.55.2 Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx=\frac{x\left(2282a^4+4879a^3bx^3+6270a^2b^2x^6+3591ab^3x^9+798b^4x^{12}+798(a+bx^3)^4\left(1+\frac{b}{}\right.\right.}{3080a^3\,(a+bx^3)^{14/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

output
```
(x*(2282*a^4 + 4879*a^3*b*x^3 + 6270*a^2*b^2*x^6 + 3591*a*b^3*x^9 + 798*b^
4*x^12 + 798*(a + b*x^3)^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/
3, 4/3, -((b*x^3)/a)]))/(3080*a^3*(a + b*x^3)^(14/3))
```

3.55.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {930, 27, 910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{6ab(2a-bx^3)}{(bx^3+a)^{14/3}}dx}{14ab}+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\int \frac{2a-bx^3}{(bx^3+a)^{14/3}}dx+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{3}{7}\left(\frac{19}{11}\int \frac{1}{(bx^3+a)^{11/3}}dx+\frac{3x}{11(a+bx^3)^{11/3}}\right)+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{3}{7}\left(\frac{19\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{11/3}}dx}{11a^3(a+bx^3)^{2/3}}+\frac{3x}{11(a+bx^3)^{11/3}}\right)+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{3}{7}\left(\frac{19x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11a^3(a+bx^3)^{2/3}}+\frac{3x}{11(a+bx^3)^{11/3}}\right)+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

output
```
(x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (3*((3*x)/(11*(a + b*x^3)^(11/3))
 + (19*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)
/a)])/(11*a^3*(a + b*x^3)^(2/3))))/7
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.55.4 Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{17}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

3.55.5 Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^6*x^18 + 6*a*b^5
*x^15 + 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 +
a^6), x)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(17/3),x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

3.55.8 Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{17/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(17/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(17/3), x)
```

3.56 $\int (a+bx^3)^{5/3} (c+dx^3)\, dx$

3.56.1 Optimal result

Integrand size = 19, antiderivative size = 174

$$\int (a+bx^3)^{5/3} (c+dx^3)\, dx = \frac{5a(9bc-ad)x(a+bx^3)^{2/3}}{162b} + \frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b} + \frac{dx(a+bx^3)^{8/3}}{9b} + \frac{5a^2(9bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}} - \frac{5a^2(9bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{162b^{4/3}}$$

output
```
5/162*a*(-a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b+1/54*(-a*d+9*b*c)*x*(b*x^3+a)^(5/
3)/b+1/9*d*x*(b*x^3+a)^(8/3)/b-5/162*a^2*(-a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3
+a)^(1/3))/b^(4/3)+5/243*a^2*(-a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(10a^2d+9b^2x^3(3c+2dx^3)+ab(72c+33dx^3)\right)-10\sqrt{3}a^2(-9bc+ad)\arctan$$

input
```
Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(10*a^2*d + 9*b^2*x^3*(3*c + 2*d*x^3) + a*b
*(72*c + 33*d*x^3)) - 10*Sqrt[3]*a^2*(-9*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 10*a^2*(-9*b*c + a*d)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] - 5*a^2*(-9*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(4/3))
```

3.56.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {913, 748, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(9bc-ad)\int \left(bx^3+a\right)^{5/3}dx}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(9bc-ad)\left(\frac{5}{6}a\int \left(bx^3+a\right)^{2/3}dx+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(9bc-ad)\left(\frac{5}{6}a\left(\frac{2}{3}a\int\frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$\downarrow$ 769

$$\frac{(9bc-ad)\left(\frac{5}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

input
```
Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(8/3))/(9*b) + ((9*b*c - a*d)*((x*(a + b*x^3)^(5/3))/6 +
(5*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
]/(2*b^(1/3))))/3))/6))/(9*b)
```

3.56.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.56.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{5\left(-\frac{216\left(\frac{11 d x^3}{24}+c\right)x\left(b x^3+a\right)^{\frac{2}{3}} a b^{\frac{4}{3}}}{5}-\frac{81 x^4\left(b x^3+a\right)^{\frac{2}{3}}\left(\frac{2 d x^3}{3}+c\right)b^{\frac{7}{3}}}{5}+a^2\left(-6\left(b x^3+a\right)^{\frac{2}{3}} d x b^{\frac{1}{3}}+(ad-9bc)\left(-2\sqrt{3}\arctan\left(\right.\right.\right.\right.}{486 b^{\frac{4}{3}}}$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-5/486*(-216/5*(11/24*d*x^3+c)*x*(b*x^3+a)^(2/3)*a*b^(4/3)-81/5*x^4*(b*x^3
+a)^(2/3)*(2/3*d*x^3+c)*b^(7/3)+a^2*(-6*(b*x^3+a)^(2/3)*d*x*b^(1/3)+(a*d-9
*b*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)
/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((
-b^(1/3)*x+(b*x^3+a)^(1/3))/x))))/b^(4/3)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.77

$$\int (a+bx^3)^{5/3}(c+dx^3)\,dx = \left[-\frac{15\sqrt{\frac{1}{3}}(9a^2b^2c-a^3bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2\right.\right.}{}\right.$$

$$-\frac{10(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{30\sqrt{\frac{1}{3}}(9a}{}}{48}$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a
)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 1
0*(9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*
(9*a^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^
4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^
2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*
b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3
)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7
 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a
)^(2/3))/b^2]
```

3.56.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \left(a + bx^3\right)^{5/3} \left(c + dx^3\right)\, dx = \frac{a^{\frac{5}{3}} cx\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{bx^{3} e^{i\pi}}{a}} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{3}} dx^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{bx^{3} e^{i\pi}}{a}} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} bcx^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{bx^{3} e^{i\pi}}{a}} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} bdx^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{bx^{3} e^{i\pi}}{a}} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

output
```
a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hy
per((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/
3)*b*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(10/3))
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.33

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx =$$

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{486}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)
/b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x
^3 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(1/3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b
^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a
^3*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4
/3) + 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a
)^(2/3)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3
)*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3
 + a)^3*b/x^9))*d
```

3.56.8 Giac [F]

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{5/3}\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(5/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

3.57 $\int (a+bx^3)^{2/3} (c+dx^3)\, dx$

3.57.1 Optimal result

Integrand size = 19, antiderivative size = 141

$$\int (a+bx^3)^{2/3} (c+dx^3)\, dx = \frac{(6bc-ad)x(a+bx^3)^{2/3}}{18b} + \frac{dx(a+bx^3)^{5/3}}{6b} + \frac{a(6bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

output
```
1/18*(-a*d+6*b*c)*x*(b*x^3+a)^(2/3)/b+1/6*d*x*(b*x^3+a)^(5/3)/b-1/18*a*(-a
*d+6*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)+1/27*a*(-a*d+6*b*c)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28

$$\int (a+bx^3)^{2/3} (c+dx^3)\, dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}(6bc+2ad+3bdx^3) - 2\sqrt{3}a(-6bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2a(-6\ldots}{54b\ldots}$$

input
```
Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*b*c + 2*a*d + 3*b*d*x^3) - 2*Sqrt[3]*a*(
-6*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
] + 2*a*(-6*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-6*b*c +
 a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(54*b^(4/3))
```

3.57.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \; 913$$

$$\frac{(6bc-ad) \int \left(bx^3+a\right)^{2/3}dx}{6b}+\frac{dx \left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \; 748$$

$$\frac{(6bc-ad) \left(\frac{2}{3} a \int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3} x \left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{dx \left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \; 769$$

$$\frac{(6bc-ad) \left(\frac{2}{3} a \left(\frac{\arctan\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3} \sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b} x\right)}{2 \sqrt[3]{b}}\right)+\frac{1}{3} x \left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{dx \left(a+bx^3\right)^{5/3}}{6b}$$

input
```
Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(5/3))/(6*b) + ((6*b*c - a*d)*((x*(a + b*x^3)^(2/3))/3 +
(2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/
3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(6*b)
```

3.57.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.57.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{3\left(b x^3+a\right)^{\frac{2}{3}} x\left(\frac{d x^3}{2}+c\right) b^{\frac{4}{3}}+\left(\left(b x^3+a\right)^{\frac{2}{3}} d x\, b^{\frac{1}{3}}+\frac{(a d-6 b c)\left(\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b x^3+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{1}{3}}}{x}\right)-\right.}{3}\right.}{9 b^{\frac{4}{3}}}$

input
```
int((b*x^3+a)^(2/3)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/9/b^(4/3)*(3*(b*x^3+a)^(2/3)*x*(1/2*d*x^3+c)*b^(4/3)+((b*x^3+a)^(2/3)*d*
x*b^(1/3)+1/3*(a*d-6*b*c)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+
a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^
2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))*a)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.01

$$\int (a+bx^3)^{2/3} (c+dx^3)\, dx = \left[-\frac{3\sqrt{\frac{1}{3}}(6\,ab^2c-a^2bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-\right.\right.}{}\right.$$

$$-\frac{2\,(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{6\sqrt{\frac{1}{3}}(6\,ab^2c-a^2}{}}{54\,b^2}$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3
*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1
/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*
a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c
- a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3
))/b^2, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(
1/3))/x) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*
b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*ar
ctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*
(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

3.57.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx=\frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

output
```
a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(114) = 228.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.28

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx = -\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*ar
ctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*
log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3)
 + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3
)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 +
(b*x^3 + a)^2*b/x^6))*d
```

3.57.8 Giac [F]

$$\int (a+bx^3)^{2/3} (c+dx^3) \, dx = \int (bx^3+a)^{\frac{2}{3}} (dx^3+c) \, dx$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{2/3} (c+dx^3) \, dx = \int (b\,x^3+a)^{2/3} \,(d\,x^3+c) \, dx$$

input
```
int((a + b*x^3)^(2/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

3.58 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

3.58.1 Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}}{3b} + \frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
1/3*d*x*(b*x^3+a)^(2/3)/b-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/
b^(4/3)+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2
))/b^(4/3)*3^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{6\sqrt[3]{b}dx\left(a+bx^3\right)^{2/3} + 2\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2(-3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(1/3)*d*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*b*c + a*d)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + (3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

3.58.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)]/(2*b^(1/3))))/(3*b)
```

3.58.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 4.05 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$\frac{6\left(b\,x^{3}+a\right)^{\frac{2}{3}} d x\, b^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a d-6\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) b c+2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x\right.}{x}\right.}{}$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*(b*x^3+a)^(2/3)*d*x*b^(1/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a*d-6*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*b*c+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a
*d-6*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*b*c-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d+3*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b*c)/b^(4/3)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx - 3\sqrt{\frac{1}{3}}\left(3\,b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx\right.\right.}{}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b
^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/
3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(
3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*
d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2
/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*
(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

3.58.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.20

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$
$$+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

3.58.8 Giac [F]

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

3.59 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

3.59.1 Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx=\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output
```
(-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4
/3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1
/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx=\frac{\frac{6\sqrt[3]{b}(bc-ad)x}{a\sqrt[3]{a+bx^3}}+2\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+d\log}{6b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((6*b^(1/3)*(b*c - a*d)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*d*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)] + d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/(6*b^(4/3))
```

3.59.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 769$$

$$\frac{d\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*(ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)]/(2*b^(1/3))))/b
```

3.59.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.59.4 Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.63

method	result
pseudoelliptic	$\frac{6b^{\frac{4}{3}}cx-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-6adx\,b^{\frac{1}{3}}-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(b^{\frac{2}{3}}x^{2}\right.}{6b^{\frac{4}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a}$

input
```
int((d*x^3+c)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*b^(4/3)*c*x-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/
3))/b^(1/3)/x)*a*d*(b*x^3+a)^(1/3)-6*a*d*x*b^(1/3)-2*ln((-b^(1/3)*x+(b*x^3
+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3))/b^(4/3)/(b*x^3+a)^(1/3)/a
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.93

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-6\,(bx^3+a)^{\frac{2}{3}}(b^2c-abd)x+2}{6\,(ab^3x^3}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x
^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b)
 + 2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^
2*b*d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2
*(a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

3.59.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

output
```
c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamm
a(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(7/3))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx =$$

$$-\frac{1}{6}d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\log\left(-b\right.}{}\right.$$

$$+\frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3
)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

3.59.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

3.60 $\int \frac{c+dx^3}{(a+bx^3)^{7/3}} \, dx$

3.60.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}} \, dx = \frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x(c+dx^3)}{4a\,(a+bx^3)^{4/3}}$$

output
```
3/4*c*x/a^2/(b*x^3+a)^(1/3)+1/4*x*(d*x^3+c)/a/(b*x^3+a)^(4/3)
```

3.60.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}} \, dx = \frac{x(4ac+3bcx^3+adx^3)}{4a^2\,(a+bx^3)^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

3.60.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3c\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3)
)
```

3.60.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.60.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(ad\,x^{3}+3bc\,x^{3}+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34
trager	$\frac{x\left(ad\,x^{3}+3bc\,x^{3}+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34
pseudoelliptic	$\frac{x\left(ad\,x^{3}+3bc\,x^{3}+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34

input
```
int((d*x^3+c)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{((3\,bc+ad)x^4+4\,acx)(bx^3+a)^{\frac{2}{3}}}{4\,(a^2b^2x^6+2\,a^3bx^3+a^4)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b
*x^3 + a^4)
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(41) = 82$.

Time = 20.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

output
```
c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)
*gamma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)cx^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{dx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b
*x^3 + a)^(4/3)*a)
```

3.60.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{4\,a\,c\,x+a\,d\,x^4+3\,b\,c\,x^4}{4\,a^2\,(b\,x^3+a)^{4/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(7/3),x)
```

output
```
(4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

3.61 $\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx$

3.61.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx=\frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}}+\frac{(6bc+ad)x}{28a^2b\,(a+bx^3)^{4/3}}+\frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

output
```
1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^(7/3)+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^(4
/3)+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^(1/3)
```

3.61.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx=\frac{28a^2cx+42abcx^4+7a^2dx^4+18b^2cx^7+3abdx^7}{28a^3\,(a+bx^3)^{7/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(28*a^2*c*x + 42*a*b*c*x^4 + 7*a^2*d*x^4 + 18*b^2*c*x^7 + 3*a*b*d*x^7)/(28
*a^3*(a + b*x^3)^(7/3))
```

3.61.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}dx$$

$$\Big\downarrow \ 910$$

$$\frac{(ad+6bc)\int \frac{1}{(bx^3+a)^{7/3}}dx}{7ab}+\frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

$$\Big\downarrow \ 749$$

$$\frac{(ad+6bc)\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7ab}+\frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

$$\Big\downarrow \ 746$$

$$\frac{\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)(ad+6bc)}{7ab}+\frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*(x/(4*a*(a + b*
x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a*b)
```

3.61.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.61.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{2}+\frac{3x^{3}\left(\frac{d\,x^{3}}{14}+c\right)ba}{2}+\frac{9b^{2}c\,x^{6}}{14}\right)}{\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	52
gosper	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57
trager	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57

input
```
int((d*x^3+c)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^3+a)^(7/3)*((1/4*d*x^3+c)*a^2+3/2*x^3*(1/14*d*x^3+c)*b*a+9/14*b^2*c
*x^6)/a^3
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(3\,(6\,b^2c+abd)x^7+7\,(6\,abc+a^2d)x^4+28\,a^2cx)(bx^3+a)^{\frac{2}{3}}}{28\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x
^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(83) = 166$.

Time = 100.01 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = c\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+}\right.$$

$$+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\right.}$$

$$+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\right.}$$

$$+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\right.}$$

$$+d\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right.$$

$$\left.+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

output
```
c*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) + d*(7*a*x**4*gamma(4/3)/(9*
a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gam
ma(10/3)) + 3*b*x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(1
0/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*
b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)dx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)cx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
-1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^
2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a
^3)
```

3.61.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{3\,a\,d\,x\,(b\,x^3+a)^2-4\,a^3\,d\,x+18\,b\,c\,x\,(b\,x^3+a)^2+a^2\,d\,x\,(b\,x^3+a)+4\,a^2\,b\,c\,x+6\,a\,b}{28\,a^3\,b\,(b\,x^3+a)^{7/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

output
```
(3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
 b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3)
)
```

3.62 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

3.62.1 Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

output
```
1/10*(-a*d+b*c)*x/a/b/(b*x^3+a)^(10/3)+1/70*(a*d+9*b*c)*x/a^2/b/(b*x^3+a)^
(7/3)+3/140*(a*d+9*b*c)*x/a^3/b/(b*x^3+a)^(4/3)+9/140*(a*d+9*b*c)*x/a^4/b/
(b*x^3+a)^(1/3)
```

3.62.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{x\left(81b^3cx^9+35a^3\left(4c+dx^3\right)+9ab^2x^6\left(30c+dx^3\right)+15a^2bx^3\left(21c+2dx^3\right)\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
(x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*
a^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

3.62.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+9bc)\int \frac{1}{(bx^3+a)^{10/3}}dx}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+9bc)\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+9bc)\left(\frac{6\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)(ad+9bc)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*(x/(7*a*(a +
b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(
1/3))))/(7*a)))/(10*a*b)
```

3.62.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.62.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{3}+\frac{9x^{3}b\left(\frac{2d\,x^{3}}{21}+c\right)a^{2}}{4}+\frac{27x^{6}b^{2}\left(\frac{d\,x^{3}}{30}+c\right)a}{14}+\frac{81b^{3}c\,x^{9}}{140}\right)}{\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	71
gosper	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}x^{3}bc+140c\,a^{3}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81
trager	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}x^{3}bc+140c\,a^{3}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81

input
```
int((d*x^3+c)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^3+a)^(10/3)*((1/4*d*x^3+c)*a^3+9/4*x^3*b*(2/21*d*x^3+c)*a^2+27/14*x
^6*b^2*(1/30*d*x^3+c)*a+81/140*b^3*c*x^9)/a^4
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(9\,(9\,b^3c+ab^2d)x^{10}+30\,(9\,ab^2c+a^2bd)x^7+140\,a^3cx+35\,(9\,a^2bc+a^3d)x^4)(bx^3+a)}{140\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3
*c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5
*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)dx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)cx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^4}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
 a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

3.62.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx=\frac{x\left(\frac{c}{10\,a}-\frac{d}{10\,b}\right)}{(b\,x^3+a)^{10/3}}+\frac{x\,(a\,d+9\,b\,c)}{70\,a^2\,b\,(b\,x^3+a)^{7/3}}+\frac{x\,(3\,a\,d+27\,b\,c)}{140\,a^3\,b\,(b\,x^3+a)^{4/3}}+\frac{x\,(9\,a\,d+81\,b\,c)}{140\,a^4\,b\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(13/3),x)
```

output
```
(x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b
*(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) +
 (x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

3.63 $\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx$

3.63.1 Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\,(a+bx^3)^{4/3}} + \frac{81(12bc+ad)x}{1820a^5b\sqrt[3]{a+bx^3}}$$

output
```
1/13*(-a*d+b*c)*x/a/b/(b*x^3+a)^(13/3)+1/130*(a*d+12*b*c)*x/a^2/b/(b*x^3+a
)^(10/3)+9/910*(a*d+12*b*c)*x/a^3/b/(b*x^3+a)^(7/3)+27/1820*(a*d+12*b*c)*x
/a^4/b/(b*x^3+a)^(4/3)+81/1820*(a*d+12*b*c)*x/a^5/b/(b*x^3+a)^(1/3)
```

3.63.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{x(972b^4cx^{12}+455a^4(4c+dx^3)+351a^2b^2x^6(20c+dx^3)+81ab^3x^9(52c+dx^3)+195a}{1820a^5\,(a+bx^3)^{13/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

output
```
(x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3
) + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a
^5*(a + b*x^3)^(13/3))
```

3.63.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}} \, dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+12bc)\int \frac{1}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{(ad+12bc)\left(\frac{9\int \frac{1}{\left(bx^3+a\right)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{(ad+12bc)\left(\frac{9\left(\frac{6\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{(ad+12bc)\left(\frac{9\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \ 746$$

$$\frac{\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)(ad+12bc)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

input

```
Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

output

```
((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*(x/(10*a*(a
+ b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^
(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(10*a)))/(13*a*b)
```

3.63. $\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.63.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.60

method	result	s
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{4}+3\left(\frac{3d\,x^{3}}{28}+c\right)x^{3}b\,a^{3}+\frac{27x^{6}b^{2}\left(\frac{d\,x^{3}}{20}+c\right)a^{2}}{7}+\frac{81x^{9}\left(\frac{d\,x^{3}}{52}+c\right)b^{3}a}{35}+\frac{243b^{4}c\,x^{12}}{455}\right)}{\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	9
gosper	$\frac{x\left(81a\,b^{3}d\,x^{12}+972b^{4}c\,x^{12}+351a^{2}b^{2}d\,x^{9}+4212a\,b^{3}c\,x^{9}+585a^{3}bd\,x^{6}+7020a^{2}b^{2}c\,x^{6}+455a^{4}d\,x^{3}+5460a^{3}bc\,x^{3}+1820a^{4}c\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	
trager	$\frac{x\left(81a\,b^{3}d\,x^{12}+972b^{4}c\,x^{12}+351a^{2}b^{2}d\,x^{9}+4212a\,b^{3}c\,x^{9}+585a^{3}bd\,x^{6}+7020a^{2}b^{2}c\,x^{6}+455a^{4}d\,x^{3}+5460a^{3}bc\,x^{3}+1820a^{4}c\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	

input
```
int((d*x^3+c)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
x*((1/4*d*x^3+c)*a^4+3*(3/28*d*x^3+c)*x^3*b*a^3+27/7*x^6*b^2*(1/20*d*x^3+c
)*a^2+81/35*x^9*(1/52*d*x^3+c)*b^3*a+243/455*b^4*c*x^12)/(b*x^3+a)^(13/3)/
a^5
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(81\,(12\,b^4c+ab^3d)x^{13}+351\,(12\,ab^3c+a^2b^2d)x^{10}+585\,(12\,a^2b^2c+a^3bd)x^7+1820\,a^4}{1820\,(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9b}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
 585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 +
 10*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = -\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)dx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)cx^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
-1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455
*(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*
(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9
+ 455*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

3.63.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{(b\,x^3+a)^{13/3}} + \frac{x\,(a\,d+12\,b\,c)}{130\,a^2\,b\,(b\,x^3+a)^{10/3}} + \frac{x\,(9\,a\,d+108\,b\,c)}{910\,a^3\,b\,(b\,x^3+a)^{7/3}} + \frac{x\,(27\,a\,d+324\,b\,c)}{1820\,a^4\,b\,(b\,x^3+a)^{4/3}} + \frac{x\,(81\,a\,d+972\,b\,c)}{1820\,a^5\,b\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

output
```
(x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2
*b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3
)) + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d +
972*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

3.64 $\int (a+bx^3)^{7/3}(c+dx^3)\,dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (a+bx^3)^{7/3}(c+dx^3)\,dx = \frac{dx(a+bx^3)^{10/3}}{11b} + \frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/11*d*x*(b*x^3+a)^(10/3)/b+1/11*a^2*(-a*d+11*b*c)*x*(b*x^3+a)^(1/3)*hyper
geom([-7/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

3.64.2 Mathematica [A] (verified)

Time = 9.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int (a+bx^3)^{7/3}(c+dx^3)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^3-\frac{a^2(-11bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{11b}$$

input
```
Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometri
c2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

3.64.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(11bc-ad)\int \left(bx^3+a\right)^{7/3}dx}{11b}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{a^2\sqrt[3]{a+bx^3}(11bc-ad)\int \left(\frac{bx^3}{a}+1\right)^{7/3}dx}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{a^2x\sqrt[3]{a+bx^3}(11bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

input
```
Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*
Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/
3))
```

3.64.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.64.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

3.64.5 Fricas [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)\, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*
c)*(b*x^3 + a)^(1/3), x)
```

3.64.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.12

$$\int (a+bx^3)^{7/3}(c+dx^3)\,dx = \frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}b^2dx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

output
```
a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*
*(4/3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*
pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3)
, (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**
10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

3.64.7 Maxima [F]

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

3.64.8 Giac [F]

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{7/3}\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(7/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(7/3)*(c + d*x^3), x)
```

3.65 $\int (a+bx^3)^{4/3} (c+dx^3)\, dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int (a+bx^3)^{4/3} (c+dx^3)\, dx = \frac{dx(a+bx^3)^{7/3}}{8b} + \frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/8*d*x*(b*x^3+a)^(7/3)/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([
-4/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

3.65.2 Mathematica [A] (verified)

Time = 8.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a+bx^3)^{4/3} (c+dx^3)\, dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^2 - \frac{a(-8bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{8b}$$

input
```
Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F
1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)
```

3.65.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(8bc-ad) \int \left(bx^3+a\right)^{4/3}dx}{8b}+\frac{dx \left(a+bx^3\right)^{7/3}}{8b}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{a \sqrt[3]{a+bx^3} (8bc-ad) \int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{8b \sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx \left(a+bx^3\right)^{7/3}}{8b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{ax \sqrt[3]{a+bx^3} (8bc-ad) \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b \sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx \left(a+bx^3\right)^{7/3}}{8b}$$

input
```
Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyper
geometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

3.65.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.65.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

3.65.5 Fricas [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)
```

3.65.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx = \frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{4}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
```

output
```
a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hy
per((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(10/3))
```

3.65.7 Maxima [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx = \int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

3.65.8 Giac [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\, dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(dx^3+c\right) dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\, dx = \int \left(b\,x^3+a\right)^{4/3} \left(d\,x^3+c\right)\, dx$$

input
```
int((a + b*x^3)^(4/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)*(c + d*x^3), x)
```

3.66 $\int \sqrt[3]{a+bx^3}(c+dx^3)\,dx$

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^3}(c+dx^3)\,dx = \frac{dx(a+bx^3)^{4/3}}{5b} + \frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1
/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

3.66.2 Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{a+bx^3}(c+dx^3)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)+\frac{(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/
3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

3.66.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(5bc-ad)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^3}(5bc-ad)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperge
ometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

3.66.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.66.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

3.66.5 Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

3.66.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

output
```
a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.66.7 Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

3.66.8 Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

3.67 $\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx$

3.67.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx = \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

output
```
1/2*d*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d+2*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([
1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

3.67.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx = \frac{dx(a+bx^3)+(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

3.67.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(2bc-ad)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2b}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2b\,(a+bx^3)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hyp
ergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

3.67.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.67.4 Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

3.67.5 Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{dx^{3}+c}{(bx^{3}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

3.67.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

3.67.7 Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

3.67.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{2/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(2/3), x)
```

3.68 $\int \frac{c+dx^3}{(a+bx^3)^{5/3}} \, dx$

3.68.1 Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}} \, dx = \frac{(bc-ad)x}{2ab(a+bx^3)^{2/3}} + \frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab(a+bx^3)^{2/3}}$$

output
```
1/2*(-a*d+b*c)*x/a/b/(b*x^3+a)^(2/3)+1/2*(a*d+b*c)*x*(1+b*x^3/a)^(2/3)*hyp
ergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

3.68.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}} \, dx = \frac{-adx+(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3
, 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

3.68.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+bc)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2ab\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)
^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^
(2/3))
```

3.68.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.68.4 Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

input
```
int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

3.68.5 Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.68.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

3.68.7 Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

3.68.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{5/3}} \,d x$$

input
```
int((c + d*x^3)/(a + b*x^3)^(5/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(5/3), x)
```

3.69 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx$

3.69.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx=\frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}$$

output
```
1/5*(-a*d+b*c)*x/a/b/(b*x^3+a)^(5/3)+1/5*(a*d+4*b*c)*x*(1+b*x^3/a)^(2/3)*h
ypergeom([1/3, 5/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

3.69.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx=\frac{x\left(-d+\frac{(4bc+ad)\left(a+bx^3\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{a^2}\right)}{4b\left(a+bx^3\right)^{5/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F
1[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

3.69.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+4bc)\int \frac{1}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}}dx}{5a^2b\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/
a)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x
^3)^(2/3))
```

3.69.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.69.4 Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}dx$$

input
```
int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

3.69.5 Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^
3 + a^3), x)
```

3.69.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 35.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{8}{3}\\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3}, \frac{8}{3}\\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
8/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

3.69.7 Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

3.69.8 Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx=\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{8/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(8/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(8/3), x)
```

3.70 $\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 262

$$\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx = \frac{5a(27b^2c^2-6abcd+a^2d^2)\, x(a+bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2-6abcd+a^2d^2)\, x(a+bx^3)^{5/3}}{162b^2} + \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} + \frac{dx(a+bx^3)^{8/3}\,(c+dx^3)}{12b} + \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{243\sqrt{3}b^{7/3}} - \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{486b^{7/3}}$$

output
```
5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^
2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/108*d*(-4*a*d+15*b*c)*x*(b
*x^3+a)^(8/3)/b^2+1/12*d*x*(b*x^3+a)^(8/3)*(d*x^3+c)/b-5/486*a^2*(a^2*d^2-
6*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+5/729*a^2*(a^
2*d^2-6*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(7/3)*3^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 dx = \frac{3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}\left(-20a^3d^2+15a^2bd(8c+dx^3)+27b^3x^3(6c^2+8cdx^3+3d^2x^6)+18ab^2(24c^2\right.}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 2
7*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3
+ 7*d^2*x^6)) + 20*Sqrt[3]*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 20*a^2*(27*b^2*c^2
 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 10*a^2*(27
*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)])/(2916*b^(7/3))
```

3.70.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {933, 913, 748, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \left(bx^3+a\right)^{5/3} \left(d(15bc-4ad)x^3+c(12bc-ad)\right)dx}{12b}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{4\left(a^2d^2-6abcd+27b^2c^2\right)\int \left(bx^3+a\right)^{5/3}dx}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}$$

$$\Big\downarrow \text{748}$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\int (bx^3+a)^{2/3}dx+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

$$\Big\downarrow \; 748$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x(a+bx^3)^{2/3}\right)+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

$$\Big\downarrow \; 769$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x(a+bx^3)^{2/3}\right)+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx^3}{}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

input
```
Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(8/3)*(c + d*x^3))/(12*b) + ((d*(15*b*c - 4*a*d)*x*(a + b
*x^3)^(8/3))/(9*b) + (4*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*((x*(a + b*x^3)
^(5/3))/6 + (5*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a +
b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6))/(9*b))/(12*b)
```

3.70. $\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx$

3.70.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.70.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{x^{4}\left(\frac{1}{2}d^{2}x^{6}+\frac{4}{3}cd\,x^{3}+c^{2}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}b^{\frac{10}{3}}}{6}+\frac{5\left(36xd\left(\frac{d\,x^{3}}{8}+c\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}a\,b^{\frac{4}{3}}+\frac{648x\left(\frac{7}{24}d^{2}x^{6}+\frac{11}{12}cd\,x^{3}+c^{2}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}b^{\frac{7}{3}}}{5}+\left(-6\left(b\,x^{3}\right.\right.\right.}{}}{}$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
5/1458/b^(7/3)*(243/5*x^4*(1/2*d^2*x^6+4/3*c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^
(10/3)+(36*x*d*(1/8*d*x^3+c)*(b*x^3+a)^(2/3)*a*b^(4/3)+648/5*x*(7/24*d^2*x
^6+11/12*c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^(7/3)+(-6*(b*x^3+a)^(2/3)*a*d^2*x*
b^(1/3)+(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/
3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln
((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a^2*d^2-6*a*b*c*d+27*b^2*c^2))*a)*a)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.74

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \left[\frac{30\sqrt{\frac{1}{3}}(27\,a^2b^3c^2-6\,a^3b^2cd+a^4bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)\right.\right.}{}\right.$$

$$-\frac{60\sqrt{\frac{1}{3}}(27\,a^2b^3c^2-6\,a^3b^2cd+a^4bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+20\,(27\,a^2b^2c^2\cdots}{}$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-
b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2
/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*
d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2
*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)
^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(1
2*b^4*c*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d
^2)*x^4 + 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^
(2/3))/b^3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*
d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(
1/3))*sqrt(-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2
 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^
4*c*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*
x^4 + 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3
))/b^3]
```

3.70.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{a^{\frac{5}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{5}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{5}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{2}{3}}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

output
```
a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma
(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*
d**2*x**10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(227) = 454$.

Time = 0.28 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.56

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx =$$

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{243}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{2916}\left(\frac{20\sqrt{3}a^4\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{10\,a^4\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{20\,a^4\log\left(-b^{\frac{1}{3}}\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

3.70. $\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx$

output
```
-1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)
/b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x
^3 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(1/3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b
^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3
*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5
*a^3*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^
(4/3) + 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 +
 a)^(2/3)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8
/3)*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x
^3 + a)^3*b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3)
 + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 +
 a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3)
 + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36
*(b*x^3 + a)^(5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^
3 + a)^(11/3)*a^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4
/x^6 - 4*(b*x^3 + a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

3.70.8 Giac [F]

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{5/3}\left(d\,x^3+c\right)^2\,dx$$

input
```
int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

3.71 $\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx$

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 219

$$\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx = \frac{(27b^2c^2-9abcd+2a^2d^2)\,x(a+bx^3)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2} + \frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b} + \frac{2a(27b^2c^2-9abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{7/3}} - \frac{a(27b^2c^2-9abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81b^{7/3}}$$

output
```
1/81*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+2/27*d*(-a*d+3
*b*c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d*x*(b*x^3+a)^(5/3)*(d*x^3+c)/b-1/81*a*(2*
a^2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2/243
*a*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1
/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx=\frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(-4a^2d^2+3abd(6c+dx^3)+9b^2(3c^2+3cdx^3+d^2x^6)\right)+2\sqrt{3}a(27b^2c^2-9ab}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2
*(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + 2*Sqrt[3]*a*(27*b^2*c^2 - 9*a*b*c*d + 2*
a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2
*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(7/3))
```

3.71.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {933, 913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \left(bx^3+a\right)^{2/3}\left(4d(3bc-ad)x^3+c(9bc-ad)\right)dx}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(2a^2d^2-9abcd+27b^2c^2\right)\int \left(bx^3+a\right)^{2/3}dx}{3b}+\frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{\left(2a^2d^2-9abcd+27b^2c^2\right)\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{3b}+\frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

$\Big\downarrow$ 769

$$\frac{\frac{\left(2a^2d^2-9abcd+27b^2c^2\right)\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{3b}+\frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

input
```
Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(5/3)*(c + d*x^3))/(9*b) + ((2*d*(3*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(3*b) + ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*((x*(a + b*x^3)^(
2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqr
t[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(3
*b))/(9*b)
```

3.71.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.71.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{4\left(-\frac{27x\left(bx^3+a\right)^{\frac{2}{3}}da\left(\frac{dx^3}{6}+c\right)b^{\frac{4}{3}}}{2}-\frac{81x\left(\frac{1}{3}d^2x^6+cdx^3+c^2\right)\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{7}{3}}}{4}+\left(3\left(bx^3+a\right)^{\frac{2}{3}}ad^2xb^{\frac{1}{3}}+\left(a^2d^2-\frac{9}{2}abcd+\frac{27}{2}b^2c^2\right)\right.\right.}{243b^{\frac{7}{3}}}$

input
```
int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/243*(-27/2*x*(b*x^3+a)^(2/3)*d*a*(1/6*d*x^3+c)*b^(4/3)-81/4*x*(1/3*d^2*
x^6+c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^(7/3)+(3*(b*x^3+a)^(2/3)*a*d^2*x*b^(1/3
)+(a^2*d^2-9/2*a*b*c*d+27/2*b^2*c^2)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*a)/b^(7/3)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.89

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \left[\frac{3\sqrt{\frac{1}{3}}(27\,ab^3c^2-9\,a^2b^2cd+2\,a^3bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)\right.\right.}{}\right.$$

$$-\frac{6\,\sqrt{\frac{1}{3}}(27\,ab^3c^2-9\,a^2b^2cd+2\,a^3bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(27\,ab^2c^2-9}{}$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)
^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3
)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2
)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9
*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d
+ a*b^2*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a
)^(2/3))/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*
d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(
1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a
^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d +
a*b^2*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^
(2/3))/b^3]
```

3.71.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.60

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{a^{\frac{2}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{2}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

output
```
a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma
(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(188) = 376.

Time = 0.28 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.52

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{27}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^3\log\left(-b^{\frac{1}{3}}+\left(\right.\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

3.71. $\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx$

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*
arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^
2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/
3) + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2
/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3
+ (b*x^3 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(
1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^
3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/
3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 1
1*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x
^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

3.71.8 Giac [F]

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{2/3}\left(d\,x^3+c\right)^2\,dx$$

input
```
int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

3.72 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

output
```
1/18*d*(-4*a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d*x*(b*x^3+a)^(2/3)*(d*x^3
+c)/b-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/
b^(7/3)+1/27*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*
x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{3\sqrt[3]{b}dx(a+bx^3)^{2/3}(-4ad+3b(4c+dx^3))+2\sqrt{3}(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
(3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + 2*Sqrt[3]*
(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
+ 2*(a + b*x^3)^(1/3))] - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(7/3))
```

3.72.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {933, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int \frac{d(9bc-4ad)x^3+c(6bc-ad)}{\sqrt[3]{bx^3+a}}dx}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$
$$\Big\downarrow \text{913}$$
$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$
$$\Big\downarrow \text{769}$$

$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d*x*(a + b*x^3)^(2/3)*(c + d*x^3))/(6*b) + ((d*(9*b*c - 4*a*d)*x*(a + b*x
^3)^(2/3))/(3*b) + (2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(ArcTan[(1 + (2*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b))/(6*b)
```

3.72.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.72.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(-9x\left(\frac{d\,x^3}{4}+c\right)d\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{4}{3}}+3\left(b\,x^3+a\right)^{\frac{2}{3}}a\,d^2x\,b^{\frac{1}{3}}+\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right.\right.\right.}{27b^{\frac{7}{3}}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-2/27*(-9*x*(1/4*d*x^3+c)*d*(b*x^3+a)^(2/3)*b^(4/3)+3*(b*x^3+a)^(2/3)*a*d^
2*x*b^(1/3)+(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1
/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-3*a*b*c*d+9/2*b^2*c^2))/b^(7
/3)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.17

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(9\,b^3c^2-6\,ab^2cd+2\,a^2bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(9\,b^3c^2-6\,ab^2cd+2\,a^2bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(9\,b^2c^2-6\,ab}{}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)
/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(
1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*s
qrt((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3
)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a
^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b
*x^3 + a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*
b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b
)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d
 + 2*a^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3
)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)
*x)*(b*x^3 + a)^(2/3))/b^3]
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper
((1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(148) = 296$.

Time = 0.28 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*
(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a
^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)
^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x
^3 + (b*x^3 + a)^2*b^2/x^6))*d^2
```

3.72.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.73 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}} dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}} dx = -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2} + \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{7/3}}$$

output
```
-1/3*d*(-4*a*d+3*b*c)*x*(b*x^3+a)^(2/3)/a/b^2+(-a*d+b*c)*x*(d*x^3+c)/a/b/(
b*x^3+a)^(1/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
+2/9*d*(-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(7/3)*3^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}} dx = \frac{\frac{3\sqrt[3]{b}x(3b^2c^2+4a^2d^2+abd(-6c+dx^3))}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}d(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2d(-3\ldots}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((3*b^(1/3)*x*(3*b^2*c^2 + 4*a^2*d^2 + a*b*d*(-6*c + d*x^3)))/(a*(a + b*x^
3)^(1/3)) + 2*Sqrt[3]*d*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3
)*x + 2*(a + b*x^3)^(1/3))] + 2*d*(-3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a +
 b*x^3)^(1/3)] + d*(3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)
^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

3.73.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{d\left(ac-(3bc-4ad)x^3\right)}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{ac-(3bc-4ad)x^3}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}-\frac{x\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}-\frac{x(a+bx^3)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(a*b*(a + b*x^3)^(1/3)) + (d*(-1/3*((3*b*c - 4
*a*d)*x*(a + b*x^3)^(2/3))/b + (2*a*(3*b*c - 2*a*d)*(ArcTan[(1 + (2*b^(1/3
)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a
 + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(a*b)
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.73. $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.73.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{2\left(d\left(ad-\frac{3bc}{2}\right)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+\right.}{x}\right.\right.\right.}{9b^{\frac{7}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}a}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(d*(a*d-3/2*b*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a
)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*a*(b*x^3+a)^(1/3)-6*x*(-3/
2*d*(-1/6*d*x^3+c)*a*b^(4/3)+a^2*d^2*b^(1/3)+3/4*b^(7/3)*c^2))/b^(7/3)/(b*
x^3+a)^(1/3)/a
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(134) = 268.

Time = 0.33 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.10

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(3\,a^2b^2cd-2\,a^3bd^2+(3\,ab^3cd-2\,a^2b^2d^2)x^3)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b\right.}{}\right.$$

$$-\frac{2\,(3\,a^2bcd-2\,a^3d^2+(3\,ab^2cd-2\,a^2bd^2)x^3)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(3\,a^2bcd-2\,a^3d^2+(3\,ab^2cd-2\,a^2b}{}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2
*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2
- 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)
*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^
2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
- (3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(
(b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(
a*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/
3))/(a*b^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d
 - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2
/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1
/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a
*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3
))/(a*b^4*x^3 + a^2*b^3)]
```

3.73. $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

3.73.6 Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(134) = 268.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,d^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{(b\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{1}{3}\,cd\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-\cdots\right.}{\cdots}\right.$$
$$+\frac{c^2x}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/
x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3)
+ (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^
(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

3.73.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.74 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{7/3}}$$

output
```
1/4*(-a*d+b*c)*(4*a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(1/3)+1/4*(-a*d+b*c)*x*(d
*x^3+c)/a/b/(b*x^3+a)^(4/3)-1/2*d^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
+1/3*d^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/
2)
```

3.74.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{\frac{3\sqrt[3]{b}(bc-ad)x(4a^2d+3b^2cx^3+ab(4c+5dx^3))}{a^2(a+bx^3)^{4/3}} + 4\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 4d^2\log\left(-\sqrt[3]{\right.}{12b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((3*b^(1/3)*(b*c - a*d)*x*(4*a^2*d + 3*b^2*c*x^3 + a*b*(4*c + 5*d*x^3)))/(
a^2*(a + b*x^3)^(4/3)) + 4*Sqrt[3]*d^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2*(a + b*x^3)^(1/3))] - 4*d^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 2*d^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(12*b^(7/3))
```

3.74.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {930, 910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{4ad^2x^3+c(3bc+ad)}{(bx^3+a)^{4/3}}dx}{4ab}+\frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{ 910}$$

$$\frac{\frac{4ad^2\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b}+\frac{x(bc-ad)(4ad+3bc)}{ab\sqrt[3]{a+bx^3}}}{4ab}+\frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{\frac{4ad^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}+\frac{x(bc-ad)(4ad+3bc)}{ab\sqrt[3]{a+bx^3}}}{4ab}+\frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (((b*c - a*d)*(3*b
*c + 4*a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (4*a*d^2*(ArcTan[(1 + (2*b^(1/3)*
x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a +
 b*x^3)^(1/3)]/(2*b^(1/3))))/b)/(4*a*b)
```

3.74.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.74.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{-3x\left(\frac{d\,x^{3}}{2}+c\right)ca\,b^{\frac{7}{3}}-\frac{9b^{\frac{10}{3}}c^{2}x^{4}}{4}+d^{2}\left(\frac{15b^{\frac{4}{3}}x^{4}}{4}+3xa\,b^{\frac{1}{3}}+\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+2\ln\left(\frac{-b^{\frac{1}{3}}x+\cdots}{\cdots}\right.\cdots}{2}\cdots}{3b^{\frac{7}{3}}\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x*(1/2*d*x^3+c)*c*a*b^(7/3)-9/4*b^(10/3)*c^2*x^4+d^2*(15/4*b^(4/3
)*x^4+3*x*a*b^(1/3)+1/2*(b*x^3+a)^(4/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(
1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-
ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))*a^2)/b^(
7/3)/(b*x^3+a)^(4/3)/a^2
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(128) = 256$.

Time = 0.40 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.73

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(a^2b^3d^2x^6+2\,a^3b^2d^2x^3+a^4bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{}\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}(a^2b^3d^2x^6+2\,a^3b^2d^2x^3+a^4bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+4\,(a^2b^2d^2x^6}{}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(
(-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/
3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^
(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3
+ a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b
^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (
b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 +
2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a
)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2
*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-
sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*
(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x
 + (b*x^3 + a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
+ a)^(2/3))/x^2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a
*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 +
 a^4*b^3)]
```

3.74.6 Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)c^2x^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{cdx^4}{2\,(bx^3+a)^{\frac{4}{3}}a} - \frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4}{}\right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4
/((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^
(4/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x
)/b^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
+ a)^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d
^2
```

3.74.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.75 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

output
```
9/14*c^2*x/a^3/(b*x^3+a)^(1/3)+3/14*c*x*(d*x^3+c)/a^2/(b*x^3+a)^(4/3)+1/7*
x*(d*x^3+c)^2/a/(b*x^3+a)^(7/3)
```

3.75.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{9b^2c^2x^7+3abcx^4(7c+dx^3)+a^2(14c^2x+7cdx^4+2d^2x^7)}{14a^3(a+bx^3)^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(9*b^2*c^2*x^7 + 3*a*b*c*x^4*(7*c + d*x^3) + a^2*(14*c^2*x + 7*c*d*x^4 + 2
*d^2*x^7))/(14*a^3*(a + b*x^3)^(7/3))
```

3.75.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{6c\int \frac{dx^3+c}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{6c\left(\frac{3c\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{6c\left(\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*c*((3*c*x)/(4*a^2*(a + b*x^
3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))))/(7*a)
```

3.75.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.75.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{\left(2 a^{2} d^{2}+3 a b c d+9 b^{2} c^{2}\right) x^{7}+7 \left(a^{2} c d+3 b \,c^{2} a \right) x^{4}+14 a^{2} c^{2} x}{14 \left(b \,x^{3}+a \right)^{\frac{7}{3}} a^{3}}$	71
gosper	$\frac{x \left(2 a^{2} d^{2} x^{6}+3 a b c d \,x^{6}+9 b^{2} c^{2} x^{6}+7 a^{2} c d \,x^{3}+21 a b \,c^{2} x^{3}+14 a^{2} c^{2}\right)}{14 \left(b \,x^{3}+a \right)^{\frac{7}{3}} a^{3}}$	76
trager	$\frac{x \left(2 a^{2} d^{2} x^{6}+3 a b c d \,x^{6}+9 b^{2} c^{2} x^{6}+7 a^{2} c d \,x^{3}+21 a b \,c^{2} x^{3}+14 a^{2} c^{2}\right)}{14 \left(b \,x^{3}+a \right)^{\frac{7}{3}} a^{3}}$	76

input
```
int((d*x^3+c)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/14*((2*a^2*d^2+3*a*b*c*d+9*b^2*c^2)*x^7+7*(a^2*c*d+3*a*b*c^2)*x^4+14*a^2
*c^2*x)/(b*x^3+a)^(7/3)/a^3
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{((9\,b^2c^2+3\,abcd+2\,a^2d^2)x^7+14\,a^2c^2x+7\,(3\,abc^2+a^2cd)x^4)(bx^3+a)^{\frac{2}{3}}}{14\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^
2 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b
*x^3 + a^6)
```

3.75.6 Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{10}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(10/3), x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)cdx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{d^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)c^2x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
-1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*
x^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3
+ a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

3.75.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{2\,a^4\,d^2\,x+2\,a^2\,d^2\,x\,{\left(b\,x^3+a\right)}^2+9\,b^2\,c^2\,x\,{\left(b\,x^3+a\right)}^2+2\,a^2\,b^2\,c^2\,x-4\,a^3\,d^2\,x\,\left(b\,x^3+a\right)}{14\,a^3\,b^2\,\left(b\,x^3\right.}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

output
```
(2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a
^2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3
*b*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^
2*(a + b*x^3)^(7/3))
```

3.76 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\,(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}$$

output
```
9/140*c^2*(-10*a*d+9*b*c)*x/a^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+3/140*c*(-10*a*
d+9*b*c)*x*(d*x^3+c)/a^3/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/70*(-10*a*d+9*b*c)*x
*(d*x^3+c)^2/a^2/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/10*b*x*(d*x^3+c)^3/a/(-a*d+b
*c)/(b*x^3+a)^(10/3)
```

3.76.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x(81b^3c^2x^9 + 18ab^2cx^6(15c+dx^3) + 10a^3(14c^2+7cdx^3+2d^2x^6) + 3a^2bx^3(105c^2 + 2}{140a^4\,(a+bx^3)^{10/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*
d*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140
*a^4*(a + b*x^3)^(10/3))
```

3.76.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {907, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{(9bc-10ad)\int \frac{(dx^3+c)^2}{(bx^3+a)^{10/3}}dx}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(9bc-10ad)\left(\frac{6c\int \frac{dx^3+c}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(9bc-10ad)\left(\frac{6c\left(\frac{3c\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{746}$$

$$\frac{(9bc-10ad)\left(\frac{6c\left(\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3)) + ((9*b*c - 10*a
*d)*((x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*c*((3*c*x)/(4*a^2*(a +
 b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))))/(7*a)))/(10*a*(
b*c - a*d))
```

3.76.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.76.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$\frac{x \left(\left(\frac{1}{7} d^{2} x^{6}+\frac{1}{2} c d \,x^{3}+c^{2}\right) a^{3}+\frac{9 x^{3} b \left(\frac{2}{105} d^{2} x^{6}+\frac{4}{21} c d \,x^{3}+c^{2}\right) a^{2}}{4}+\frac{27 x^{6} \left(\frac{d \,x^{3}}{15}+c \right) b^{2} c a}{14}+\frac{81 b^{3} c^{2} x^{9}}{140}\right)}{\left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{4}}$	96
gosper	$\frac{x \left(6 a^{2} b \,d^{2} x^{9}+18 a \,b^{2} c d \,x^{9}+81 b^{3} c^{2} x^{9}+20 a^{3} d^{2} x^{6}+60 a^{2} b c d \,x^{6}+270 a \,b^{2} c^{2} x^{6}+70 a^{3} c d \,x^{3}+315 a^{2} b \,c^{2} x^{3}+140 a^{3} c^{2}\right)}{140 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{4}}$	115
trager	$\frac{x \left(6 a^{2} b \,d^{2} x^{9}+18 a \,b^{2} c d \,x^{9}+81 b^{3} c^{2} x^{9}+20 a^{3} d^{2} x^{6}+60 a^{2} b c d \,x^{6}+270 a \,b^{2} c^{2} x^{6}+70 a^{3} c d \,x^{3}+315 a^{2} b \,c^{2} x^{3}+140 a^{3} c^{2}\right)}{140 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{4}}$	115

input
```
int((d*x^3+c)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^3+9/4*x^3*b*(2/105*d^2*x^6+4/21*c*d*x^3
+c^2)*a^2+27/14*x^6*(1/15*d*x^3+c)*b^2*c*a+81/140*b^3*c^2*x^9)/(b*x^3+a)^(
10/3)/a^4
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(3\,(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2)x^{10}+10\,(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2)x^7+140\,a^3c^2x+}{140\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2
+ 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c
*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 +
 4*a^7*b*x^3 + a^8)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)d^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}+\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)cdx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)c^2x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^4}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*
(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 +
a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^
2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

3.76.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{(b\,x^3+a)^{10/3}}-\frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{(b\,x^3+a)^{7/3}} + \frac{x\,(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2)}{140\,a^3\,b^2\,(b\,x^3+a)^{4/3}}+\frac{x\,(6\,a^2\,d^2+18\,a\,b\,c\,d+81\,b^2\,c^2)}{140\,a^4\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

output
```
(x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (
x*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x
^3)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x
^3)^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b
*x^3)^(1/3))
```

3.76. $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$

3.77 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2(a+bx^3)^{7/3}} + \frac{3(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^4b^2(a+bx^3)^{4/3}} + \frac{9(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{13ab(a+bx^3)^{13/3}}$$

output
```
2/65*(-a*d+b*c)*(a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(10/3)+1/455*(2*a^2*d^2+9*
a*b*c*d+54*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(7/3)+3/910*(2*a^2*d^2+9*a*b*c*d+5
4*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(4/3)+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2
)*x/a^5/b^2/(b*x^3+a)^(1/3)+1/13*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(13/
3)
```

3.77.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{x(486b^4c^2x^{12}+81ab^3cx^9(26c+dx^3)+65a^4(14c^2+7cdx^3+2d^2x^6)+39a^3bx^3(70c^2+}{910a^5(a+bx^3)^{13/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*
c*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*
a^2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/
3))
```

3.77.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {930, 910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(9bc+4ad)x^3+c(12bc+ad)}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{(2a^2d^2+9abcd+54b^2c^2)\int \frac{1}{(bx^3+a)^{10/3}}dx}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\frac{(2a^2d^2+9abcd+54b^2c^2)\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\frac{\left(2a^2d^2+9abcd+54b^2c^2\right)\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$$\Big\downarrow \; 746$$

$$\frac{\frac{\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)\left(2a^2d^2+9abcd+54b^2c^2\right)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3)) + ((2*(b*c - a*d)*
(3*b*c + a*d)*x)/(5*a*b*(a + b*x^3)^(10/3)) + ((54*b^2*c^2 + 9*a*b*c*d + 2
*a^2*d^2)*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*
x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(5*a*b))/(13*a*b)
```

3.77.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.77.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}d^{2}x^{6}+\frac{1}{2}cd\,x^{3}+c^{2}\right)a^{4}+3x^{3}b\left(\frac{1}{35}d^{2}x^{6}+\frac{3}{14}cd\,x^{3}+c^{2}\right)a^{3}+\frac{27x^{6}b^{2}\left(\frac{1}{195}d^{2}x^{6}+\frac{1}{10}cd\,x^{3}+c^{2}\right)a^{2}}{7}+\frac{81x^{9}b^{3}c\left(\frac{d\,x^{3}}{26}+c\right)a}{35}+\frac{243b^{4}c^{2}}{455}}{(b\,x^{3}+a)^{\frac{13}{3}}a^{5}}$
gosper	$\frac{x(18a^{2}b^{2}d^{2}x^{12}+81a\,b^{3}cd\,x^{12}+486b^{4}c^{2}x^{12}+78a^{3}b\,d^{2}x^{9}+351a^{2}b^{2}cd\,x^{9}+2106a\,b^{3}c^{2}x^{9}+130a^{4}d^{2}x^{6}+585a^{3}bcd\,x^{6}+3510a^{2}b^{2}}{910(b\,x^{3}+a)^{\frac{13}{3}}a^{5}}$
trager	$\frac{x(18a^{2}b^{2}d^{2}x^{12}+81a\,b^{3}cd\,x^{12}+486b^{4}c^{2}x^{12}+78a^{3}b\,d^{2}x^{9}+351a^{2}b^{2}cd\,x^{9}+2106a\,b^{3}c^{2}x^{9}+130a^{4}d^{2}x^{6}+585a^{3}bcd\,x^{6}+3510a^{2}b^{2}}{910(b\,x^{3}+a)^{\frac{13}{3}}a^{5}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^4+3*x^3*b*(1/35*d^2*x^6+3/14*c*d*x^3+c^
2)*a^3+27/7*x^6*b^2*(1/195*d^2*x^6+1/10*c*d*x^3+c^2)*a^2+81/35*x^9*b^3*c*(
1/26*d*x^3+c)*a+243/455*b^4*c^2*x^12)/(b*x^3+a)^(13/3)/a^5
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(9\,(54\,b^4c^2+9\,ab^3cd+2\,a^2b^2d^2)x^{13}+39\,(54\,ab^3c^2+9\,a^2b^2cd+2\,a^3bd^2)x^{10}+65\,(54\,a^2}{910\,(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^
2 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d +
 2*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3
+ a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^
6 + 5*a^9*b*x^3 + a^10)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx &= \frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)d^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} \\ &\quad-\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)cdx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^4} \\ &\quad+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)c^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}\end{aligned}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*
x^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x
^3 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4)
 + 1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 4
55*(b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13
/3)*a^5)
```

3.77.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{(b\,x^3+a)^{13/3}} - \frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{(b\,x^3+a)^{10/3}} + \frac{x\,(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2)}{455\,a^3\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(6\,a^2\,d^2+27\,a\,b\,c\,d+162\,b^2\,c^2)}{910\,a^4\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(18\,a^2\,d^2+81\,a\,b\,c\,d+486\,b^2\,c^2)}{910\,a^5\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

output
```
(x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3)
- (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a
 + b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(
a + b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^
2*(a + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^
5*b^2*(a + b*x^3)^(1/3))
```

3.77. $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$

3.78 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} dx$

3.78.1 Optimal result

Integrand size = 21, antiderivative size = 253

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} dx = \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2+6abcd+a^2d^2)x}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{81(45b^2c^2+6abcd+a^2d^2)x}{7280a^6b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}}$$

output
```
1/208*(-a*d+b*c)*(4*a*d+15*b*c)*x/a^2/b^2/(b*x^3+a)^(13/3)+1/520*(a^2*d^2+
6*a*b*c*d+45*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(10/3)+9/3640*(a^2*d^2+6*a*b*c*d
+45*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(7/3)+27/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c
^2)*x/a^5/b^2/(b*x^3+a)^(4/3)+81/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^6
/b^2/(b*x^3+a)^(1/3)+1/16*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(16/3)
```

3.78.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(3645b^5c^2x^{15}+486ab^4cx^{12}(40c+dx^3)+81a^2b^3x^9(520c^2+32cdx^3+d^2x^6)+520a^5(\ldots}{728\ldots}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(
520*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)
 + 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175
*c^2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

3.78.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{4d(3bc+ad)x^3+c(15bc+ad)}{(bx^3+a)^{16/3}}dx}{16ab}+\frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\int \frac{1}{(bx^3+a)^{13/3}}dx+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\int\frac{1}{\left(bx^3+a\right)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$$\downarrow \text{749}$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\left(\frac{6\int\frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$$\downarrow \text{749}$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\left(\frac{6\left(\frac{3\int\frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$$\downarrow \text{746}$$

3.78. $\int \frac{\left(c+dx^3\right)^2}{(a+bx^3)^{19/3}}\,dx$

$$\frac{\frac{4}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3)) + ((((15*b*c^2)/a
- 11*c*d - (4*a*d^2)/b)*x)/(13*(a + b*x^3)^(13/3)) + (4*((45*b*c^2)/a + 6*
c*d + (a*d^2)/b)*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/
3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7
*a)))/(10*a)))/13)/(16*a*b)
```

3.78.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.78.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}d^2x^6+\frac{1}{2}cd\,x^3+c^2\right)a^5+\frac{15x^3b\left(\frac{6}{175}d^2x^6+\frac{8}{35}cd\,x^3+c^2\right)a^4}{4}+\frac{45x^6\left(\frac{3}{325}d^2x^6+\frac{3}{25}cd\,x^3+c^2\right)b^2a^3}{7}+\frac{81x^9b^3\left(\frac{1}{520}d^2x^6+\frac{4}{65}cd\,x^3+\right.}{14}\right.}{\left(b\,x^3+a\right)^{\frac{16}{3}}a^6}$
gosper	$\frac{x\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2cd\right.}{7280\left(b\,x^3+a\right)^{\frac{16}{3}}a}$
trager	$\frac{x\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2cd\right.}{7280\left(b\,x^3+a\right)^{\frac{16}{3}}a}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^5+15/4*x^3*b*(6/175*d^2*x^6+8/35*c*d*x^
3+c^2)*a^4+45/7*x^6*(3/325*d^2*x^6+3/25*c*d*x^3+c^2)*b^2*a^3+81/14*x^9*b^3
*(1/520*d^2*x^6+4/65*c*d*x^3+c^2)*a^2+243/91*x^12*b^4*(1/40*d*x^3+c)*c*a+7
29/1456*b^5*c^2*x^15)/(b*x^3+a)^(16/3)/a^6
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(81\,(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2)x^{16}+432\,(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2)x^{13}+936\,(45\,a}{7280\,(a^6b^6x^{18}+6\,a^7b^5x}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

output
```
1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c
^2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c
*d + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d
 + a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(
a^6*b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10
*b^2*x^6 + 6*a^11*b*x^3 + a^12)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

output
```
Timed out
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = -\frac{\left(455\,b^3 - \frac{1680\,(bx^3+a)b^2}{x^3} + \frac{2184\,(bx^3+a)^2b}{x^6} - \frac{1040\,(bx^3+a)^3}{x^9}\right)d^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4} + \frac{\left(455\,b^4 - \frac{2240\,(bx^3+a)b^3}{x^3} + \frac{4368\,(bx^3+a)^2b^2}{x^6} - \frac{4160\,(bx^3+a)^3b}{x^9} + \frac{1820\,(bx^3+a)^4}{x^{12}}\right)cdx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^5} - \frac{\left(91\,b^5 - \frac{560\,(bx^3+a)b^4}{x^3} + \frac{1456\,(bx^3+a)^2b^3}{x^6} - \frac{2080\,(bx^3+a)^3b^2}{x^9} + \frac{1820\,(bx^3+a)^4b}{x^{12}} - \frac{1456\,(bx^3+a)^5}{x^{15}}\right)c^2x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^6}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

output
```
-1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 1
040*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4
 - 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a
)^3*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1
/1456*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 208
0*(b*x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x
^15)*c^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

3.78.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x\left(\frac{c^2}{16\,a}+\frac{a\left(\frac{d^2}{16\,b}-\frac{c\,d}{8\,a}\right)}{b}\right)}{(b\,x^3+a)^{16/3}} - \frac{x\left(\frac{d^2}{13\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{(b\,x^3+a)^{13/3}} + \frac{x\,(a^2\,d^2+6\,a\,b\,c\,d+45\,b^2\,c^2)}{520\,a^3\,b^2\,(b\,x^3+a)^{10/3}} + \frac{x\,(9\,a^2\,d^2+54\,a\,b\,c\,d+405\,b^2\,c^2)}{3640\,a^4\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(27\,a^2\,d^2+162\,a\,b\,c\,d+1215\,b^2\,c^2)}{7280\,a^5\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(81\,a^2\,d^2+486\,a\,b\,c\,d+3645\,b^2\,c^2)}{7280\,a^6\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

output
```
(x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (
x*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a +
b*x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b
*x^3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(
a + b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^
5*b^2*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(
7280*a^6*b^2*(a + b*x^3)^(1/3))
```

3.78. $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx$

3.79 $\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx$

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} + \frac{a^2(77b^2c^2-14abcd+2a^2d^2)\, x\sqrt[3]{a+bx^3}\ \text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/154*d*(-4*a*d+17*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d*x*(b*x^3+a)^(10/3)*(
d*x^3+c)/b+1/77*a^2*(2*a^2*d^2-14*a*b*c*d+77*b^2*c^2)*x*(b*x^3+a)^(1/3)*hy
pergeom([-7/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

3.79.2 Mathematica [A] (warning: unable to verify)

Time = 12.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.31

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{ax\sqrt[3]{a+bx^3}\Big(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{7}{3}\right)\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{}$$

input
```
Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*
Hypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*
d*x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3
)/a)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}
, {1, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

3.79.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \left(bx^3+a\right)^{7/3} \left(d(17bc-4ad)x^3+c(14bc-ad)\right)dx}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{2\left(2a^2d^2-14abcd+77b^2c^2\right)\int \left(bx^3+a\right)^{7/3}dx}{11b}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{2a^2\sqrt[3]{a+bx^3}\left(2a^2d^2-14abcd+77b^2c^2\right)\int \left(\frac{bx^3}{a}+1\right)^{7/3}dx}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{2a^2x\sqrt[3]{a+bx^3}(2a^2d^2-14abcd+77b^2c^2)\text{ Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b}$$

input
```
Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(10/3)*(c + d*x^3))/(14*b) + ((d*(17*b*c - 4*a*d)*x*(a +
b*x^3)^(10/3))/(11*b) + (2*a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a
+ b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 +
 (b*x^3)/a)^(1/3)))/(14*b)
```

3.79.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.79.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)^{2}dx$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

3.79.5 Fricas [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx=\int \left(bx^{3}+a\right)^{\frac{7}{3}}\left(dx^{3}+c\right)^{2}dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x
)
```

3.79.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.10

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2\,dx=\frac{a^{\frac{7}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{7}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{7}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^{\frac{4}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{4}{3}}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^{\frac{4}{3}}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{\sqrt[3]{a}b^2c^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2\sqrt[3]{a}b^2cdx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{\sqrt[3]{a}b^2d^2x^{13}\Gamma\left(\frac{13}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{13}{3}\\ \frac{16}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

input
```
integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

output
```
a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + 2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((
-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3
)*b*d**2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I
*pi)/a)/(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2
*c*d*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13
/3), (16/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

3.79. $\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2\,dx$

3.79.7 Maxima [F]

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \int (bx^3+a)^{\frac{7}{3}} (dx^3+c)^2 \, dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

3.79.8 Giac [F]

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \int (bx^3+a)^{\frac{7}{3}} (dx^3+c)^2 \, dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \int (b\,x^3+a)^{7/3} (d\,x^3+c)^2 \, dx$$

input
```
int((a + b*x^3)^(7/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(7/3)*(c + d*x^3)^2, x)
```

3.80 $\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b} + \frac{a(44b^2c^2-11abcd+2a^2d^2)\, x\sqrt[3]{a+bx^3}\, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/44*d*(-2*a*d+7*b*c)*x*(b*x^3+a)^(7/3)/b^2+1/11*d*x*(b*x^3+a)^(7/3)*(d*x^
3+c)/b+1/44*a*(2*a^2*d^2-11*a*b*c*d+44*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeo
m([-4/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

3.80.2 Mathematica [A] (warning: unable to verify)

Time = 12.00 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\,\Gamma\left(-\frac{4}{3}\right)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right) - \cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-4/3]*Hy
pergeometric2F1[-4/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-1/3]*Hypergeometric2F1[-1/3, 4/3, 13/3, -((b*x^3)/
a)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-1/3]*HypergeometricPFQ[{-1/3, 4/3, 2},
{1, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-4/3])
```

3.80.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int \left(bx^3+a\right)^{4/3} \left(2d(7bc-2ad)x^3+c(11bc-ad)\right)dx}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{\left(2a^2d^2-11abcd+44b^2c^2\right)\int \left(bx^3+a\right)^{4/3}dx}{4b}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{a\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right)\int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{4b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{ax\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

input
```
Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(7/3)*(c + d*x^3))/(11*b) + ((d*(7*b*c - 2*a*d)*x*(a + b*
x^3)^(7/3))/(4*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4*b*(1 + (b*x^3)/
a)^(1/3)))/(11*b)
```

3.80.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.80.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)^{2}dx$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

3.80.5 Fricas [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^{3}+a\right)^{\frac{4}{3}} \left(dx^{3}+c\right)^{2} dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^
2)*(b*x^3 + a)^(1/3), x)
```

3.80.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.03

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx = \frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2\sqrt[3]{a}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

output
```
a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*
d**2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

3.80.7 Maxima [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx = \int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

3.80.8 Giac [F]

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \int (bx^3+a)^{\frac{4}{3}} (dx^3+c)^2 \, dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \int (b\,x^3+a)^{4/3} (d\,x^3+c)^2 \, dx$$

input
```
int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

3.81 $\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2 \, dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2 \, dx = \frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\left(10b^2c^2-4abcd+a^2d^2\right)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/40*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d*x*(b*x^3+a)^(4/3)*(d*x^
3+c)/b+1/10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1
/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

3.81.2 Mathematica [A] (verified)

Time = 9.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2 \, dx = \frac{x\sqrt[3]{a+bx^3}\left(20a\left(14c^2+7cdx^3+2d^2x^6\right)\text{Gamma}\left(-\frac{1}{3}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)-3bx^3(11c^2\right.}{280a}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hy
pergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)
] - 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

3.81.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2 dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \sqrt[3]{bx^3+a}\left(d(11bc-4ad)x^3+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{4\left(a^2d^2-4abcd+10b^2c^2\right)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{4\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{4x\sqrt[3]{a+bx^3}(a^2d^2-4abcd+10b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(4/3)*(c + d*x^3))/(8*b) + ((d*(11*b*c - 4*a*d)*x*(a + b*
x^3)^(4/3))/(5*b) + (4*(10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1
/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^
(1/3)))/(8*b)
```

3.81.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.81.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{2}dx$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

3.81.5 Fricas [F]

$$\int \sqrt[3]{a+bx^3}(c+dx^3)^2\,dx=\int \left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)^{2}dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

3.81.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

output
```
a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
)
```

3.81.7 Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

3.81.8 Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int {\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)
```

3.82 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} + \frac{(5b^2c^2-5abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2\,(a+bx^3)^{2/3}}$$

output
```
2/5*d*(-a*d+2*b*c)*x*(b*x^3+a)^(1/3)/b^2+1/5*d*x*(b*x^3+a)^(1/3)*(d*x^3+c)
/b+1/5*(2*a^2*d^2-5*a*b*c*d+5*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3,
 2/3],[4/3],-b*x^3/a)/b^2/(b*x^3+a)^(2/3)
```

3.82.2 Mathematica [A] (verified)

Time = 15.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(-d(a+bx^3)\,(2ad-b(5c+dx^3))+(5b^2c^2-5abcd+2a^2d^2)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeor}\right.}{5b^2\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(x*(-(d*(a + b*x^3)*(2*a*d - b*(5*c + d*x^3))) + (5*b^2*c^2 - 5*a*b*c*d +
2*a^2*d^2)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)]))/(5*b^2*(a + b*x^3)^(2/3))
```

3.82.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \frac{4d(2bc-ad)x^3+c(5bc-ad)}{(bx^3+a)^{2/3}}dx}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{(2a^2d^2-5abcd+5b^2c^2)\int \frac{1}{(bx^3+a)^{2/3}}dx}{b}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \ 779$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{b(a+bx^3)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{b(a+bx^3)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3)^(1/3)*(c + d*x^3))/(5*b) + ((2*d*(2*b*c - a*d)*x*(a + b*x
^3)^(1/3))/b + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(b*(a + b*x^3)^(2/3)))/(
5*b)
```

3.82.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.82.4 Maple [F]

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

3.82.5 Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

3.82.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper
((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

3.82.7 Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

3.82.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.83 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x(c+dx^3)}{2ab\,(a+bx^3)^{2/3}} + \frac{(b^2c^2+2abcd-2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab^2\,(a+bx^3)^{2/3}}$$

output
```
-1/2*d*(-2*a*d+b*c)*x*(b*x^3+a)^(1/3)/a/b^2+1/2*(-a*d+b*c)*x*(d*x^3+c)/a/b
/(b*x^3+a)^(2/3)+1/2*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b^2/(b*x^3+a)^(2/3)
```

3.83.2 Mathematica [A] (warning: unable to verify)

Time = 12.87 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(4a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{10}{3},-\right.\right.}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*
Hypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*
(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/
(84*a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

3.83.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(-2a^2d^2+2abcd+b^2c^2)\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{b}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(-2a^2d^2+2abcd+b^2c^2)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{b(a+bx^3)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(-2a^2d^2+2abcd+b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{b(a+bx^3)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(2*a*b*(a + b*x^3)^(2/3)) + (-((d*(b*c - 2*a*d
)*x*(a + b*x^3)^(1/3))/b) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (b*x
^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(b*(a + b*x^3
)^(2/3)))/(2*a*b)
```

3.83.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.83.4 Maple [F]

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{5}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

3.83.5 Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^
3 + a^2), x)
```

3.83.6 Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

3.83.7 Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

3.83.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(5/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.84 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\,(a+bx^3)^{2/3}}+\frac{(bc-ad)x(c+dx^3)}{5ab\,(a+bx^3)^{5/3}} + \frac{(2b^2c^2+abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b^2\,(a+bx^3)^{2/3}}$$

output
```
2/5*(c^2/a^2-d^2/b^2)*x/(b*x^3+a)^(2/3)+1/5*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*
x^3+a)^(5/3)+1/5*(2*a^2*d^2+a*b*c*d+2*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hyperge
om([1/3, 2/3],[4/3],-b*x^3/a)/a^2/b^2/(b*x^3+a)^(2/3)
```

3.84.2 Mathematica [A] (verified)

Time = 14.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(5a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{10}{3},-\right.\right.}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(5*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*
Hypergeometric2F1[1/3, 8/3, 10/3, -((b*x^3)/a)] - 2*b*x^3*(11*c^2 + 16*c*d
*x^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 11/3, 13/3, -((b*x^3)/a)] - 6*b*x
^3*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 11/3}, {1, 13/3}, -((b*x^3)/a)
]))/(63*a^3*(a + b*x^3)^(2/3)*Gamma[8/3])
```

3.84.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{d(bc+4ad)x^3+c(4bc+ad)}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \ 910$$

$$\frac{\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\int \frac{1}{(bx^3+a)^{2/3}}dx+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \ 779$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*((b*c^2)/a - (
a*d^2)/b)*x)/(a + b*x^3)^(2/3) + (((2*b*c^2)/a + c*d + (2*a*d^2)/b)*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x
^3)^(2/3))/(5*a*b)
```

3.84.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.84.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{2}}{(b\,x^{3}+a)^{\frac{8}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

3.84.5 Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*
x^6 + 3*a^2*b*x^3 + a^3), x)
```

3.84.6 Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(8/3), x)
```

3.84.7 Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

3.84.8 Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(8/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.85 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

output
```
1/10*x*(b*x^3+a)^3/c/(d*x^3+c)^(10/3)+9/70*a*x*(b*x^3+a)^2/c^2/(d*x^3+c)^(
7/3)+27/140*a^2*x*(b*x^3+a)/c^3/(d*x^3+c)^(4/3)+81/140*a^3*x/c^4/(d*x^3+c)
^(1/3)
```

3.85.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x(14b^3c^3x^9 + 6ab^2c^2x^6(10c+3dx^3) + 3a^2bcx^3(35c^2+30cdx^3+9d^2x^6) + a^3(140c^3+3}{140c^4\,(c+dx^3)^{10/3}}$$

input
```
Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

output
```
(x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*
c^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x
^6 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

3.85.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\int \frac{(bx^3+a)^2}{(dx^3+c)^{10/3}}dx}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\left(\frac{6a\int \frac{bx^3+a}{(dx^3+c)^{7/3}}dx}{7c}+\frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\left(\frac{6a\left(\frac{3a\int \frac{1}{(dx^3+c)^{4/3}}dx}{4c}+\frac{x(a+bx^3)}{4c(c+dx^3)^{4/3}}\right)}{7c}+\frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{9a\left(\frac{6a\left(\frac{x(a+bx^3)}{4c(c+dx^3)^{4/3}}+\frac{3ax}{4c^2\sqrt[3]{c+dx^3}}\right)}{7c}+\frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

input
```
Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

output
```
(x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*((x*(a + b*x^3)^2)/(7*c
*(c + d*x^3)^(7/3)) + (6*a*((x*(a + b*x^3))/(4*c*(c + d*x^3)^(4/3)) + (3*a
*x)/(4*c^2*(c + d*x^3)^(1/3))))/(7*c)))/(10*c)
```

3.85.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.85.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{x \left(\left(\frac{1}{10} b^{3} x^{9}+\frac{3}{7} a \,b^{2} x^{6}+\frac{3}{4} a^{2} b \,x^{3}+a^{3}\right) c^{3}+\frac{9 x^{3} \left(\frac{2}{35} b^{2} x^{6}+\frac{2}{7} a b \,x^{3}+a^{2}\right) d a \,c^{2}}{4}+\frac{27 x^{6} \left(\frac{b \,x^{3}}{10}+a \right) d^{2} a^{2} c}{14}+\frac{81 a^{3} d^{3} x^{9}}{140}\right)}{\left(d \,x^{3}+c \right)^{\frac{10}{3}} c^{4}}$
gosper	$\frac{x(81a^{3}d^{3}x^{9}+27a^{2}bc\,d^{2}x^{9}+18a\,b^{2}c^{2}d\,x^{9}+14b^{3}c^{3}x^{9}+270a^{3}c\,d^{2}x^{6}+90a^{2}b\,c^{2}d\,x^{6}+60a\,b^{2}c^{3}x^{6}+315a^{3}c^{2}d\,x^{3}+105a^{2}b\,c^{3}x^{3}+1}{140(d\,x^{3}+c)^{\frac{10}{3}}c^{4}}$
trager	$\frac{x(81a^{3}d^{3}x^{9}+27a^{2}bc\,d^{2}x^{9}+18a\,b^{2}c^{2}d\,x^{9}+14b^{3}c^{3}x^{9}+270a^{3}c\,d^{2}x^{6}+90a^{2}b\,c^{2}d\,x^{6}+60a\,b^{2}c^{3}x^{6}+315a^{3}c^{2}d\,x^{3}+105a^{2}b\,c^{3}x^{3}+1}{140(d\,x^{3}+c)^{\frac{10}{3}}c^{4}}$

input
```
int((b*x^3+a)^3/(d*x^3+c)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
x/(d*x^3+c)^(10/3)*((1/10*b^3*x^9+3/7*a*b^2*x^6+3/4*a^2*b*x^3+a^3)*c^3+9/4
*x^3*(2/35*b^2*x^6+2/7*a*b*x^3+a^2)*d*a*c^2+27/14*x^6*(1/10*b*x^3+a)*d^2*a
^2*c+81/140*a^3*d^3*x^9)/c^4
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{((14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3)x^{10}+30\,(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2)x^7+}{140\,(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7dx^3}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

output
```
1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 +
30*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(
a^2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*
x^9 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

output
```
Timed out
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.67

$$\begin{aligned}\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx &= \frac{b^3x^{10}}{10\,(dx^3+c)^{\frac{10}{3}}\,c} - \frac{3\,ab^2\left(7\,d-\frac{10\,(dx^3+c)}{x^3}\right)x^{10}}{70\,(dx^3+c)^{\frac{10}{3}}\,c^2} \\ &+ \frac{3\left(14\,d^2-\frac{40\,(dx^3+c)d}{x^3}+\frac{35\,(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140\,(dx^3+c)^{\frac{10}{3}}\,c^3} \\ &- \frac{\left(14\,d^3-\frac{60\,(dx^3+c)d^2}{x^3}+\frac{105\,(dx^3+c)^2d}{x^6}-\frac{140\,(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140\,(dx^3+c)^{\frac{10}{3}}\,c^4}\end{aligned}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

output
```
1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^
3)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 +
35*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3
- 60*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9
)*a^3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

3.85.8 Giac [F]

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx=\int \frac{(bx^3+a)^3}{(dx^3+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.49

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx=\frac{x\left(\frac{a^3}{10\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{(d\,x^3+c)^{10/3}}-\frac{x\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{(d\,x^3+c)^{4/3}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{70\,c^2\,d^3}\right)}{(d\,x^3+c)^{7/3}}+\frac{x\,(81\,a^3\,d^3+27\,a^2\,b\,c\,d^2+18\,a\,b^2\,c^2\,d+14\,b^3\,c^3)}{140\,c^4\,d^3\,(d\,x^3+c)^{1/3}}$$

input
```
int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

output
```
(x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10
*c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 +
 6*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c
*(b^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*
a^3*d^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c +
 d*x^3)^(1/3))
```

3.85. $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx$

3.86 $\int \frac{(a+bx^3)^{8/3}}{c+dx^3} dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 331

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3} dx = -\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2} + \frac{bx(a+bx^3)^{5/3}}{6d} + \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^3} - \frac{(bc-ad)^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^3} - \frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3} + \frac{(bc-ad)^{8/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3} - \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^3}$$

output
```
-1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^(2/3)/d^2+1/6*b*x*(b*x^3+a)^(5/3)/d-1/
6*(-a*d+b*c)^(8/3)*ln(d*x^3+c)/c^(2/3)/d^3+1/2*(-a*d+b*c)^(8/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^3-1/18*b^(2/3)*(20*a^2*d^2-
24*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/27*b^(2/3)*(20*
a^2*d^2-24*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/d^3*3^(1/2)-1/3*(-a*d+b*c)^(8/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d^3*3^(1/2)
```

3.86.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.83 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{3b\sqrt[3]{bc-ad}\,(9b^2c^2-24abcd+20a^2d^2)\,x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+}{}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

output
```
(3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^
(1/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a
*d)^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b
*c - a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*
(3*b^2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c
- a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7
*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c -
 a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*
x^3)^(1/3)] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + 9*a^3*d^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c
*d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

3.86.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {933, 25, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3} \, dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{\left(bx^3+a\right)^{2/3}\left(b(6bc-11ad)x^3+a(bc-6ad)\right)}{dx^3+c}dx}{6d}+\frac{bx\left(a+bx^3\right)^{5/3}}{6d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(b(6bc-11ad)x^3+a(bc-6ad)\right)}{dx^3+c}dx}{6d}$$

$$\Big\downarrow \; 1025$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{\int -\frac{2\left(b\left(9b^2c^2-24abdc+20a^2d^2\right)x^3+a\left(3b^2c^2-7abdc+9a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}+\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}}{6d}$$

$$\Big\downarrow \; 27$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}-\frac{2\int \frac{b\left(9b^2c^2-24abdc+20a^2d^2\right)x^3+a\left(3b^2c^2-7abdc+9a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}}{6d}$$

$$\Big\downarrow \; 1026$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}-\frac{2\left(\frac{b\left(20a^2d^2-24abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{9(bc-ad)^3\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}\right)}{3d}}{6d}$$

$$\Big\downarrow \; 769$$

$$\frac{\frac{bx(a+bx^3)^{5/3}}{6d}-\frac{\frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{3d}-\frac{2\frac{b(20a^2d^2-24abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{9(bc-ad)^3\int\frac{1}{\sqrt[3]{bx^3+a}(d\ldots}}{d}}{3d}}{6d}$$

$\downarrow$ 901

$$\frac{\frac{bx(a+bx^3)^{5/3}}{6d}-\frac{\frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{3d}-\frac{2\frac{b(20a^2d^2-24abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-9(bc-ad)^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{b\ldots}}{\sqrt[3]{c}\sqrt[3]{\ldots}}\ldots}{\ldots}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{\ldots}}\ldots\right)}{3d}}{6d}$$

input
```
Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

3.86. $\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx$

output
```
(b*x*(a + b*x^3)^(5/3))/(6*d) - ((b*(6*b*c - 11*a*d)*x*(a + b*x^3)^(2/3))/
(3*d) - (2*((-9*(b*c - a*d)^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
 + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + (b*(9*b^2*c^2 -
 24*a*b*c*d + 20*a^2*d^2)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sq
rt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3
))))/d))/(3*d))/(6*d)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.86.4 Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{\frac{(ad-bc)^{3}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\frac{\left(\frac{20a^{2}d^{2}b^{\frac{2}{3}}}{9}+b^{\frac{5}{3}}c\left(bc-\frac{8ad}{3}\right)\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}(b}{}\right.}{2}}{}$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*d-b*c)/c)^(1/3)*(1/2*(a*d-b*c)^3*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*
d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-1/2*(20/9*a^2*d^2*
b^(2/3)+b^(5/3)*c*(b*c-8/3*a*d))*c*((a*d-b*c)/c)^(1/3)*ln((b^(2/3)*x^2+b^(
1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(a*d-b*c)^3*ln(((((a*d-b*c)/c)
^(1/3)*x+(b*x^3+a)^(1/3))/x)+(20/9*a^2*d^2*b^(2/3)+b^(5/3)*c*(b*c-8/3*a*d)
)*3^(1/2)*c*((a*d-b*c)/c)^(1/3)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^
(1/3))/b^(1/3)/x)+(20/9*a^2*d^2*b^(2/3)+b^(5/3)*c*(b*c-8/3*a*d))*c*((a*d-b
*c)/c)^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-7/3*x*b*d*(-3/7*b*c+d*(3/1
4*b*x^3+a))*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)-3^(1/2)*arctan(1/3*3^(1/
2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*(a*d-b
*c)^3)/c/d^3
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(273) = 546$.

Time = 6.48 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx =$$

$$-\frac{18\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}}{}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x))
 + 2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/
3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*l
og((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b
*c - a*d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-
((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a
^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(
2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^
2*d^2*x^4 - 2*(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

3.86.6 Sympy [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{8}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(8/3)/(c + d*x**3), x)
```

3.86.7 Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

3.86.8 Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

3.87 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx = &\frac{bx(a+bx^3)^{2/3}}{3d} - \frac{b^{2/3}(3bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} \\ &+ \frac{(bc-ad)^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^2} + \frac{(bc-ad)^{5/3}\log(c+dx^3)}{6c^{2/3}d^2} \\ &- \frac{(bc-ad)^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2} \\ &+ \frac{b^{2/3}(3bc-5ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^2}\end{aligned}$$

output
```
1/3*b*x*(b*x^3+a)^(2/3)/d+1/6*(-a*d+b*c)^(5/3)*ln(d*x^3+c)/c^(2/3)/d^2-1/2
*(-a*d+b*c)^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d
^2+1/6*b^(2/3)*(-5*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^2-1/9*b^(2/
3)*(-5*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*
3^(1/2)+1/3*(-a*d+b*c)^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b
*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d^2*3^(1/2)
```

3.87.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.51 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{3b\sqrt[3]{bc-ad}(-3bc+5ad)x^4\sqrt[3]{1+\frac{bx^3}{a}}\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2\sqrt[3]{c}\Bigg(6abc^{2/}}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

output
```
(3*b*(b*c - a*d)^(1/3)*(-3*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1
[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1/3)*(6*a*b*c^(2/3)*
(b*c - a*d)^(1/3)*x + 6*b^2*c^(2/3)*(b*c - a*d)^(1/3)*x^4 + 2*Sqrt[3]*a*(-
(b*c) + 3*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*(b*c - 3*a*d)*(a + b*x^3)^(1/3)*Log
[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*b*c*(a + b*x^3)^(1
/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a^2*d*(a + b*x^3)^(1/3)*Log[c^(2/
3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3
)*x)/(b + a*x^3)^(1/3)]))/(36*c*d*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

3.87.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {933, 25, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{b(3bc-5ad)x^3+a(bc-3ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3d}+\frac{bx(a+bx^3)^{2/3}}{3d}$$

$\Big\downarrow$ 25

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\int \frac{b(3bc-5ad)x^3+a(bc-3ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3d}$$

$\Big\downarrow$ 1026

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3d}$$

$\Big\downarrow$ 769

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3d}$$

$\Big\downarrow$ 901

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3(bc-ad)^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\Big(}{}\right)}{d}}{3d}$$

input `Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]`

3.87. $\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx$

output
```
(b*x*(a + b*x^3)^(2/3))/(3*d) - ((-3*(b*c - a*d)^2*(ArcTan[(1 + (2*(b*c -
a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
- a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
 a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3)))
)/d + (b*(3*b*c - 5*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)))/d)/(3*d)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.87.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.47

method	result
pseudoelliptic	$-\frac{-\frac{5\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c \left(a d b^{\frac{2}{3}}-\frac{3 b^{\frac{5}{3}} c}{5}\right) \ln\left(\frac{b^{\frac{2}{3}} x^2+b^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{3}-2(ad-bc)^2 \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{1}{3}}}{x}\right)+\frac{10\sqrt{3}\,(\ldots}{\ldots}}{\ldots}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-5/3*((a*d-b*c)/c)^(1/3)*c*(a*d*b^(2/3)-3/5*b^(5/3)*c)*ln((b^(2/3)*x
^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*(a*d-b*c)^2*ln((((a*d
-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+10/3*3^(1/2)*((a*d-b*c)/c)^(1/3)*c*(a
*d*b^(2/3)-3/5*b^(5/3)*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))
/b^(1/3)/x)+10/3*((a*d-b*c)/c)^(1/3)*c*(a*d*b^(2/3)-3/5*b^(5/3)*c)*ln((-b^
(1/3)*x+(b*x^3+a)^(1/3))/x)-2*(b*x^3+a)^(2/3)*x*b*c*((a*d-b*c)/c)^(1/3)*d+
(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c
)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c)^2)/((a*d-b*c)/c)^(1/3)/c/d^
2
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(220) = 440$.

Time = 0.76 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{6\left(bx^3+a\right)^{\frac{2}{3}} bdx + 6\sqrt{3}(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}} c\left(\ldots\right.}{3\,(bc-ad)x}\ldots\right.}{\ldots}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*
d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x -
 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c
 - 5*a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3
*b*c - 5*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x
- (b*x^3 + a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3)
+ (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d^2
```

3.87.6 Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

3.87.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

3.87.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx=\int \frac{(b\,x^3+a)^{5/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

3.88 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

output
```
-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+
b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

3.88.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.46 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\ldots}\right)}{c^{2/3}}\ldots}{\ldots}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
(4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] -
((2*I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c
 - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)])/c^(2/3))/(12*d)
```

3.88.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {916, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{ 916}$$

$$\frac{b\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \; 901$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
-(((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^
(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)
^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (b*(ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)]/(2*b^(1/3))))/d
```

3.88. $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

3.88.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

3.88.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{2}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c \left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+\left(ad-bc\right) \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\sqrt{3}\, b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\right.}{3b}\right.}{}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(1/2*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1/3)+(a*d-b*c)*ln((((a*d-b*c)/
c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-3^(1/2)*b^(2/3)*arctan(1/3*3^(1/2)*(b^(1/3)
*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c*((a*d-b*c)/c)^(1/3)-b^(2/3)*ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b
*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln(((
(a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/
3))/x^2))*(a*d-b*c))/d/c
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.

Time = 0.30 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(s
qrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-
1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*
log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2
) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2
- 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

3.88.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.88.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.88.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.89 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)
-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^
(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$=\frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\right.\right.\right.}{12c^2}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt
[3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(
12*c^(2/3)*(b*c - a*d)^(1/3))
```

3.89.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \ 901$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/
(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d
)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/
3)*(b*c - a*d)^(1/3))
```

3.89.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.89.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+(b}{x^{2}}\right)}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^
3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^
3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/c
```

3.89.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.89.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.89.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.89.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.90 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3
)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^
(4/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^
(1/2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

3.90.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]
]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*
(-I + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d
)^(4/3)))/12
```

3.90.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)
^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c
 - a*d)
```

3.90.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.90.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(ad-bc)ca}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d
-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(
1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2
*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/
c/a
```

3.90.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.90.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.90.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.90.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.91 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)} dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)} dx = \frac{bx}{4a(bc-ad)(a+bx^3)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}} + \frac{d^2\log(c+dx^3)}{6c^{2/3}(bc-ad)^{7/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}$$

output
```
1/4*b*x/a/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/4*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)
^2/(b*x^3+a)^(1/3)+1/6*d^2*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(7/3)-1/2*d^2*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(7/3)+1/3*
d^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c
^(2/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{3bx(-8a^2d+3b^2cx^3+ab(4c-7dx^3))}{a^2(bc-ad)^2(a+bx^3)^{4/3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{7/3}} + \frac{2(1+i\sqrt{3})d^2\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{7/3}} - \frac{i(-i+\sqrt{3})d^2\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{7/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

output
```
((3*b*x*(-8*a^2*d + 3*b^2*c*x^3 + a*b*(4*c - 7*d*x^3)))/(a^2*(b*c - a*d)^2
*(a + b*x^3)^(4/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x
^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(7/3)) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b
*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*(
b*c - a*d)^(7/3)) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-
1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt
[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(7/3)))/12
```

3.91.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {931, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}-\frac{\int -\frac{3bdx^3+3bc-4ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^3+3bc-4ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{4a^2d^2}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4ad^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}+\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{4ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(3*b*c - 7*a*d)*x)/(a*(b*c
 - a*d)*(a + b*x^3)^(1/3)) + (4*a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)
/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))
 + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c - a*d)
)/(4*a*(b*c - a*d))
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.91.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{2}d^{2}\left(b\,x^{3}+a\right)^{\frac{4}{3}}+12xbc\left(a^{2}d-\frac{\left(-\frac{7d\,x^{3}}{4}+c\right)ba}{2}-\frac{3b^{2}c\,x^{3}}{8}\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+\left(-2\arctan\left(\frac{\sqrt{3}}{\right.\right.}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(a\right.}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)
*a^2*d^2*(b*x^3+a)^(4/3)+12*x*b*c*(a^2*d-1/2*(-7/4*d*x^3+c)*b*a-3/8*b^2*c*
x^3)*((a*d-b*c)/c)^(1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(
b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2
-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d^2*(b*x^3+a
)^(4/3)*a^2)/(b*x^3+a)^(4/3)/(a*d-b*c)^2/c/a^2
```

3.91.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.91.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

3.91.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

3.91.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

3.92 $\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx$

3.92.1 Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^3)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2(a+bx^3)^{4/3}} + \frac{b(18b^2c^2-57abcd+67a^2d^2)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{10/3}} - \frac{d^3\log(c+dx^3)}{6c^{2/3}(bc-ad)^{10/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}$$

output
```
1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^3+a)^(4/3)+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d
+b*c)^3/(b*x^3+a)^(1/3)-1/6*d^3*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(10/3)+1/2*
d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(10/
3)-1/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(2/3)/(-a*d+b*c)^(10/3)*3^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.43 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{1}{84}\left(-\frac{3bx(84a^4d^2+18b^4c^2x^6+3ab^3cx^3(14c-19dx^3)+21a^3bd(-4c+7dx^3)\cdots}{a^3(-bc+ad)^3(a+bx^3)^{7/3}}\right.$$
$$+\frac{14\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{10/3}}$$
$$-\frac{14i(-i+\sqrt{3})d^3\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{10/3}}$$
$$\left.+\frac{7(1+i\sqrt{3})d^3\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{10/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

output
```
((-3*b*x*(84*a^4*d^2 + 18*b^4*c^2*x^6 + 3*a*b^3*c*x^3*(14*c - 19*d*x^3) +
21*a^3*b*d*(-4*c + 7*d*x^3) + a^2*b^2*(28*c^2 - 133*c*d*x^3 + 67*d^2*x^6))
)/(a^3*(-(b*c) + a*d)^3*(a + b*x^3)^(7/3)) + (14*Sqrt[-6 + (6*I)*Sqrt[3]]*
d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(10/3)) - ((14*I
)*(-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)]])/(c^(2/3)*(b*c - a*d)^(10/3)) + (7*(1 + I*Sqrt[3])*d^3*L
og[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*
(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(
b*c - a*d)^(10/3)))/84
```

3.92.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {931, 25, 1024, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}-\frac{\int -\frac{6bdx^3+6bc-7ad}{(bx^3+a)^{7/3}(dx^3+c)}dx}{7a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6bdx^3+6bc-7ad}{(bx^3+a)^{7/3}(dx^3+c)}dx}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}-\frac{\int -\frac{3bd(6bc-13ad)x^3+18b^2c^2+28a^2d^2-39abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3bd(6bc-13ad)x^3+18b^2c^2+28a^2d^2-39abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{28a^3d^3}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{bx(67a^2d^2-57abcd+18b^2c^2)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{28a^2d^3\int\frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

$\downarrow$ 901

$$\frac{\frac{\frac{bx(67a^2d^2-57abcd+18b^2c^2)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{28a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(7*a*(b*c - a*d)*(a + b*x^3)^(7/3)) + ((b*(6*b*c - 13*a*d)*x)/(4*a*(
b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2)
*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (28*a^2*d^3*(ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
 - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))
))/(b*c - a*d))/(4*a*(b*c - a*d)))/(7*a*(b*c - a*d))
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.92.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{3}d^{3}\left(b\,x^{3}+a\right)^{\frac{7}{3}}-9xbc\left(a^{4}d^{2}-\left(-\frac{7d\,x^{3}}{4}+c\right)bd\,a^{3}+\frac{b^{2}\left(\frac{67}{28}d^{2}x^{6}-\frac{19}{4}cd\,x^{3}+c^{2}\right)a^{2}}{3}+\frac{x^{3}b^{3}\left(-\frac{19d\,x^{3}}{14}\right.}{2}}{3\left(\right.}$

input
```
int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(7/3)*(ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x)*a^3*d^3*(b*x^3+a)^(7/3)-9*x*b*c*(a^4*d^2-(-7/4*d*x^3+c)*b*d*a
^3+1/3*b^2*(67/28*d^2*x^6-19/4*c*d*x^3+c^2)*a^2+1/2*x^3*b^3*(-19/14*d*x^3+
c)*c*a+3/14*b^4*c^2*x^6)*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b*
c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((
a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2))*(b*x^3+a)^(7/3)*d^3*a^3)/(a*d-b*c)^3/c/a^3
```

3.92.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.92.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{10}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)
```

3.92.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

3.92.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{10/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

3.93 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3} \, dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3} \, dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/
a)^(1/3)
```

3.93.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.

Time = 10.46 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3} \, dx = \frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + \frac{4\left(-4ac(2a^2d+abdx^3+b^2x^3(c+dx^3))\,\text{AppellF1}\cdots\right.}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\cdots\right.\right.}\right.}{8d\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*
x^3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
b*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/
(8*d*(a + b*x^3)^(2/3))
```

3.93.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \ 937$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

3.93.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.93.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

3.93.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.93.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.93.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.93.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.94 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/3)
```

3.94.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 10.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$=\frac{4acx\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\text{App}\right.\right.}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.94.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^3} \text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*(1 + (b*x^3)/a)^(1/3))
```

3.94.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.94.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

3.94.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.94.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.94.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.94.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.95 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^
(2/3)
```

3.95.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$

$$-\frac{4acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*
x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(a + b*x^3)^(2/3))
```

3.95.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.95.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

3.95.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.95.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.95.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.95.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.96 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a
)^(2/3)
```

3.96.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(62) = 124.

Time = 10.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\frac{x\left(-\frac{bdx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac(2ad-b(2c+dx^3))\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\right.\right.}{(c+dx^3)\left(4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\right.\right.}\right.}{8a(-bc+}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

output
```
(x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3,
2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*Appell
F1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3,
 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3,
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

3.96.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}(dx^3+c)}dx}{a(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*(a + b*x^3)^(2/3))
```

3.96.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.96.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)
```

3.96.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

3.96.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)), x)
```

3.97 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3
+a)^(2/3)
```

3.97.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 429 vs. 2(62) = 124.

Time = 10.78 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.92

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx =$$

$$-\frac{x\left(\frac{bd(-4bc+9ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac\left(10a^3d^2+4b^3cx^3\left(2c+dx^3\right)-a^2bd\left(20c+dx^3\right)+ab^2\left(10c^2-12cdx^3-\right.\right.\right.}{(a+bx^3)(c+dx^3)\left(-4c\right.}\right.}{40a}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

output
```
-1/40*(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/
3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*
c*x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3
- 9*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*Appe
llF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appe
llF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^3
)^(2/3))
```

3.97.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

3.97.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.97.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}} \left(d\,x^{3}+c\right)}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

3.97.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.97.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{8}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)), x)
```

3.97.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

3.97.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{8/3}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)), x)
```

3.98 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2} dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 351

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2} dx &= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} \\ &- \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd(c+dx^3)} - \frac{2b^{5/3}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^3} \\ &+ \frac{2(bc-ad)^{5/3}(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^3} \\ &+ \frac{(bc-ad)^{5/3}(3bc+ad)\log(c+dx^3)}{9c^{5/3}d^3} \\ &- \frac{(bc-ad)^{5/3}(3bc+ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3} \\ &+ \frac{b^{5/3}(3bc-4ad)\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{3d^3}\end{aligned}$$

output
```
1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3
)/c/d/(d*x^3+c)+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1
/3*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1
/3))/c^(5/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))
/d^3-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/d^3*3^(1/2)+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1/3*(1+2*(-a
*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^3*3^(1/2)
```

3.98.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.94 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{1}{18}\left(\frac{6x(a+bx^3)^{2/3}\left(b^2+\frac{(bc-ad)^2}{c(c+dx^3)}\right)}{d^2}\right.$$

$$-\frac{9b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}$$

$$+\frac{12ab^2x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd\sqrt[3]{a+bx^3}}$$

$$+\frac{2a^3\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax}}\right.\right.}{c^{5/3}\sqrt[3]{bc-ad}}$$

$$-\frac{2ab^2\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{b}}{\sqrt[3]{b}}\right.\right.}{d^2\sqrt[3]{bc-ad}}$$

$$+\frac{2a^2b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{2/3}d\sqrt[3]{bc-ad}}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

3.98. $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx$

output
```
((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^
3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/
3)) + (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3))
- (2*a*b^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) +
 (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)
) + (2*a^2*b*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)
))/18
```

3.98.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(3b(2bc-ad)x^3+a(bc+2ad)\right)}{dx^3+c}dx}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{\frac{\int -\frac{6\left(b^2c(3bc-4ad)x^3+a\left(b^2c^2-abdc-a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}+\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\int \frac{b^2c(3bc-4ad)x^3+a\left(b^2c^2-abdc-a^2d^2\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\,(c+dx^3)}$$

$\Big\downarrow$ 1026

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\int\frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)^2(ad+3bc)\int\frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}\right)}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\,(c+dx^3)}$$

$\Big\downarrow$ 769

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2(ad+3bc)\int\frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}\right)}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\,(c+dx^3)}$$

$\Big\downarrow$ 901

3.98. $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx$

$$\frac{\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-a\ldots}}{\sqrt[3]{c}\sqrt[3]{a+b\ldots}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-\ldots}}\ldots\right.}{\ldots}\right)}{d}}{3cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

input

```
Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

output

```
-1/3*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)) + ((b*(2*b*c - a*
d)*x*(a + b*x^3)^(2/3))/d - (2*(-(((b*c - a*d)^2*(3*b*c + a*d)*(ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c
^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) -
 Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c -
a*d)^(1/3))))/d) + (b^2*c*(3*b*c - 4*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)]/(2*b^(1/3))))/d))/d)/(3*c*d)
```

3.98.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

3.98. $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx$

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.98.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$\frac{\frac{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(a\,b^{\frac{5}{3}}d-\frac{3b^{\frac{8}{3}}c}{4}\right)c^{2}\left(d\,x^{3}+c\right)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{9}+\frac{2\left(d\,x^{3}+c\right)(ad+3bc)(ad-bc)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(\right.}{x}\right.}{9}}{}$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*(2*((a*d-b*c)/c)^(1/3)*(a*b^(5/3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*ln((b
^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(d*x^3+c)*(a*d+
3*b*c)*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-4*((a*d-b
*c)/c)^(1/3)*3^(1/2)*(a*b^(5/3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*arctan(1/3*
3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-4*((a*d-b*c)/c)^(1/3)*(a*
b^(5/3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+
3/2*x*(2*b^2*c^2-2*b*d*(-1/2*b*x^3+a)*c+a^2*d^2)*d*c*(b*x^3+a)^(2/3)*((a*d
-b*c)/c)^(1/3)+(a*d+3*b*c)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b
*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*
x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c)
*(a*d-b*c)^2)/((a*d-b*c)/c)^(1/3)/d^3/c^2/(d*x^3+c)
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(291) = 582$.

Time = 3.35 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{2\sqrt{3}(3\,b^2c^3-2\,abc^2d-a^2cd^2+(3\,b^2c^2d-2\,abcd^2-a^2d^3)x^3)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan}{}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b
*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(
-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4
*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sq
rt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^
3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 -
 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*
d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^
(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*
d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 +
(2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c
^2*d^3)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

3.98.8 Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

3.99 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx$

3.99.1 Optimal result

Integrand size = 21, antiderivative size = 301

$$
\begin{aligned}
\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx &= -\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)} + \frac{b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} \\
&- \frac{(bc-ad)^{2/3}(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^2} \\
&- \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(c+dx^3\right)}{18c^{5/3}d^2} \\
&+ \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2} \\
&- \frac{b^{5/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^2}
\end{aligned}
$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^3+a)^(2/3)/c/d/(d*x^3+c)-1/18*(-a*d+b*c)^(2/3)*(2*a
*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^2+1/6*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/d^2-1/2*b^(5/3)*ln(-b^(1
/3)*x+(b*x^3+a)^(1/3))/d^2+1/3*b^(5/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))/d^2*3^(1/2)-1/9*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^2*3^(
1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.12 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \frac{-\frac{12d(bc-ad)x(a+bx^3)^{2/3}}{c(c+dx^3)} + 12\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \frac{2i\left(3i+\sqrt{3}\right)\left(3b^2c^2-abcd-2a^2d^2\right)\cdots}{c\cdots}\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

output
```
((-12*d*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*(c + d*x^3)) + 12*Sqrt[3]*b^(5
/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + ((2*I)
*(3*I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*ArcTanh[(I + ((-I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(c^(5/3)
*(b*c - a*d)^(1/3)) - 12*b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - (
(2*I)*(-I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^(
1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(5/3)*(b*c - a*d)^
(1/3)) + 6*b^(5/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] + ((1 + I*Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*
c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x
^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(5/3)*(b*c - a*
d)^(1/3)))/(36*d^2)
```

3.99.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {930, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{3b^2cx^3+a(bc+2ad)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{\frac{3b^2c\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)(2ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{\frac{3b^2c\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)(2ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{\frac{3b^2c\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)(2ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\right.}{}\right.}{d}}{3cd}$$

$$\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

input
```
Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

output
```
-1/3*((b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (-(((b*c - a*d)
*(3*b*c + 2*a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)
^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*
c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^
3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (3*b^2*c*(ArcTan[(1 + (2*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(3*c*d)
```

3.99.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.99.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$-\frac{-\frac{3 b^{\frac{5}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}} x+\left(b x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c^{2} \left(d x^{3}+c\right) \left(\frac{a d -b c}{c}\right)^{\frac{1}{3}}}{2}-2 \left(a d +\frac{3 b c}{2}\right) \left(d x^{3}+c\right) \left(a d -b c \right) \ln\left(\frac{\left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} x+\left(b x^{3}+\cdots\right.}{x}\right.\cdots}{\cdots}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3/2*b^(5/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/
3))/x^2)*c^2*(d*x^3+c)*((a*d-b*c)/c)^(1/3)-2*(a*d+3/2*b*c)*(d*x^3+c)*(a*d-
b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3*3^(1/2)*b^(5/3)*arcta
n(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c^2*(d*x^3+c)*((a*d
-b*c)/c)^(1/3)+3*b^(5/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*c^2*(d*x^3+c)*
((a*d-b*c)/c)^(1/3)+(-3*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)*c*d*x+(a*d+3/2
*b*c)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a
*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3
)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c))*(a*d-b*c))/((a*d-b*c
)/c)^(1/3)/d^2/c^2/(d*x^3+c)
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(248) = 496$.

Time = 0.52 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.10

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx =$$

$$-\frac{2\sqrt{3}((3\,bcd+2\,ad^2)x^3+3\,bc^2+2\,acd)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right.}{3\,(bc-ad)x}\right.}{}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sq
rt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b
*c - a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sq
rt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*
d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x
^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) +
3*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^
(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^
3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((
b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(
1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(
b*c - a*d))/x^2))/(c*d^3*x^3 + c^2*d^2)
```

3.99.6 Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(5/3)/(c + d*x**3)**2, x)
```

3.99.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

3.99.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

3.100 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx = \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{2a\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)+1/9*a*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/
3)-1/3*a*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)
^(1/3)+2/9*a*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3
^(1/2))/c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}}{c+dx^3} - \frac{2\sqrt{-6+6i\sqrt{3}}a\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2(a+i\sqrt{3}a)\log(2}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

output
```
((6*c^(2/3)*x*(a + b*x^3)^(2/3))/(c + d*x^3) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*a*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(1/3) + (2*(a + I*Sqrt[3]*
a)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(1/3) - (I*(-I + Sqrt[3])*a*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[
3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(18*c^(5/3))
```

3.100.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}dx$$

$$\Big\downarrow \; 903$$

$$\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3c} + \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

$$\Big\downarrow \; 901$$

$$\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*
d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d
)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3
*c)
```

3.100.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.100.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$\frac{-\frac{a \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) \left(d\,x^{3}+c\right)}{9}+\frac{2 a \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) \left(d\,x^{3}+c\right)}{9}+\frac{x \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3}}{c^{2} \left(d\,x^{3}+c\right) \left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*(-1/2*a*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2)*(d*x^3+c)+a*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)*(d*x^3+c)+3/2*x*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+arctan(1/3
*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*
3^(1/2)*a*(d*x^3+c))/((a*d-b*c)/c)^(1/3)/c^2/(d*x^3+c)
```

3.100.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.100.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

3.100.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

3.100.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx &= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)\,(c+dx^3)} \\ &\quad + \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} \\ &\quad + \frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}} \\ &\quad - \frac{(3bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}\end{aligned}$$

output
```
-1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/18*(-2*a*d+3*b*c)*ln(d*x
^3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^
(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/9*(-2*a*d+3*b*c)*arctan(
1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*
d+b*c)^(4/3)*3^(1/2)
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$= \frac{-12c^{2/3}d\sqrt[3]{bc-ad}x(a+bx^3)^{2/3}+2\big(3-i\sqrt{3}\big)(3bc-2ad)(c+dx^3)\operatorname{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)+}{}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
(-12*c^(2/3)*d*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) + 2*(3 - I*Sqrt[3])*(
3*b*c - 2*a*d)*(c + d*x^3)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]] + 2*(1 + I*Sqrt[3])*(3*b*c - 2*a*
d)*(c + d*x^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3)] - I*(-I + Sqrt[3])*(3*b*c - 2*a*d)*(c + d*x^3)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1
/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^
(4/3)*(c + d*x^3))
```

3.101.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a(dx^3+c)}}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c(bc-ad)}-\frac{dx(a+bx^3)^{2/3}}{3c(c+dx^3)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d
)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c -
a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d))
```

3.101.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.101.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\dfrac{-\dfrac{\left(ad-\frac{3bc}{2}\right)\left(d\,x^3+c\right)\ln\left(\dfrac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9}+\dfrac{2\left(ad-\frac{3bc}{2}\right)\left(d\,x^3+c\right)\ln\left(\dfrac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right.}{9}}{c^2(ad-bc)(d\,x^3+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-3/2*b*c)*(d*x^3+c)*ln((((a*d-b*c)/c)^(2
/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-3
/2*b*c)*(d*x^3+c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*(b*x^3
+a)^(2/3)*((a*d-b*c)/c)^(1/3)*c*d*x+arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3
)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)*(a*d-3/2*b*c)*(d*x^3
+c))/c^2/(a*d-b*c)/(d*x^3+c)
```

3.101.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.101.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

3.101.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

3.101.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx=\int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

3.102 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2} dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2} dx = \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}(c+dx^3)} - \frac{2d(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}$$

output
```
1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/
(b*x^3+a)^(1/3)/(d*x^3+c)-1/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c
)^(7/3)+1/3*d*(-a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/
c^(5/3)/(-a*d+b*c)^(7/3)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/
3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a^2d^2+abd^2x^3+3b^2c(c+dx^3))}{a(bc-ad)^2\sqrt[3]{a+bx^3}(c+dx^3)} + \frac{2i\left(3i+\sqrt{3}\right)d(3bc-ad)\operatorname{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+b}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right.}{(bc-ad)^{7/3}}}{}$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output
```
((6*c^(2/3)*x*(a^2*d^2 + a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3)))/(a*(b*c - a*d
)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((2*I)*(3*I + Sqrt[3])*d*(3*b*c - a*d
)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/
3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*d*(-3*b*c + a*d)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(7/3) + ((1 + I*Sqrt[3])*d*(3*b*c - a*d)*Log[2*(b*c - a*d)^(2/3)*
x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(
I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(18*c^(5/3))
```

3.102.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {931, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^3+3bc-2ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{2ad(3bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{2d(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{2d(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((b*(3*b*c + a*
d)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (2*d*(3*b*c - a*d)*(ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(
2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - L
og[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*
d)^(1/3))))/(b*c - a*d))/(3*c*(b*c - a*d))
```

3.102. $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.102.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}} a d\left(d\,x^{3}+c\right)(ad-3bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{9}+\frac{xc\left(a\left(b\,x^{3}+a\right)d^{2}+3x^{3}b^{2}cd+3b^{2}c^{2}\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{3}+\frac{2(ad-3bc)da\left(d\,x^{3}\cdots\right.}{\cdots}}{c^{2}(d\,x^{3}+c)(ad-bc)^{2}\left(\frac{ad\cdots}{\cdots}\right.}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*((b*x^3+a)^(1/3)*a*d*(d*x^3+c)*(a*
d-3*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*x*c*(a*(b*x^3+a
)*d^2+3*x^3*b^2*c*d+3*b^2*c^2)*((a*d-b*c)/c)^(1/3)+(a*d-3*b*c)*d*a*(d*x^3+
c)*(b*x^3+a)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^
(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*
d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/c^2/(d*x^3+c)/(a
*d-b*c)^2/a
```

3.102.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.102.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

3.102.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

3.102.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

3.103 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2} dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 324

$$\begin{aligned}\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2} dx = &\frac{b(3bc+4ad)x}{12ac(bc-ad)^2(a+bx^3)^{4/3}} \\ &+ \frac{b(9b^2c^2-33abcd-4a^2d^2)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)} \\ &+ \frac{d^2(9bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{10/3}} \\ &- \frac{d^2(9bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}\end{aligned}$$

output
```
1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)+1/12*b*(-4*a^2*d^2
-33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*
d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)+1/18*d^2*(-2*a*d+9*b*c)*ln(d*x^3+c)/c^(5/
3)/(-a*d+b*c)^(10/3)-1/6*d^2*(-2*a*d+9*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(10/3)+1/9*d^2*(-2*a*d+9*b*c)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d
+b*c)^(10/3)*3^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.11 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \frac{\frac{3c^{2/3}x(4a^4d^3+8a^3bd^3x^3-9b^4c^2x^3(c+dx^3)+4a^2b^2d(9c^2+9cdx^3+d^2x^6)+3ab^3c(-4c^2+7cdx^3+11d^2x^6)}{a^2(-bc+ad)^3(a+bx^3)^{4/3}(c+dx^3)}}{}$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

output
```
((3*c^(2/3)*x*(4*a^4*d^3 + 8*a^3*b*d^3*x^3 - 9*b^4*c^2*x^3*(c + d*x^3) + 4
*a^2*b^2*d*(9*c^2 + 9*c*d*x^3 + d^2*x^6) + 3*a*b^3*c*(-4*c^2 + 7*c*d*x^3 +
 11*d^2*x^6)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((2*
I)*(3*I + Sqrt[3])*d^2*(-9*b*c + 2*a*d)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/
3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(10/3)
+ (2*(1 + I*Sqrt[3])*d^2*(9*b*c - 2*a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(10/3) + ((1 + I*Sqrt[3
])*d^2*(-9*b*c + 2*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(
1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a +
b*x^3)^(2/3)])/(b*c - a*d)^(10/3))/(36*c^(5/3))
```

3.103.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {931, 1024, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-6bdx^3+3bc-2ad}{(bx^3+a)^{7/3}(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}-\frac{\int -\frac{3bd(3bc+4ad)x^3+9b^2c^2+8a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3bd(3bc+4ad)x^3+9b^2c^2+8a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}+\frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{4a^2d^2(9bc-2ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}}{4a(bc-ad)}+\frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{4ad^2(9bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{bc-ad}+\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)}+\frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}+\frac{4ad^2(9bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{bx(}{4a(a+b}$$
$$\overline{3c(bc-ad)}$$
$$\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

3.103. $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((b*(3*b*c + 4*
a*d)*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(9*b^2*c^2 - 33*a*b*c*d
- 4*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + (4*a*d^2*(9*b*c - 2*a*
d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c -
 a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c
^(2/3)*(b*c - a*d)^(1/3))))/(b*c - a*d))/(4*a*(b*c - a*d)))/(3*c*(b*c - a*
d))
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.103.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{-\frac{3x\left(4a^2b^2d^3x^6+33a\,b^3c\,d^2x^6-9b^4c^2d\,x^6+8a^3b\,d^3x^3+36a^2b^2c\,d^2x^3+21a\,b^3c^2d\,x^3-9b^4c^3x^3+4a^4d^3+36a^2b^2c^2d-12a\,b^3c^3\right)c\left(\frac{ad-b}{c}\right.}{2}}{}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/18/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(4/3)*(-3/2*x*(4*a^2*b^2*d^3*x^6+33*a*
b^3*c*d^2*x^6-9*b^4*c^2*d*x^6+8*a^3*b*d^3*x^3+36*a^2*b^2*c*d^2*x^3+21*a*b^
3*c^2*d*x^3-9*b^4*c^3*x^3+4*a^4*d^3+36*a^2*b^2*c^2*d-12*a*b^3*c^3)*c*((a*d
-b*c)/c)^(1/3)+(b*x^3+a)^(4/3)*a^2*d^2*(d*x^3+c)*(2*a*d-9*b*c)*(-2*arctan(
1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/
x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))
/c^2/(d*x^3+c)/(a*d-b*c)^3/a^2
```

3.103.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.103.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

3.103.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

3.103.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

3.104 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^
3/a)^(1/3)
```

3.104.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(60) = 120.

Time = 10.34 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.68

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{x\left(b(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(3a^2d-b^2cx^3+abd\cdots\right.}{(c+dx^3)\left(\cdots\right.}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

output
```
(x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3
)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*
x^3*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c
+ d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a +
 b*x^3)^(2/3))
```

3.104.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{\left(dx^3+c\right)^2}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

3.104.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.104.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)^{2}}d x$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

3.104.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.104.6 Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3)**2, x)
```

3.104.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

3.104.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^2, x)
```

3.105 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$

3.105.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \frac{x\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/
a)^(1/3)
```

3.105.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.22 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$
$$= \frac{x\left(\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\cdots\right.}{-4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\cdots\right.\right.}\right.\cdots}{c+dx^3}\cdots\right)}{12\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -
((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
]))))/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

3.105.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{(dx^3+c)^2}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

3.105.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.105.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{(d\,x^{3}+c)^{2}}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

3.105.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.105.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

3.105.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

3.105.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

3.106 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a
)^(2/3)
```

3.106.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 393 vs. $2(59) = 118$.

Time = 10.30 (sec) , antiderivative size = 393, normalized size of antiderivative = 6.66

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{4acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(4c(-3bc+3ad+bdx^3)+bdx^3\left(1+\right.\right.}{12c^2(bc-}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

output
```
(4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b
*c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1
[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) +
 b*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x
^3)/a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])
)/(12*c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3,
2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2
, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b
*x^3)/a), -((d*x^3)/c)])))
```

3.106.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}\left(dx^3+c\right)^2}dx}{\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(a + b*x^3)^(2/3))
```

3.106.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.106.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

3.106.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.106.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)
```

3.106.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

3.106.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)
```

3.107 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3
+a)^(2/3)
```

3.107.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 386 vs. 2(62) = 124.

Time = 10.53 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.23

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(bd(3bc+2ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c(16ac(6\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

output
```
(x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d
*x^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a
*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1
[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)
^(2/3))
```

3.107.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}dx$$

$$\Big\downarrow \; 937$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}(dx^3+c)^2}dx}{a(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

3.107.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.107.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

3.107.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.107.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)
```

3.107.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

3.107.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)
```

3.108 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x
^3+a)^(2/3)
```

3.108.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 550 vs. 2(62) = 124.

Time = 10.98 (sec) , antiderivative size = 550, normalized size of antiderivative = 8.87

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \frac{\frac{bd(-6b^2c^2+21abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3} + \frac{4c\left(-4acx(15a^4d^3-6b\right.}{}}{}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

output
```
((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appe
llF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*
c*(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c
+ 4*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c
^2 + 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((
d*x^3)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3)
 + a^2*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d
*x^3 + 7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^
3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(
(b*c - a*d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3
)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)]))))/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

3.108.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}dx$$

$$\Big\downarrow \ 937$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)^2}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \ 936$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

3.108.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.108.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}\left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} \, dx = \int \frac{1}{(a+bx^3)^{\frac{8}{3}}(c+dx^3)^2} \, dx$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**2), x)
```

3.108.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} \, dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^2} \, dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

3.108.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} \, dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^2} \, dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx=\int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2), x)
```

3.109 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

3.109.1 Optimal result

Integrand size = 21, antiderivative size = 541

$$\begin{aligned}
\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx &= -\frac{b(2bc-ad)\left(18b^2c^2-18abcd-5a^2d^2\right)x(a+bx^3)^{2/3}}{18c^2d^4} \\
&+ \frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3} \\
&- \frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} \\
&+ \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^5} \\
&- \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5} \\
&- \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^5} \\
&+ \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5} \\
&- \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^5}
\end{aligned}$$

output
```
-1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/
c^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^
3-1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d
+12*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(8/3)*(5*a^2*
d^2+18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(
5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(
1/3))/c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(
1/3)*x+(b*x^3+a)^(1/3))/d^5+1/27*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^
2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^5*3^(1/2)-1/27*(-
a*d+b*c)^(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c
)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^5*3^(1/2)
```

3.109.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 12.28 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \frac{1}{108}\left(\frac{6x(a+bx^3)^{2/3}\left(-2b^3(9bc-13ad)+3b^4dx^3+\frac{3(bc-ad)^4}{c(c+dx^3)^2}-\frac{(bc-ad)^3(21bc+5ad)}{c^2(c+dx^3)}\right)}{d^4}\right.$$

$$+\frac{162b^5cx^4\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^4\sqrt[3]{a+bx^3}}$$

$$-\frac{378ab^4x^4\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^3\sqrt[3]{a+bx^3}}$$

$$+\frac{231a^2b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd^2\sqrt[3]{a+bx^3}}$$

$$+\frac{10a^5\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{8/3}\sqrt[3]{bc-ad}}$$

$$+\frac{36ab^4c^{4/3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{}}{\sqrt[3]{b}}\right.\right.}{d^4\sqrt[3]{bc-ad}}$$

$$-\frac{72a^2b^3\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}}{\sqrt[3]{}}\right.\right.}{d^3\sqrt[3]{bc-ad}}$$

$$+\frac{30a^3b^2\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc}}{\sqrt[3]{b+}}\right.\right.}{c^{2/3}d^2\sqrt[3]{bc-ad}}$$

$$+\frac{6a^4b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{5/3}d\sqrt[3]{bc-ad}}$$

3.109. $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

input
```
Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

output
```
((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
 a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*
x^3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*
(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3
)) + (10*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3))
+ (36*a*b^4*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
+ a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3
)) - (72*a^2*b^3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c
^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)
^(1/3)) + (30*a^3*b^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c...
```

3.109.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {930, 1023, 27, 1025, 27, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{\left(bx^3+a\right)^{8/3}\left(6b(2bc-ad)x^3+a(bc+5ad)\right)}{\left(dx^3+c\right)^2}dx}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{ 1023}$$

$$\frac{\frac{x\left(a+bx^3\right)^{8/3}\left(\frac{5a^2d}{c}+7ab-\frac{12b^2c}{d}\right)}{3(c+dx^3)}-\frac{\int -\frac{2\left(bx^3+a\right)^{5/3}\left(3b\left(18b^2c^2-10abdc-5a^2d^2\right)x^3+a\left(6b^2c^2-2abdc+5a^2d^2\right)\right)}{dx^3+c}dx}{3cd}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{\left(bx^3+a\right)^{5/3}\left(3b\left(18b^2c^2-10abdc-5a^2d^2\right)x^3+a\left(6b^2c^2-2abdc+5a^2d^2\right)\right)}{dx^3+c}dx}{3cd}+\frac{x\left(a+bx^3\right)^{8/3}\left(\frac{5a^2d}{c}+7ab-\frac{12b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{2\left(\frac{\int -\frac{3\left(bx^3+a\right)^{2/3}\left(3b(2bc-ad)\left(18b^2c^2-18abdc-5a^2d^2\right)x^3+a\left(18b^3c^3-22ab^2dc^2-a^2bd^2c-10a^3d^3\right)\right)}{dx^3+c}dx}{6d}+\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}\right)}{3cd}+\frac{x(\ldots}{\ldots}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(3b(2bc-ad)\left(18b^2c^2-18abdc-5a^2d^2\right)x^3+a\left(18b^3c^3-22ab^2dc^2-a^2bd^2c-10a^3d^3\right)\right)}{dx^3+c}dx}{2d}\right)}{3cd}+\frac{x(a\ldots}{\ldots}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\frac{\int -\frac{6\left(b^3c^2\left(54b^2c^2-126abdc+77a^2d^2\right)x^3+a\left(18b^4c^4-36ab^3dc^3+15a^2b^2d^2c^2+3a^3bd^3c+5a^4d^4\right)\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3d}+\frac{bx(a+bx^3\ldots}{\ldots}}{2d}\right)}{3cd}\ldots}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

3.109. $\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx$

$$\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{\frac{2\int\frac{b^3c^2\left(54b^2c^2-126abdc+77a^2d^2\right)x^3+a\left(18b^4c^4-36ab^3c\cdots\right.}{\sqrt[3]{bx^3+a}\left(dx^3+\cdots\right.}}{d}}{2d}\cdots\right.}{3cd}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 1026

$$\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\int\frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\cdots\right.}{2d}\cdots\right.}{3cd}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 769

3.109. $\int\frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx$

$$\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+b\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\cdots}\cdots\right)}{d}\cdots\right)}{2\cdots}\cdots\right)}{3cd\cdots}}{6cd\cdots}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 901

3.109. $\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx$

$$2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}\right.}{d}\right.}{}\right.$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

3.109. $\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx$

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(11/3))/(c*d*(c + d*x^3)^2) + (((7*a*b - (
12*b^2*c)/d + (5*a^2*d)/c)*x*(a + b*x^3)^(8/3))/(3*(c + d*x^3)) + (2*((b*(
18*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(5/3))/(2*d) - ((b*(2*b
*c - a*d)*(18*b^2*c^2 - 18*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/d - (
2*(-(((b*c - a*d)^3*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*
(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[
((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^
(1/3))))/d) + (b^3*c^2*(54*b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*(ArcTan[(1
+ (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d))/d)/(2*d)))/(3*c*d))/(6*c*d)
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.109.4 Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{8}{3}}\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^3\left(d\,x^3+c\right)^2\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}+\left(5a^2d^2+18abcd+54b^2c^2\right)\left(d\,x^3+\right.}{}$

input
```
int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/27/((a*d-b*c)/c)^(1/3)*(1/2*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*
((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2)+(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*(d*x^3+c)^2*(a
*d-b*c)^3*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-3^(1/2)*b^(8/3)*(7
7*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*arct
an(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-b^(8/3)*(77*a^2*d^
2-126*a*b*c*d+54*b^2*c^2)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((-b^(1/3)
*x+(b*x^3+a)^(1/3))/x)+3/2*x*(3*b^4*c^2*d^3*x^9+26*a*b^3*c^2*d^3*x^6-12*b^
4*c^3*d^2*x^6+5*a^4*d^5*x^3+6*a^3*b*c*d^4*x^3-48*a^2*b^2*c^2*d^3*x^3+110*a
*b^3*c^3*d^2*x^3-54*b^4*c^4*d*x^3+8*a^4*c*d^4-6*a^3*b*c^2*d^3-30*a^2*b^2*c
^3*d^2+72*a*b^3*c^4*d-36*b^4*c^5)*d*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)-
1/2*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c
)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)
/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))
*(d*x^3+c)^2*(a*d-b*c)^3)/d^5/c^3/(d*x^3+c)^2
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. $2(473) = 946$.

Time = 94.18 (sec) , antiderivative size = 1555, normalized size of antiderivative = 2.87

$$\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-1/54*(2*sqrt(3)*(54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3
*b*c^3*d^3 + 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b
^2*c^2*d^4 + 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c
^4*d^2 + 23*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/
((b*c - a*d)*x)) + 2*sqrt(3)*(54*b^4*c^6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^
4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^2*d^4)*x^6 + 2*
(54*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3)*(-b^2)^(1/3)*
arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)
) - 2*(54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d^3
+ 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b^2*c^2*d^4
+ 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c^4*d^2 + 23
*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(54*b^4*c^6 - 126*a*b^3*c^5*d
+ 77*a^2*b^2*c^4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^
2*d^4)*x^6 + 2*(54*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3
)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (54*b^4*c^
6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*...
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

3.109.8 Giac [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{14/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

3.110 $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 458

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx = \frac{b(18b^2c^2-7abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2\,(c+dx^3)} - \frac{b^{8/3}(9bc-11ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^4} + \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^4} + \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}d^4} - \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4} + \frac{b^{8/3}(9bc-11ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^4}$$

output
```
1/18*b*(-5*a^2*d^2-7*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c^2/d^3-1/6*(-a
*d+b*c)*x*(b*x^3+a)^(8/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+9*b*c)*x*
(b*x^3+a)^(5/3)/c^2/d^2/(d*x^3+c)+1/54*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*
c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^4-1/18*(-a*d+b*c)^(5/3)*(5*a^2*d^2+1
2*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/
3)/d^4+1/6*b^(8/3)*(-11*a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^4-1/9*
b^(8/3)*(-11*a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2)
)/d^4*3^(1/2)+1/27*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*arct
an(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d
^4*3^(1/2)
```

3.110.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.74 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \frac{1}{108}\left(\frac{6x(a+bx^3)^{2/3}\left(6b^3-\frac{3(bc-ad)^3}{c(c+dx^3)^2}+\frac{5(bc-ad)^2(3bc+ad)}{c^2(c+dx^3)}\right)}{d^3}\right.$$

$$-\frac{81b^4x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^3\sqrt[3]{a+bx^3}}$$

$$+\frac{99ab^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd^2\sqrt[3]{a+bx^3}}$$

$$+\frac{10a^4\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-\cdots}}{\sqrt[3]{b+a\cdots}}\right.\right.}{c^{8/3}\sqrt[3]{bc-ad}}$$

$$-\frac{18ab^3\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{\cdots}}{\sqrt[3]{b\cdots}}\right.\right.}{d^3\sqrt[3]{bc-ad}}$$

$$+\frac{16a^2b^2\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc\cdots}}{\sqrt[3]{b+\cdots}}\right.\right.}{c^{2/3}d^2\sqrt[3]{bc-ad}}$$

$$+\frac{4a^3b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-\cdots}}{\sqrt[3]{b+a\cdots}}\right.\right.}{c^{5/3}d\sqrt[3]{bc-ad}}$$

input
```
Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

3.110. $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx$

output
```
((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*
(b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b
*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^
3*(a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a
^4*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/
3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] +
 Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(
1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)
*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a
^2*b^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)
^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) +
(4*a^3*b*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/...
```

3.110.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {930, 1023, 25, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{\left(bx^3+a\right)^{5/3}\left(3b(3bc-ad)x^3+a(bc+5ad)\right)}{\left(dx^3+c\right)^2}dx}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{1023}$$

$$\frac{\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}-\frac{\int -\frac{(bx^3+a)^{2/3}\left(3b\left(18b^2c^2-7abdc-5a^2d^2\right)x^3+a\left(9b^2c^2-abdc+10a^2d^2\right)\right)}{dx^3+c}dx}{3cd}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(bx^3+a)^{2/3}\left(3b\left(18b^2c^2-7abdc-5a^2d^2\right)x^3+a\left(9b^2c^2-abdc+10a^2d^2\right)\right)}{dx^3+c}dx}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{\frac{\int -\frac{6\left(3b^3c^2(9bc-11ad)x^3+a\left(9b^3c^3-8ab^2dc^2-2a^2bd^2c-5a^3d^3\right)\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3d}+\frac{bx(a+bx^3)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{bx(a+bx^3)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\int \frac{3b^3c^2(9bc-11ad)x^3+a\left(9b^3c^3-8ab^2dc^2-2a^2bd^2c-5a^3d^3\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1026$$

$$\frac{\frac{\frac{bx(a+bx^3)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+27b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}\right)}{d}}{3cd}+\frac{x(\cdots}{\cdots}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 769$$

3.110. $\displaystyle\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx$

$$\frac{\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+27b\cdots\right.}{\cdots}\right)}{d}}{3cd}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 901

$$\frac{\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+27b\cdots\right.}{\cdots}\right)}{d}}{3cd}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input

```
Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

3.110. $\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx$

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(8/3))/(c*d*(c + d*x^3)^2) + (((4*a*b - (9
*b^2*c)/d + (5*a^2*d)/c)*x*(a + b*x^3)^(5/3))/(3*(c + d*x^3)) + ((b*(18*b^
2*c^2 - 7*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/d - (2*(-(((b*c - a*d)
^2*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)
*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/
3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3
)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (3*
b^3*c^2*(9*b*c - 11*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)))/d))/d)/(3*c*d))/(6*c*d)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.110.4 Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{-3 b^{\frac{8}{3}} \left(11 a d -9 b c \right) \left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} c^{3} \left(d \,x^{3}+c \right)^{2} \ln \left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2}}\right)-2 \left(5 a^{2} d^{2}+12 a b c d +27 b^{2} c^{2}\right) \left(d \,x^{3}+c \right)^{2} (}{}$

input
```
int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-3*b^(8/3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((b
^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*(5*a^2*d^2+12
*a*b*c*d+27*b^2*c^2)*(d*x^3+c)^2*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*
x^3+a)^(1/3))/x)+6*b^(8/3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*3^(1/2)*c^3*
(d*x^3+c)^2*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+6*
b^(8/3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((-b^(1/3)*x+
(b*x^3+a)^(1/3))/x)-3*x*d*c*(6*b^3*c^2*d^2*x^6+5*a^3*d^4*x^3+5*a^2*b*c*d^3
*x^3-25*a*b^2*c^2*d^2*x^3+27*b^3*c^3*d*x^3+8*a^3*c*d^3-4*a^2*b*c^2*d^2-16*
a*b^2*c^3*d+18*b^3*c^4)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+(5*a^2*d^2+12*
a*b*c*d+27*b^2*c^2)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3
+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*
d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c)^2*(a*d-
b*c)^2)/((a*d-b*c)/c)^(1/3)/(d*x^3+c)^2/d^4/c^3
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. $2(394) = 788$.

Time = 18.18 (sec) , antiderivative size = 1246, normalized size of antiderivative = 2.72

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2
*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6
 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3
)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c -
a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^
3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3
)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^
2)^(1/3))/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^
3*c^2*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5
)*x^6 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4
)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b
^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^
3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^
3 + a)^(1/3)*b)/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a
*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*l
og(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(
2/3)*b)/x^2) + (27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x...
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.110.7 Maxima [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

3.110.8 Giac [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{11/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

3.111 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 391

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx &= -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd(c+dx^3)^2} \\ &- \frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2(c+dx^3)} + \frac{b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} \\ &- \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^3} \\ &- \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}d^3} \\ &+ \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3} \\ &- \frac{b^{8/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^3}\end{aligned}$$

output
```
-1/6*(-a*d+b*c)*x*(b*x^3+a)^(5/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6
*b*c)*x*(b*x^3+a)^(2/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(2/3)*(5*a^2*d^2
+6*a*b*c*d+9*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^3+1/18*(-a*d+b*c)^(2/3)*(5*a^2
*d^2+6*a*b*c*d+9*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c
^(8/3)/d^3-1/2*b^(8/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/3*b^(8/3)*arct
an(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)-1/27*(-a*d+b*c
)^(2/3)*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^3*3^(1/2)
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.00 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(-bc+ad)x(a+bx^3)^{2/3}(3bc(2c+3dx^3)+ad(8c+5dx^3))}{d^2(c+dx^3)^2} + \frac{27b^3c^{5/3}x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\right.}{d^2\sqrt[3]{a+bx^3}}}{}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*
d*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)
/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a
+ b*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^
(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/
3) + (6*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b +
a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) +
(c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3))
 + (2*a^2*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
 a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^
3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(
1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108
*c^(8/3))
```

3.111.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {930, 1023, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{(bx^3+a)^{2/3}(6b^2cx^3+a(bc+5ad))}{(dx^3+c)^2}dx}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \ 1023$$

$$\frac{-\frac{\int -\frac{2\left(9b^3c^2x^3+a\left(3b^2c^2+ad(bc+5ad)\right)\right)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\int \frac{9b^3c^2x^3+a\left(3b^2c^2+ad(bc+5ad)\right)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \ 1026$$

$$\frac{\frac{2\left(\frac{9b^3c^2\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}\right)}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \ 769$$

$$\frac{2\left(\frac{9b^3c^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\int\frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}\right)}{3cd} - \frac{x\left(a+bx^3\right)^{2/3}(b}{3cd(c}$$

$$\frac{}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 901

$$\frac{2\left(\frac{9b^3c^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-}}\right.}{d}\right)}{3cd}$$

$$\frac{}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

3.111. $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx$

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)^2) + (-1/3*((b*c -
 a*d)*(6*b*c + 5*a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (2*(-(((b*c
 - a*d)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^
(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(
1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) +
(9*b^3*c^2*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]
*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d))/(3*c*d
))/(6*c*d)
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.111. $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3} dx$

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.111.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{\frac{9 b^{\frac{8}{3}} c^{3} \left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} \left(d \,x^{3}+c \right)^{2} \ln \left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2}}\right)}{2}+\left(5 a^{2} d^{2}+6 a b c d +9 b^{2} c^{2}\right) \left(d \,x^{3}+c \right)^{2} \left(a d -b c \right) \ln \left(\frac{\left(\frac{a d -b c}{c}\right)^{\frac{1}{3}}}{\cdots}\right.}{\cdots}$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/27*(9/2*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)^2*ln((b^(2/3)*x^2+b^(1
/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2
)*(d*x^3+c)^2*(a*d-b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-9*3^
(1/2)*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)^2*arctan(1/3*3^(1/2)*(b^(1
/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-9*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x
^3+c)^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/2*(3*(b*x^3+a)^(2/3)*x*d*(5*a
*d^2*x^3+9*b*c*d*x^3+8*a*c*d+6*b*c^2)*c*((a*d-b*c)/c)^(1/3)-(-2*arctan(1/3
*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*
3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+
(b*x^3+a)^(2/3))/x^2))*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*(d*x^3+c)^2)*(a*d-b
*c))/((a*d-b*c)/c)^(1/3)/(d*x^3+c)^2/d^3/c^3
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(334) = 668$.

Time = 1.95 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.44

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx =$$

$$-\frac{2\sqrt{3}((9\,b^2c^2d^2+6\,abcd^3+5\,a^2d^4)x^6+9\,b^2c^4+6\,abc^3d+5\,a^2c^2d^2+2\,(9\,b^2c^3d+6\,abc^2d^2+5\,a^2cd^3)x^3)\Big(\frac{b^2}{}\cdots}{\cdots}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^
4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c
*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3
)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*
c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*
a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d +
6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/
3)*(b*c - a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^
2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x
^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b
*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d
^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^
2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a
*b*c*d^3 - 5*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)...
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^3}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

3.111.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^3}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

3.112 $\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx=\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{5a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}$$

output
```
1/6*x*(b*x^3+a)^(5/3)/c/(d*x^3+c)^2+5/18*a*x*(b*x^3+a)^(2/3)/c^2/(d*x^3+c)
+5/54*a^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-5/18*a^2*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+5/27*a^2*arctan(1/3*
(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*
c)^(1/3)*3^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(8acx+3bcx^4+5adx^4)}{(c+dx^3)^2} - \frac{10\sqrt{-6+6i\sqrt{3}}a^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(8*a*c*x + 3*b*c*x^4 + 5*a*d*x^4))/(c + d*x^
3)^2 - (10*Sqrt[-6 + (6*I)*Sqrt[3]]*a^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sq
rt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(
b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*a^2*Log[2*(b*c - a*d)^(1/3)*x + (1
+ I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) - ((5*I)*(-I +
Sqrt[3])*a^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3
)])/(b*c - a*d)^(1/3))/(108*c^(8/3))
```

3.112.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {903, 903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{5a\int \frac{(bx^3+a)^{2/3}}{(dx^3+c)^2}dx}{6c} + \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

$$\Big\downarrow \; 903$$

$$\frac{5a\left(\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c}+\frac{x\left(a+bx^3\right)^{2/3}}{3c(c+dx^3)}\right)}{6c}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{901}$$

$$\frac{5a\left(\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x\left(a+bx^3\right)^{2/3}}{3c(c+dx^3)}\right)}{6c}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*((x*(a + b*x^3)^(2/3))/(3
*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^
3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c)))/(6*c)
```

3.112. $\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}dx$

3.112.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.112.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{-\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)a^{2}\left(d\,x^{3}+c\right)^{2}}{54}+\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{2}\left(d\,x^{3}+c\right)^{2}}{27}+\frac{4x\left(\frac{(5c}{}\right.}{}}{c^{3}\left(d\,x^{3}+c\right)^{2}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)*a^2*(d*x^3+c)^2+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x)*a^2*(d*x^3+c)^2+12/5*x*(1/8*(5*a*d+3*b*c)*x^3+a*c)*c*(b*x^3+a
)^(2/3)*((a*d-b*c)/c)^(1/3)+3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3
)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a^2*(d*x^3+c)^2)/((a*d-b*c)/
c)^(1/3)/c^3/(d*x^3+c)^2
```

3.112.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

3.112.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

3.113 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$

3.113.1 Optimal result

Integrand size = 21, antiderivative size = 267

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2}+\frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{a(6bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc-5ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}$$

output
```
-1/6*d*x*(b*x^3+a)^(5/3)/c/(-a*d+b*c)/(d*x^3+c)^2+1/18*(-5*a*d+6*b*c)*x*(b
*x^3+a)^(2/3)/c^2/(-a*d+b*c)/(d*x^3+c)+1/54*a*(-5*a*d+6*b*c)*ln(d*x^3+c)/c
^(8/3)/(-a*d+b*c)^(4/3)-1/18*a*(-5*a*d+6*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3
)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)+1/27*a*(-5*a*d+6*b*c)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d
+b*c)^(4/3)*3^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}(3bc(2c+dx^3)-ad(8c+5dx^3))}{(bc-ad)(c+dx^3)^2} + \frac{2i\left(3i+\sqrt{3}\right)a(-6bc+5ad)\operatorname{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+b}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + d*x^3) - a*d*(8*c + 5*d*x^3)
))/((b*c - a*d)*(c + d*x^3)^2) + ((2*I)*(3*I + Sqrt[3])*a*(-6*b*c + 5*a*d)
*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3
)*x))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*a*(6*b*c - 5*a*d)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(4/3) + ((1 + I*Sqrt[3])*a*(-6*b*c + 5*a*d)*Log[2*(b*c - a*d)^(2/
3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(108*c^(8/3
))
```

3.113.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {907, 903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{(6bc-5ad)\int \frac{(bx^3+a)^{2/3}}{(dx^3+c)^2}dx}{6c(bc-ad)} - \frac{dx(a+bx^3)^{5/3}}{6c(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(6bc-5ad)\left(\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3c}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}\right)}{6c(bc-ad)}-\frac{dx(a+bx^3)^{5/3}}{6c(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{(6bc-5ad)\left(\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}\right)}{6c(bc-ad)}-\frac{dx(a+bx^3)^{5/3}}{6c(c+dx^3)^2(bc-ad)}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

output
```
-1/6*(d*x*(a + b*x^3)^(5/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a
*d)*((x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
 - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))
))/(3*c)))/(6*c*(b*c - a*d))
```

3.113. $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.113.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{\frac{5\left(ad-\frac{6bc}{5}\right)a\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54}+\frac{5\left(ad-\frac{6bc}{5}\right)a\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3 \cdots\right.}{x}\right.}{27}}{(ad-bc)c^3 \cdots}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-6/5*b*c)*a*(d*x^3+c)^2*ln((((a*d-b*c)/
c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(
a*d-6/5*b*c)*a*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+1
2/5*(1/8*(5*a*d^2-3*b*c*d)*x^3+c*(a*d-3/4*b*c))*x*c*(b*x^3+a)^(2/3)*((a*d-
b*c)/c)^(1/3)+(a*d-6/5*b*c)*3^(1/2)*a*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1
/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*(d*x^3+c)^2)/(a*d-b*c)/c^3
/(d*x^3+c)^2
```

3.113.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.113.6 Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3)**3, x)
```

3.113.7 Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

3.113.8 Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^3}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 307

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = -\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\,(c+dx^3)^2} - \frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\,(c+dx^3)} + \frac{(9b^2c^2-12abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{7/3}} + \frac{(9b^2c^2-12abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{7/3}} - \frac{(9b^2c^2-12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}$$

output
```
-1/6*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*
(b*x^3+a)^(2/3)/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^
2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(7/3)-1/18*(5*a^2*d^2-12*a*b*c*d+9*b
^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^
(7/3)+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/
3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$

$$= \frac{\frac{6c^{2/3}dx(a+bx^3)^{2/3}(-3bc(4c+3dx^3)+ad(8c+5dx^3))}{(bc-ad)^2(c+dx^3)^2} + \frac{2\left(3-i\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\,\text{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}} + 2\ldots}{\ldots}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

output
```
((6*c^(2/3)*d*x*(a + b*x^3)^(2/3)*(-3*b*c*(4*c + 3*d*x^3) + a*d*(8*c + 5*d
*x^3)))/((b*c - a*d)^2*(c + d*x^3)^2) + (2*(3 - I*Sqrt[3])*(9*b^2*c^2 - 12
*a*b*c*d + 5*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3]
)*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(7/3) - (I*(-I + Sqrt[3])
*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 -
I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])
*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(108*c^(8/3))
```

3.114.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {931, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^3+6bc-5ad}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{2\left(9b^2c^2-12abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c\left(c+dx^3\right)(bc-ad)}}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(5a^2d^2-12abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c\left(c+dx^3\right)(bc-ad)}}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{2\left(5a^2d^2-12abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c\left(c+dx^3\right)(bc-ad)}}{6c(bc-ad)}$$

$$\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + (-1/3*(d*(9*b
*c - 5*a*d)*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + (2*(9*b^2*c
^2 - 12*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
+ d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d)))/(
6*c*(b*c - a*d))
```

3.114. $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}dx$

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.114.4 Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\frac{5\left(a^{2}d^{2}-\frac{12}{5}abcd+\frac{9}{5}b^{2}c^{2}\right)\left(d\,x^{3}+c\right)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{27}+\frac{4x\left(-\frac{3b\,c^{2}}{2}+d\left(-\frac{9b\,x^{3}}{8}+a\right)c+\frac{5a\,d^{2}x^{3}}{8}\right)dc\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)}{9}}{\left(\frac{ad-bc}{c}\right.}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*((a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*(d*x^3+c)^2*l
n((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5*x*(-3/2*b*c^2+d*(-9/8*b*
x^3+a)*c+5/8*a*d^2*x^3)*d*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+(arctan(1/
3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)
*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*(d*x^3+c)^2
)/(a*d-b*c)^2/c^3/(d*x^3+c)^2
```

3.114.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

3.114.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

3.115 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3} dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 377

$$\begin{aligned}\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3} dx &= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2} \\ &+ \frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)} + \frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18ac^2(bc-ad)^3\,(c+dx^3)} \\ &- \frac{d(27b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{10/3}} \\ &- \frac{d(27b^2c^2-18abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}} \\ &+ \frac{d(27b^2c^2-18abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}}\end{aligned}$$

output
```
-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(1/3)/(d*x^3+c)^2+1/6*b*(a*d+6*b*c)*x/a/c/
(-a*d+b*c)^2/(b*x^3+a)^(1/3)/(d*x^3+c)+1/18*d*(-5*a^2*d^2+15*a*b*c*d+18*b^
2*c^2)*x*(b*x^3+a)^(2/3)/a/c^2/(-a*d+b*c)^3/(d*x^3+c)-1/54*d*(5*a^2*d^2-18
*a*b*c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(10/3)+1/18*d*(5*a^2*d
^2-18*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c
^(8/3)/(-a*d+b*c)^(10/3)-1/27*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d
+b*c)^(10/3)*3^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 13.68 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx =$$

$$-\frac{65c^2(a+bx^3)^2\left(-14000a^2c^5-21896abc^5x^3-48104a^2c^4dx^3-8391b^2c^5x^6-70802abc^4dx^6-60807a^2c^3d^2\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

output
```
-1/16380*(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104
*a^2*c^4*d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^
2*x^6 - 24417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^
9 - 23409*b^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12
- 7425*b^2*c^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c +
 d*x^3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*
x^3 + 33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*
x^9))*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) +
 486*(b*c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1
, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)^3*x^8*
(a + b*x^3)^(7/3)*(c + d*x^3)^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {931, 1024, 27, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-6bdx^3+6bc-5ad}{(bx^3+a)^{4/3}(dx^3+c)^2}dx}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\,(c+dx^3)^2\,(bc-ad)}$$

$$\Big\downarrow \ 1024$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{\int \frac{d\left(a(12bc-5ad)-3b(6bc+ad)x^3\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\int \frac{a(12bc-5ad)-3b(6bc+ad)x^3}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \ 1024$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\left(\frac{\int \frac{2a\left(27b^2c^2-18abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{x\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\left(\frac{2a\left(5a^2d^2-18abcd+27b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{x\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \ 901$$

3.115. $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)} - \frac{d\left(\frac{2a(5a^2d^2-18abcd+27b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+\cdots}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\cdots\right)}{3c(bc-ad)}\cdots\right)}{a(bc-ad)}\cdots}{6c(bc-ad)}$$

$$\frac{dx}{6c\sqrt[3]{a+bx^3}\,(c+dx^3)^2\,(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + ((b*(6*b*c +
a*d)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) - (d*(-1/3*((18*b^2*
c^2 + 15*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x
^3)) + (2*a*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
 a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))
/(3*c*(b*c - a*d))))/(a*(b*c - a*d)))/(6*c*(b*c - a*d))
```

3.115. $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.115.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{-2\left(5a^2d^2-18abcd+27b^2c^2\right)da\left(dx^3+c\right)^2\left(bx^3+a\right)^{\frac{1}{3}}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-3xc\left(5a^2bd^4x^6-15ab^2cd^3x^6-18b^3c\right.}{}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-2*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*d*a*(d*x^3+c)^2*(b*x^3+a)^(1/3
)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-3*x*c*(5*a^2*b*d^4*x^6-15*
a*b^2*c*d^3*x^6-18*b^3*c^2*d^2*x^6+5*a^3*d^4*x^3-7*a^2*b*c*d^3*x^3-18*a*b^
2*c^2*d^2*x^3-36*b^3*c^3*d*x^3+8*a^3*c*d^3-18*a^2*b*c^2*d^2-18*b^3*c^4)*((
a*d-b*c)/c)^(1/3)+(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*d*a*(-2*arctan(1/3*3^(
1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1
/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x
^3+a)^(2/3))/x^2))*(d*x^3+c)^2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/(b*x^3
+a)^(1/3)/(d*x^3+c)^2/c^3/(a*d-b*c)^3/a
```

3.115.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

3.115.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

3.116 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 463

$$\begin{aligned}\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx &= -\frac{dx}{6c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)^2} \\ &+ \frac{b(3bc+2ad)x}{12ac(bc-ad)^2(a+bx^3)^{4/3}(c+dx^3)} + \frac{b(9b^2c^2-42abcd-2a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\,(c+dx^3)} \\ &+ \frac{d(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3)\,x(a+bx^3)^{2/3}}{36a^2c^2(bc-ad)^4(c+dx^3)} \\ &+ \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{13/3}} \\ &+ \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{13/3}} \\ &- \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}}\end{aligned}$$

output
```
-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)^2+1/12*b*(2*a*d+3*b*c)*x/a
/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)/(d*x^3+c)+1/12*b*(-2*a^2*d^2-42*a*b*c*d+9*
b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)/(d*x^3+c)+1/36*d*(10*a^3*d^3
-42*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x*(b*x^3+a)^(2/3)/a^2/c^2/(-a*
d+b*c)^4/(d*x^3+c)+1/54*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/
c^(8/3)/(-a*d+b*c)^(13/3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(13/3)+1/27*d
^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^
(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(13/3)*3^(1/2)
```

3.116.2 Mathematica [A] (warning: unable to verify)

Time = 15.75 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{1}{36}x(a+bx^3)^{2/3}\left(-\frac{9b^3}{a(-bc+ad)^3(a+bx^3)^2}+\frac{27b^3(bc-5ad)}{a^2(bc-ad)^4(a+bx^3)}-\frac{6d^3}{c(bc-ad)^3(c+dx^3)^2}+\frac{2d^3(-21bc+5ad}{c^2(bc-ad)^4(c+d\ldots}\right. + \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\ldots\right.}{54c^{8/3}(bc-ad)^{13/3}}$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

output
```
(x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^
3*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^
3*(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3
))))/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^
(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a
*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^
3)^(1/3)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

3.116.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {931, 1024, 27, 1024, 25, 27, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-9bdx^3+6bc-5ad}{(bx^3+a)^{7/3}(dx^3+c)^2}dx}{6c(bc-ad)}-\frac{dx}{6c(a+bx^3)^{4/3}(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}-\frac{\int -\frac{2\left(6bd(3bc+2ad)x^3+9b^2c^2+10a^2d^2-24abcd\right)}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)^2}dx}{4a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c(a+bx^3)^{4/3}(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{6bd(3bc+2ad)x^3+9b^2c^2+10a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)^2}dx}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c(a+bx^3)^{4/3}(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}-\frac{\int -\frac{d\left(3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c(a+bx^3)^{4/3}(c+dx^3)^2(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{d\left(3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{d\int \frac{3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\downarrow$ 1024

$$\frac{\frac{\frac{d\left(\frac{\int \frac{4a^2d\left(54b^2c^2-24abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}+\frac{x\left(a+bx^3\right)^{2/3}\left(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3b}{2a(a+bx^3)^{4/3}(c+d}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{d\left(\frac{4a^2d\left(5a^2d^2-24abcd+54b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}+\frac{x\left(a+bx^3\right)^{2/3}\left(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}-}{6c(bc-ad)}$$

$$\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\downarrow$ 901

3.116. $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx$

$$\frac{\frac{\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}+\frac{d\left(\frac{4a^2d\left(5a^2d^2-24abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\cdots}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\cdots}{3c(bc-ad)}\cdots\right)}{a(bc-ad)}\cdots}{2a(bc-ad)}\cdots}{6c(bc-ad)}$$

$$\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)^2) + ((b*(3*b*c +
2*a*d)*x)/(2*a*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((b*(9*b^2*c^2
 - 42*a*b*c*d - 2*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)
) + (d*(((27*b^3*c^3 - 135*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 10*a^3*d^3)*x*(a
 + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + (4*a^2*d*(54*b^2*c^2 - 24
*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3
]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a +
 b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d))))/(a*(b*c
 - a*d)))/(2*a*(b*c - a*d)))/(6*c*(b*c - a*d))
```

3.116. $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx$

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.116.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\frac{5\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(a^{2}d^{2}-\frac{24}{5}abcd+\frac{54}{5}b^{2}c^{2}\right)d^{2}a^{2}\left(d\,x^{3}+c\right)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{27}+\frac{4\left(\frac{5a^{3}x^{3}\left(b\,x^{3}+a\right)^{2}d^{5}}{8}+\left(b\,x^{3}+a\right)^{2}\left(-\frac{21b\,x^{3}}{8}\right.\right.}{}}{}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(4/3)*((b*x^3+a)^(4/3)*(a^2*d^2-24/5*a*
b*c*d+54/5*b^2*c^2)*d^2*a^2*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a
)^(1/3))/x)+12/5*(5/8*a^3*x^3*(b*x^3+a)^2*d^5+(b*x^3+a)^2*(-21/8*b*x^3+a)*
c*a^2*d^4-3*b*(45/16*b^3*x^9+4*a*b^2*x^6+2*a^2*b*x^3+a^3)*c^2*a*d^3-18*x^3
*b^3*(-3/32*b^2*x^6+13/16*a*b*x^3+a^2)*c^3*d^2-9*b^3*(-3/8*b^2*x^6+7/16*a*
b*x^3+a^2)*c^4*d+9/4*b^4*(3/4*b*x^3+a)*c^5)*x*c*((a*d-b*c)/c)^(1/3)+(arcta
n(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3
)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)
^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-24/5*a*b*c*d+54/5*b^2*c^2)*d^2*a^
2*(b*x^3+a)^(4/3)*(d*x^3+c)^2)/(d*x^3+c)^2/c^3/(a*d-b*c)^4/a^2
```

3.116.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.116.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

3.116.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

3.117 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx = \frac{ax\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^
3/a)^(1/3)
```

3.117.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(60) = 120.

Time = 10.47 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx = \frac{x\left(b(2bc+5ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left((a+bx^3)(-bc(c-2dx\cdots\right.}{\cdots}\right.}{72\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

output
```
(x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + (4*c*((a + b*x^3)*(-(b*c*(c - 2*d*x^3)) + a
*d*(8*c + 5*d*x^3)) + (4*a^2*c*(b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
-((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)]))))/(c + d*x^3)^2))/(72*c^3*d*(a + b*x^3)^(2/3))
```

3.117.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{\left(dx^3+c\right)^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

3.117.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.117.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{(d\,x^{3}+c)^{3}}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.117.7 Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

3.117.8 Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^3, x)
```

3.118 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/
a)^(1/3)
```

3.118.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 407 vs. $2(59) = 118$.

Time = 10.62 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \frac{-b(-4bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\mathrm{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16acx\left(-b^2cx^3(7c+4dx^3)+3a^2d(6c+5dx^3)+ab(\right.\right.}{}}{}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

output
```
(-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3))
 + 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(a + b*x^3)*(-(b*
c*(7*c + 4*d*x^3)) + a*d*(8*c + 5*d*x^3))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3
)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)]))))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3))
```

3.118.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{(dx^3+c)^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

3.118.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.118.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{(d\,x^{3}+c)^{3}}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

3.118.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.118.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3)**3, x)
```

3.118.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3} \, dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

3.118.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3} \, dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^3, x)
```

3.119 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx$

3.119.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a
)^(2/3)
```

3.119.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(59) = 118.

Time = 10.68 (sec) , antiderivative size = 418, normalized size of antiderivative = 7.08

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \frac{x\left(5bd(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c(-4c}{\quad}\right.}{\quad}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

output
```
(x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3)
+ b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 +
5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3
*(a + b*x^3)*(a*d*(8*c + 5*d*x^3) - b*c*(13*c + 10*d*x^3))*(3*a*d*AppellF1
[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3,
 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)]))))/(72*c^3*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

3.119.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)^3}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(a + b*x^3)^(2/3))
```

3.119.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.119.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{3}}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

3.119.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

3.119.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)
```

3.120 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3
+a)^(2/3)
```

3.120.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 531 vs. 2(62) = 124.

Time = 10.92 (sec) , antiderivative size = 531, normalized size of antiderivative = 8.56

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx=\frac{\frac{bd(-9b^2c^2-16abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3}-\frac{4c\left(4acx(3a^3d^3(6c+5\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

output
```
((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appe
llF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*
c*(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 1
6*d^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2
- 43*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c
*d^2*x^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(
3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Appel
lF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d
*x^3)^2*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^
3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*
x^3)^(2/3))
```

3.120.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}(dx^3+c)^3}dx}{a(a+bx^3)^{2/3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

3.120.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.120.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{3}}dx$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

3.120.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{5/3}\left(d\,x^3+c\right)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3), x)
```

3.121 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx$

3.121.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x
^3+a)^(2/3)
```

3.121.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(62) = 124$.

Time = 11.48 (sec) , antiderivative size = 515, normalized size of antiderivative = 8.31

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx=\frac{x\left(bd(36b^3c^3-171ab^2c^2d-110a^2bcd^2+25a^3d^3)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

output
```
(x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*
(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c
 + d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*
c*d*x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^
2*b^3*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*
x^3) + (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^
3*b*c*d^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*
x^3)^2))/(360*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

3.121.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)^3}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

3.121.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.121.4 Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{8}{3}}\,(d\,x^3+c)^3}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

3.121.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{8/3}\,\left(d\,x^3+c\right)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3), x)
```

3.122 $\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

output
```
4/25*x*(b*x^3+a)^(7/4)/c/(d*x^3+c)^(25/12)+84/325*a*x*(b*x^3+a)^(3/4)/c^2/
(d*x^3+c)^(13/12)+189/325*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom(
[1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(1/4)/(d*x^3+c
)^(1/12)
```

3.122.2 Mathematica [A] (warning: unable to verify)

Time = 5.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \frac{ax(a+bx^3)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

input
```
Integrate[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

output
```
(a*x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b*c) + a*d)*x
^3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (
d*x^3)/c)^(1/4))
```

3.122.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{7/4}}{\left(c+dx^3\right)^{37/12}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{21a\int \frac{\left(bx^3+a\right)^{3/4}}{\left(dx^3+c\right)^{25/12}}dx}{25c}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

$$\Big\downarrow \text{903}$$

$$\frac{21a\left(\frac{9a\int \frac{1}{\sqrt[4]{bx^3+a}\left(dx^3+c\right)^{13/12}}dx}{13c}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}\right)}{25c}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

$$\Big\downarrow \text{905}$$

$$\frac{21a\left(\frac{9ax\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{13c^2\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}\right)}{25c}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

input
```
Int[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

output
```
(4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (21*a*((4*x*(a + b*x^
3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3))/(a*(c + d*x
^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))))/(25*c)
```

3.122.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.122.4 Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{7}{4}}}{\left(d\,x^{3}+c\right)^{\frac{37}{12}}}d x$$

input
```
int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

output
```
int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

3.122.5 Fricas [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(7/4)*(d*x^3 + c)^(11/12)/(d^4*x^12 + 4*c*d^3*x^9 + 6
*c^2*d^2*x^6 + 4*c^3*d*x^3 + c^4), x)
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)
```

output
```
Timed out
```

3.122.7 Maxima [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

3.122.8 Giac [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(b\,x^3+a)^{7/4}}{(d\,x^3+c)^{37/12}}\,dx$$

input
```
int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x)
```

output
```
int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12), x)
```

3.123 $\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}} + \frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{133c^3\,(a+bx^3)^{3/4}}$$

output
```
4/19*x*(b*x^3+a)^(5/4)/c/(d*x^3+c)^(19/12)+60/133*a*x*(b*x^3+a)^(1/4)/c^2/
(d*x^3+c)^(7/12)+45/133*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5
/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)
^(3/4)
```

3.123.2 Mathematica [A] (warning: unable to verify)

Time = 5.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \frac{ax\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{7/12}}$$

input
```
Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

output
```
(a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c
+ d*x^3)^(7/12))
```

3.123.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{5/4}}{\left(c+dx^3\right)^{31/12}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{15a\int \frac{\sqrt[4]{bx^3+a}}{\left(dx^3+c\right)^{19/12}}dx}{19c}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

$$\Big\downarrow \text{903}$$

$$\frac{15a\left(\frac{3a\int \frac{1}{\left(bx^3+a\right)^{3/4}\left(dx^3+c\right)^{7/12}}dx}{7c}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}\right)}{19c}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

$$\Big\downarrow \text{905}$$

$$\frac{15a\left(\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}\right)}{19c}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

input
```
Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

output
```
(4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (15*a*((4*x*(a + b*x^
3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/(a*(c + d*x^3
)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))))/(19*c)
```

3.123.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.123.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{5}{4}}}{(d\,x^{3}+c)^{\frac{31}{12}}}dx$$

input
```
int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

output
```
int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

3.123.5 Fricas [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

3.123.8 Giac [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx=\int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx=\int \frac{(b\,x^3+a)^{5/4}}{(d\,x^3+c)^{31/12}}\,dx$$

input
```
int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)
```

output
```
int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)
```

3.124 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx=\frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}}+\frac{9ax\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

output
```
4/13*x*(b*x^3+a)^(3/4)/c/(d*x^3+c)^(13/12)+9/13*a*x*(c*(b*x^3+a)/a/(d*x^3+
c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x
^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

3.124.2 Mathematica [A] (warning: unable to verify)

Time = 5.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx=\frac{x(a+bx^3)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

input
```
Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

output
```
(x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3
)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

3.124.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$$

$$\Big\downarrow \text{ 903}$$

$$\frac{9a\int \frac{1}{\sqrt[4]{bx^3+a}(dx^3+c)^{13/12}}dx}{13c}+\frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

$$\Big\downarrow \text{ 905}$$

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}+\frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

input
```
Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

output
```
(4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3
))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*
x^3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

3.124.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.124.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{3}{4}}}{(d\,x^{3}+c)^{\frac{25}{12}}}dx$$

input
```
int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

output
```
int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

3.124.5 Fricas [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*
c^2*d*x^3 + c^3), x)
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
```

output
```
Timed out
```

3.124.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

3.124.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(b\,x^3+a)^{3/4}}{(d\,x^3+c)^{25/12}}\,dx$$

input
```
int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x)
```

output
```
int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12), x)
```

3.125 $\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} \, dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} \, dx = \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}} + \frac{3ax\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{7c^2(a+bx^3)^{3/4}}$$

output
```
4/7*x*(b*x^3+a)^(1/4)/c/(d*x^3+c)^(7/12)+3/7*a*x*(c*(b*x^3+a)/a/(d*x^3+c))
^(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^
3+c))/c^2/(b*x^3+a)^(3/4)
```

3.125.2 Mathematica [A] (warning: unable to verify)

Time = 3.78 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} \, dx = \frac{x\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{7/12}}$$

input
```
Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

output
```
(x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*
x^3)^(7/12))
```

3.125.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3a\int \frac{1}{(bx^3+a)^{3/4}(dx^3+c)^{7/12}}dx}{7c}+\frac{4x\sqrt[4]{a+bx^3}}{7c\,(c+dx^3)^{7/12}}$$

$$\Big\downarrow \text{905}$$

$$\frac{3ax\,(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{7c^2\,(a+bx^3)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\,(c+dx^3)^{7/12}}$$

input
```
Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

output
```
(4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

3.125.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.125.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{4}}}{(d\,x^{3}+c)^{\frac{19}{12}}}dx$$

input
```
int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

output
```
int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

3.125.5 Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
 x)
```

3.125.6 Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
```

output
```
Integral((a + b*x**3)**(1/4)/(c + d*x**3)**(19/12), x)
```

3.125.7 Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

3.125.8 Giac [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/4}}{{\left(d\,x^3+c\right)}^{19/12}}\,dx$$

input
```
int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x)
```

output
```
int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12), x)
```

3.126 $\int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}}\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

output
```
x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x
^3/a/(d*x^3+c))/c/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

3.126.2 Mathematica [A] (warning: unable to verify)

Time = 3.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\left(1+\frac{dx^3}{c}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}$$

input
```
Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3
)^(13/12))
```

3.126.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx$$

$$\Big\downarrow \text{905}$$

$$\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^
(1/12))
```

3.126.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.126.4 Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{1}{4}}\,(d\,x^3+c)^{\frac{13}{12}}}dx$$

input
```
int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

output
```
int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

3.126.5 Fricas [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d
^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

3.126.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
```

output
```
Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

3.126.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

3.126.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\int \frac{1}{(b\,x^3+a)^{1/4}\,(d\,x^3+c)^{13/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x)
```

output
```
int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)), x)
```

3.127 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx=\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{3/4}}$$

output
```
x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4
/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(3/4)
```

3.127.2 Mathematica [A] (warning: unable to verify)

Time = 5.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}$$

input
```
Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3
)^(7/12))
```

3.127.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$$

$$\Big\downarrow \text{ 905}$$

$$\frac{x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{3/4}}$$

input
```
Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometr
ic2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)
^(3/4))
```

3.127.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.127.4 Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{3}{4}}\,(d\,x^3+c)^{\frac{7}{12}}}dx$$

input
```
int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

output
```
int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

3.127.5 Fricas [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
 a*c), x)
```

3.127.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{4}}\,(c+dx^3)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
```

output
```
Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

3.127.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

3.127.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(b\,x^3+a)^{3/4}\,(d\,x^3+c)^{7/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x)
```

output
```
int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)), x)
```

3.128 $\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{5/4}}$$

output
```
x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[
4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(5/4)
```

3.128.2 Mathematica [A] (warning: unable to verify)

Time = 3.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}\,\sqrt[4]{1+\frac{dx^3}{c}}}$$

input
```
Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*
x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3
)/c)^(1/4))
```

3.128.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$$

$$\Big\downarrow \ 905$$

$$\frac{x\left(c+dx^3\right)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\left(a+bx^3\right)^{5/4}}$$

input
```
Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeomet
ric2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3
)^(5/4))
```

3.128.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.128.4 Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{5}{4}}\,(d\,x^3+c)^{\frac{1}{12}}}\,dx$$

input
```
int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

output
```
int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

3.128.5 Fricas [F]

$$\int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b
*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

3.128.6 Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{4}} \sqrt[12]{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

output
```
Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

3.128.7 Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

3.128.8 Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/4}\,(d\,x^3+c)^{1/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x)
```

output
```
int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)), x)
```

3.129 $\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \frac{4x(c+dx^3)^{5/12}}{9a\,(a+bx^3)^{3/4}} + \frac{5x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a\,(a+bx^3)^{3/4}}$$

output
```
4/9*x*(d*x^3+c)^(5/12)/a/(b*x^3+a)^(3/4)+5/9*x*(c*(b*x^3+a)/a/(d*x^3+c))^(
3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+
c))/a/(b*x^3+a)^(3/4)
```

3.129.2 Mathematica [A] (warning: unable to verify)

Time = 5.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\,(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

input
```
Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

output
```
(x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3
)/c)^(3/4))
```

3.129.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{5c\int \frac{1}{(bx^3+a)^{3/4}(dx^3+c)^{7/12}}dx}{9a}+\frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{5x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a(a+bx^3)^{3/4}}+\frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

input
```
Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

output
```
(4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(
a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

3.129.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.129.4 Maple [F]

$$\int \frac{\left(d\,x^{3}+c\right)^{\frac{5}{12}}}{\left(b\,x^{3}+a\right)^{\frac{7}{4}}}d x$$

input
```
int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

output
```
int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

3.129.5 Fricas [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx=\int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
 x)
```

3.129.6 Sympy [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(c+dx^3)^{\frac{5}{12}}}{(a+bx^3)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
```

output
```
Integral((c + d*x**3)**(5/12)/(a + b*x**3)**(7/4), x)
```

3.129.7 Maxima [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

3.129.8 Giac [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(d\,x^3+c)^{5/12}}{(b\,x^3+a)^{7/4}}\,dx$$

input
```
int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x)
```

output
```
int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4), x)
```

3.130 $\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}} + \frac{11x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a(a+bx^3)^{5/4}}$$

output
```
4/15*x*(d*x^3+c)^(11/12)/a/(b*x^3+a)^(5/4)+11/15*x*(c*(b*x^3+a)/a/(d*x^3+c
))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d
*x^3+c))/a/(b*x^3+a)^(5/4)
```

3.130.2 Mathematica [A] (warning: unable to verify)

Time = 5.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{x\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{9}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\sqrt[4]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

input
```
Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*
x^3)/c)^(5/4))
```

3.130.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{11c\int \frac{1}{\left(bx^3+a\right)^{5/4}\sqrt[12]{dx^3+c}}dx}{15a}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

input
```
Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

output
```
(4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4
/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

3.130.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.130.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{11}{12}}}{(b\,x^{3}+a)^{\frac{9}{4}}}dx$$

input
```
int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

output
```
int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

3.130.5 Fricas [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*
a^2*b*x^3 + a^3), x)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
```

output
```
Timed out
```

3.130.7 Maxima [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

3.130.8 Giac [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(d\,x^3+c)^{11/12}}{(b\,x^3+a)^{9/4}}\,dx$$

input
```
int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x)
```

output
```
int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4), x)
```

3.131 $\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}} + \frac{85cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{189a^2(a+bx^3)^{3/4}}$$

output
```
68/189*c*x*(d*x^3+c)^(5/12)/a^2/(b*x^3+a)^(3/4)+4/21*x*(d*x^3+c)^(17/12)/a
/(b*x^3+a)^(7/4)+85/189*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/1
2)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(
3/4)
```

3.131.2 Mathematica [A] (warning: unable to verify)

Time = 5.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{cx\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

input
```
Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

output
```
(c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (
d*x^3)/c)^(3/4))
```

3.131.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{17c\int \frac{\left(dx^3+c\right)^{5/12}}{\left(bx^3+a\right)^{7/4}}dx}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{903}$$

$$\frac{17c\left(\frac{5c\int \frac{1}{\left(bx^3+a\right)^{3/4}\left(dx^3+c\right)^{7/12}}dx}{9a}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}\right)}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{17c\left(\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}\right)}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

input
```
Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

output
```
(4*x*(c + d*x^3)^(17/12))/(21*a*(a + b*x^3)^(7/4)) + (17*c*((4*x*(c + d*x^
3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a*(c + d*x^3))
)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))))/(21*a)
```

3.131.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.131.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{17}{12}}}{(b\,x^{3}+a)^{\frac{11}{4}}}dx$$

input
```
int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

output
```
int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

3.131.5 Fricas [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*
a^2*b*x^3 + a^3), x)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)
```

output
```
Timed out
```

3.131.7 Maxima [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

3.131.8 Giac [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(d\,x^3+c)^{17/12}}{(b\,x^3+a)^{11/4}}\,dx$$

input
```
int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)
```

output
```
int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)
```

3.132 $\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}} + \frac{253cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{405a^2(a+bx^3)^{5/4}}$$

output
```
92/405*c*x*(d*x^3+c)^(11/12)/a^2/(b*x^3+a)^(5/4)+4/27*x*(d*x^3+c)^(23/12)/
a/(b*x^3+a)^(9/4)+253/405*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(1
1/12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a
)^(5/4)
```

3.132.2 Mathematica [A] (warning: unable to verify)

Time = 5.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \frac{cx\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{11/12}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^3\sqrt[4]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

input
```
Integrate[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

output
```
(c*x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 13/4
, 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^3*(a + b*x^3)^(1/4)*(1 +
(d*x^3)/c)^(5/4))
```

3.132.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{23/12}}{\left(a+bx^3\right)^{13/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{23c\int \frac{\left(dx^3+c\right)^{11/12}}{\left(bx^3+a\right)^{9/4}}dx}{27a}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

$$\Big\downarrow \text{903}$$

$$\frac{23c\left(\frac{11c\int \frac{1}{\left(bx^3+a\right)^{5/4}\sqrt[12]{dx^3+c}}dx}{15a}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}\right)}{27a}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{23c\left(\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}\right)}{27a}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

input
```
Int[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

output
```
(4*x*(c + d*x^3)^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (23*c*((4*x*(c + d*x^
3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c -
a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))))/(27*a)
```

3.132.3.1 Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.132.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{23}{12}}}{(b\,x^{3}+a)^{\frac{13}{4}}}dx$$

input
```
int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

output
```
int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

3.132.5 Fricas [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(23/12)/(b^4*x^12 + 4*a*b^3*x^9 + 6
*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(23/12)/(b*x**3+a)**(13/4),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

3.132.8 Giac [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(d\,x^3+c)^{23/12}}{(b\,x^3+a)^{13/4}}\,dx$$

input
```
int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x)
```

output
```
int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4), x)
```

3.133 $\int (a+bx^3)^m (c+dx^3)^p \, dx$

3.133.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

output
```
x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3,-m,-p,4/3,-b*x^3/a,-d*x^3/c)/((1+b*
x^3/a)^m)/((1+d*x^3/c)^p)
```

3.133.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = \frac{4acx(a+bx^3)^m (c+dx^3)^p \operatorname{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac \operatorname{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(bcm \operatorname{AppellF1}\left(\frac{4}{3},1-m,-p,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adp \operatorname{Appe}\right.}$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

output
```
(4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/
a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^
3)/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

3.133.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \left(\frac{bx^3}{a}+1\right)^m \left(dx^3+c\right)^p dx$$

$$\Big\downarrow \text{ 937}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \int \left(\frac{bx^3}{a}+1\right)^m \left(\frac{dx^3}{c}+1\right)^p dx$$

$$\Big\downarrow \text{ 936}$$

$$x\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

output
```
(x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

3.133.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.133.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{p}dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

3.133.5 Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

3.133.8 Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^p \, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^p,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^p, x)
```

3.134 $\int (a+bx^3)^2 (c+dx^3)^q \, dx$

3.134.1 Optimal result

Integrand size = 19, antiderivative size = 167

$$\int (a+bx^3)^2 (c+dx^3)^q \, dx = -\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2-2abcd(7+3q)+a^2d^2(28+33q+9q^2))\,x(c+dx^3)^{1+q}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+q,\frac{4}{3},-\frac{dx^3}{c}\right)}{cd^2(4+3q)(7+3q)}$$

output
```
-b*(4*b*c-a*d*(10+3*q))*x*(d*x^3+c)^(1+q)/d^2/(9*q^2+33*q+28)+b*x*(b*x^3+a
)*(d*x^3+c)^(1+q)/d/(7+3*q)+(4*b^2*c^2-2*a*b*c*d*(7+3*q)+a^2*d^2*(9*q^2+33
*q+28))*x*(d*x^3+c)^(1+q)*hypergeom([1, 4/3+q],[4/3],-d*x^3/c)/c/d^2/(9*q^
2+33*q+28)
```

3.134.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int (a+bx^3)^2 (c+dx^3)^q \, dx = \frac{1}{14}x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} \left(14a^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right) + bx^3\left(7a\,\text{Hypergeometric2F1}\left(\frac{4}{3},-q,\frac{7}{3},-\frac{dx^3}{c}\right) + 2bx^3\,\text{Hypergeometric2F1}\left(\frac{7}{3},-q,\frac{10}{3},-\frac{dx^3}{c}\right)\right)\right)$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

output
```
(x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)] + b
*x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)] + 2*b*x^3*Hypergeo
metric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(1 + (d*x^3)/c)^q)
```

3.134.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\left(dx^3+c\right)^q \left(b(4bc-ad(3q+10))x^3+a(bc-ad(3q+7))\right)dx}{d(3q+7)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}-\frac{\int \left(dx^3+c\right)^q \left(b(4bc-ad(3q+10))x^3+a(bc-ad(3q+7))\right)dx}{d(3q+7)}$$

$$\Big\downarrow \text{913}$$

$$\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}-\frac{\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad(3q+10))}{d(3q+4)}-\frac{\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right)\int \left(dx^3+c\right)^q dx}{d(3q+4)}}{d(3q+7)}$$

$$\Big\downarrow \text{779}$$

$$\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}-\frac{\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad(3q+10))}{d(3q+4)}-\frac{\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right)\int \left(\frac{dx^3}{c}+1\right)^q dx}{d(3q+4)}}{d(3q+7)}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{bx(a+bx^3)(c+dx^3)^{q+1}}{d(3q+7)}-\frac{\frac{bx(c+dx^3)^{q+1}(4bc-ad(3q+10))}{d(3q+4)}-\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}(a^2d^2(9q^2+33q+28)-2abcd(3q+7)+4b^2c^2)\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right)}{d(3q+4)}}{d(3q+7)}}{}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

output
```
(b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) - ((b*(4*b*c - a*d*(10
 + 3*q))*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) - ((4*b^2*c^2 - 2*a*b*c*d*(7
 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*Hypergeometric2F1[1
/3, -q, 4/3, -((d*x^3)/c)])/(d*(4 + 3*q)*(1 + (d*x^3)/c)^q))/(d*(7 + 3*q))
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.134.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{2} \left(d\,x^{3}+c\right)^{q} dx$$

input
```
int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

output
```
int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

3.134.5 Fricas [F]

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^2 \left(dx^3+c\right)^q \, dx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)
```

3.134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 90.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \frac{a^2c^qx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -q \\ \frac{4}{3} \end{matrix} \middle| \frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2abc^qx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -q \\ \frac{7}{3} \end{matrix} \middle| \frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2c^qx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{7}{3}, -q \\ \frac{10}{3} \end{matrix} \middle| \frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c)**q,x)
```

output
```
a**2*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/
(3*gamma(4/3)) + 2*a*b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**
3*exp_polar(I*pi)/c)/(3*gamma(7/3)) + b**2*c**q*x**7*gamma(7/3)*hyper((7/3
, -q), (10/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(10/3))
```

3.134.7 Maxima [F]

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^2 \left(dx^3+c\right)^q \, dx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

3.134.8 Giac [F]

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^2 \left(dx^3+c\right)^q \, dx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(b\,x^3+a\right)^2 \left(d\,x^3+c\right)^q \, dx$$

input
```
int((a + b*x^3)^2*(c + d*x^3)^q,x)
```

output
```
int((a + b*x^3)^2*(c + d*x^3)^q, x)
```

3.135 $\int (a+bx^3)(c+dx^3)^q \, dx$

3.135.1 Optimal result

Integrand size = 17, antiderivative size = 84

$$\int (a+bx^3)(c+dx^3)^q \, dx = \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \frac{(bc-ad(4+3q))x(c+dx^3)^{1+q}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+q,\frac{4}{3},-\frac{dx^3}{c}\right)}{cd(4+3q)}$$

output
```
b*x*(d*x^3+c)^(1+q)/d/(4+3*q)-(b*c-a*d*(4+3*q))*x*(d*x^3+c)^(1+q)*hypergeo
m([1, 4/3+q],[4/3],-d*x^3/c)/c/d/(4+3*q)
```

3.135.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int (a+bx^3)(c+dx^3)^q \, dx = \frac{x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\left(b(c+dx^3)\left(1+\frac{dx^3}{c}\right)^q + (-bc+ad(4+3q))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right)\right)}{d(4+3q)}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
```

output
```
(x*(c + d*x^3)^q*(b*(c + d*x^3)*(1 + (d*x^3)/c)^q + (-(b*c) + a*d*(4 + 3*q
))*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]))/(d*(4 + 3*q)*(1 + (d*x^
3)/c)^q)
```

3.135.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(a-\frac{bc}{3dq+4d}\right)\int \left(dx^3+c\right)^q dx+\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}$$

$$\Big\downarrow \text{779}$$

$$\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right)\int \left(\frac{dx^3}{c}+1\right)^q dx+\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}$$

$$\Big\downarrow \text{778}$$

$$x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right)+\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

output
```
(b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*q))*x*(c
+ d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(1 + (d*x^3)/c)^
q
```

3.135.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.135.4 Maple [F]

$$\int (b\,x^{3}+a)\,(d\,x^{3}+c)^{q}\,dx$$

input
```
int((b*x^3+a)*(d*x^3+c)^q,x)
```

output
```
int((b*x^3+a)*(d*x^3+c)^q,x)
```

3.135.5 Fricas [F]

$$\int (a+bx^3)\,(c+dx^3)^q\,dx=\int (bx^3+a)\,(dx^3+c)^q\,dx$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

3.135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx=\frac{ac^qx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-q\\\frac{4}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{bc^qx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-q\\\frac{7}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

output
```
a*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*
gamma(4/3)) + b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_p
olar(I*pi)/c)/(3*gamma(7/3))
```

3.135.7 Maxima [F]

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx=\int \left(bx^3+a\right)\left(dx^3+c\right)^q\,dx$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

3.135.8 Giac [F]

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx=\int \left(bx^3+a\right)\left(dx^3+c\right)^q\,dx$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)(c+dx^3)^q\,dx = \int (b\,x^3+a)\,(d\,x^3+c)^q\,dx$$

input
```
int((a + b*x^3)*(c + d*x^3)^q,x)
```

output
```
int((a + b*x^3)*(c + d*x^3)^q, x)
```

3.136 $\int \frac{(c+dx^3)^q}{a+bx^3}\,dx$

3.136.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \frac{x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{3},1,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

output

```
x*(d*x^3+c)^q*AppellF1(1/3,1,-q,4/3,-b*x^3/a,-d*x^3/c)/a/((1+d*x^3/c)^q)
```

3.136.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx$$
$$= \frac{4acx(c+dx^3)^q\operatorname{AppellF1}\left(\frac{1}{3},-q,1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(4ac\operatorname{AppellF1}\left(\frac{1}{3},-q,1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(adq\operatorname{AppellF1}\left(\frac{4}{3},1-q,1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bc\,\mathrm{Ap}\right.\right.}$$

input

```
Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

output

```
(4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/
a)] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)
] - b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

3.136.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^q}{a+bx^3}dx$$

$$\Big\downarrow \text{ 937}$$

$$(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\int \frac{\left(\frac{dx^3}{c}+1\right)^q}{bx^3+a}dx$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{3},1,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

input
```
Int[(c + d*x^3)^q/(a + b*x^3),x]
```

output
```
(x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a
*(1 + (d*x^3)/c)^q)
```

3.136.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.136.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{q}}{b\,x^{3}+a}dx$$

input
```
int((d*x^3+c)^q/(b*x^3+a),x)
```

output
```
int((d*x^3+c)^q/(b*x^3+a),x)
```

3.136.5 Fricas [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**q/(b*x**3+a),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx=\int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

3.136.8 Giac [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx=\int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx=\int \frac{(d\,x^3+c)^q}{b\,x^3+a}\,dx$$

input
```
int((c + d*x^3)^q/(a + b*x^3),x)
```

output
```
int((c + d*x^3)^q/(a + b*x^3), x)
```

3.137 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx$

3.137.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx = \frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

output
```
x*(d*x^3+c)^q*AppellF1(1/3,2,-q,4/3,-b*x^3/a,-d*x^3/c)/a^2/((1+d*x^3/c)^q)
```

3.137.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx$$
$$= \frac{4acx(c+dx^3)^q \text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^2 \left(4ac \text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(adq \text{AppellF1}\left(\frac{4}{3},2,1-q,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bc\right.\right.}$$

input
```
Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

output
```
(4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/
c)] - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.137.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}dx$$

$$\Big\downarrow \ 937$$

$$(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\int \frac{\left(\frac{dx^3}{c}+1\right)^q}{(bx^3+a)^2}dx$$

$$\Big\downarrow \ 936$$

$$\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

input
```
Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

output
```
(x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a
^2*(1 + (d*x^3)/c)^q)
```

3.137.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.137.4 Maple [F]

$$\int \frac{(d\,x^{3}+c)^{q}}{(b\,x^{3}+a)^{2}}dx$$

input
```
int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

output
```
int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

3.137.5 Fricas [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.137.7 Maxima [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

3.137.8 Giac [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

input
```
integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^q}{(b\,x^3+a)^2}\,dx$$

input
```
int((c + d*x^3)^q/(a + b*x^3)^2,x)
```

output
```
int((c + d*x^3)^q/(a + b*x^3)^2, x)
```

3.138 $\int (a+bx^3)^m (c+dx^3)^3 \, dx$

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 298

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx$$
$$= \frac{d(28a^2d^2 - abcd(92+15m) + b^2c^2(118+60m+9m^2))\, x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)}$$
$$- \frac{d(7ad - bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)}$$
$$- \frac{(28a^3d^3 - 12a^2bcd^2(10+3m) + 3ab^2c^2d(70+51m+9m^2) - b^3c^3(280+414m+189m^2+27m^3))\, x(a}{b^3(4+3m)(7+3m)(10+3m)}$$

output
```
d*(28*a^2*d^2-a*b*c*d*(92+15*m)+b^2*c^2*(9*m^2+60*m+118))*x*(b*x^3+a)^(1+m
)/b^3/(10+3*m)/(9*m^2+33*m+28)-d*(7*a*d-b*c*(16+3*m))*x*(b*x^3+a)^(1+m)*(d
*x^3+c)/b^2/(9*m^2+51*m+70)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)^2/b/(10+3*m)-(28
*a^3*d^3-12*a^2*b*c*d^2*(10+3*m)+3*a*b^2*c^2*d*(9*m^2+51*m+70)-b^3*c^3*(27
*m^3+189*m^2+414*m+280))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)
/b^3/(10+3*m)/(9*m^2+33*m+28)/((1+b*x^3/a)^m)
```

3.138.2 Mathematica [A] (verified)

Time = 8.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.46

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx$$
$$= \frac{1}{140}x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(140c^3 \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right) + dx^3\left(105c^2 \text{Hypergeometric2F1}\left(\frac{4}{3},-m,\frac{7}{3},-\frac{bx^3}{a}\right) +2dx^3\left(30c\,\text{Hypergeometric2F1}\left(\frac{7}{3},-m,\frac{10}{3},-\frac{bx^3}{a}\right)+7dx^3 \text{Hypergeometric2F1}\left(\frac{10}{3},-m,\frac{13}{3},-\frac{bx^3}{a}\right)\right.\right.\right.$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] +
d*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30
*c*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric
2F1[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

3.138.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^3 \left(a+bx^3\right)^m \, dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\left(bx^3+a\right)^m \left(dx^3+c\right) \left(d(7ad-bc(3m+16))x^3+c(ad-bc(3m+10))\right)dx}{b(3m+10)}+\frac{dx\left(c+dx^3\right)^2 \left(a+bx^3\right)^{m+1}}{b(3m+10)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\int\left(bx^3+a\right)^m\left(dx^3+c\right)\left(d(7ad-bc(3m+16))x^3+c(ad-bc(3m+10))\right)dx}{b(3m+10)}$$

$\downarrow$ 1025

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{\int-\left(bx^3+a\right)^m\left(d\left(b^2\left(9m^2+60m+118\right)c^2-abd(15m+92)c+28a^2d^2\right)x^3+c\left(b^2\left(9m^2+51m+70\right)c^2-abd(6m+23)c+7a^2d^2\right)\right)dx}{b(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+bx^3\right.}{b(}}{b(3m+10)}$$

$\downarrow$ 25

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\int\left(bx^3+a\right)^m\left(d\left(b^2\left(9m^2+60m+118\right)c^2-abd(15m+92)c+28a^2d^2\right)x^3+c\left(b^2\left(9m^2+51m+70\right)c^2-abd\right.}{b(3m+7)}}{b(3m+10)}$$

$\downarrow$ 913

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{\left(28a^3d^3-12a^2bcd^2(3m+10)+3ab^2c^2d\right.}{}}{b(3m+7)}}{b(3m+10)}$$

$\downarrow$ 779

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-12a\right.}{}}{b(3m+7)}}{b(3m+10)}$$

$\downarrow$ 778

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-12\right.}{}}{b}}{b(3m+10)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

3.138. $\int\left(a+bx^3\right)^m\left(c+dx^3\right)^3\,dx$

output
```
(d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((d*(7*a*d - b*c*
(16 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) - ((d*(28*a^2
*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a + b*x^3)^(
1 + m))/(b*(4 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*a*b^2
*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2 + 27*m^3))*x*(
a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b*(4 + 3*m)*(
1 + (b*x^3)/a)^m))/(b*(7 + 3*m)))/(b*(10 + 3*m))
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.138.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{3} dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

3.138.5 Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \int \left(dx^{3}+c\right)^{3}\left(bx^{3}+a\right)^{m} \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \,dx = \int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m \,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

3.138.8 Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \,dx = \int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m \,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \,dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^3 \,dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^3, x)
```

3.139 $\int (a+bx^3)^m (c+dx^3)^2 \, dx$

3.139.1 Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = -\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)} + \frac{(4a^2d^2-2abcd(7+3m)+b^2c^2(28+33m+9m^2))\,x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{3},-\right.}{b^2(4+3m)(7+3m)}$$

output
```
-d*(4*a*d-b*c*(10+3*m))*x*(b*x^3+a)^(1+m)/b^2/(9*m^2+33*m+28)+d*x*(b*x^3+a
)^(1+m)*(d*x^3+c)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33
*m+28))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^2/(9*m^2+33*m+
28)/((1+b*x^3/a)^m)
```

3.139.2 Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 dx = \frac{1}{14}x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(14c^2 \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)+dx^3\left(7c \text{Hypergeometric2F1}\left(\frac{4}{3},-m,\frac{7}{3},-\frac{bx^3}{a}\right)+2dx^3 \text{Hypergeometric2F1}\left(\frac{7}{3},-m,\frac{10}{3},-\frac{bx^3}{a}\right)\right)\right)$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeo
metric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

3.139.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^2 \left(a+bx^3\right)^m dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\left(bx^3+a\right)^m \left(d(4ad-bc(3m+10))x^3+c(ad-bc(3m+7))\right)dx}{b(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\int \left(bx^3+a\right)^m \left(d(4ad-bc(3m+10))x^3+c(ad-bc(3m+7))\right)dx}{b(3m+7)}$$

$$\Big\downarrow \text{913}$$

$$\frac{dx(c+dx^3)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\int\left(bx^3+a\right)^mdx}{b(3m+4)}}{b(3m+7)}$$

$$\Big\downarrow \textbf{779}$$

$$\frac{dx(c+dx^3)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\int\left(\frac{bx^3}{a}+1\right)^mdx}{b(3m+4)}}{b(3m+7)}$$

$$\Big\downarrow \textbf{778}$$

$$\frac{dx(c+dx^3)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)}{b(3m+4)}}{b(3m+7)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) - ((d*(4*a*d - b*c*(10
 + 3*m))*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) - ((4*a^2*d^2 - 2*a*b*c*d*(7
 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeometric2F1[1
/3, -m, 4/3, -((b*x^3)/a)])/(b*(4 + 3*m)*(1 + (b*x^3)/a)^m))/(b*(7 + 3*m))
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.139.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{2}dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

3.139.5 Fricas [F]

$$\int (a+bx^3)^m\,(c+dx^3)^2\,dx=\int (dx^3+c)^2(bx^3+a)^m\,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

3.139.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 89.96 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \frac{a^m c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^m c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^m d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{7}{3}, -m \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**2,x)
```

output
```
a**m*c**2*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + 2*a**m*c*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**m*d**2*x**7*gamma(7/3)*hyper((7/3
, -m), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.139.7 Maxima [F]

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \int (dx^3+c)^2 (bx^3+a)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

3.139.8 Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(dx^3+c\right)^2 \left(bx^3+a\right)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^2 \, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^2, x)
```

3.140 $\int (a+bx^3)^m (c+dx^3) \, dx$

3.140.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^3)^m (c+dx^3) \, dx = \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \frac{(ad-bc(4+3m))x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)}{b(4+3m)}$$

output
```
d*x*(b*x^3+a)^(1+m)/b/(4+3*m)-(a*d-b*c*(4+3*m))*x*(b*x^3+a)^m*hypergeom([1
/3, -m],[4/3],-b*x^3/a)/b/(4+3*m)/((1+b*x^3/a)^m)
```

3.140.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^3)^m (c+dx^3) \, dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^m + (-ad+bc(4+3m)) \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},\right.\right.}{b(4+3m)}$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m
))*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^
3)/a)^m)
```

3.140.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^3)(a+bx^3)^m\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{3bm+4b}\right)\int (bx^3+a)^m\,dx+\frac{dx(a+bx^3)^{m+1}}{b(3m+4)}$$

$$\Big\downarrow \text{779}$$

$$(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right)\int\left(\frac{bx^3}{a}+1\right)^m dx+\frac{dx(a+bx^3)^{m+1}}{b(3m+4)}$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{dx(a+bx^3)^{m+1}}{b(3m+4)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a
+ b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^
m
```

3.140.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.140.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c),x)
```

3.140.5 Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^m\,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

3.140.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int (a+bx^3)^m (c+dx^3)\, dx = \frac{a^m cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^m dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c),x)
```

output
```
a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(7/3))
```

3.140.7 Maxima [F]

$$\int (a+bx^3)^m (c+dx^3)\, dx = \int (dx^3+c)(bx^3+a)^m\, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

3.140.8 Giac [F]

$$\int (a+bx^3)^m (c+dx^3)\, dx = \int (dx^3+c)(bx^3+a)^m\, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3), x)
```

3.141 $\int (a+bx^3)^m \, dx$

3.141.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^3)^m \, dx = x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)$$

output
```
x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/((1+b*x^3/a)^m)
```

3.141.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.61

$$\int (a+bx^3)^m \, dx = \frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-m}(a+bx^3)^m \text{AppellF1}\left(1+m,-m,-m\right.}{\sqrt[3]{b}(1+m)}$$

input
```
Integrate[(a + b*x^3)^m,x]
```

output
```
(((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2
+ m, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (-1)^(1/3))*a^(
1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^m*b
^(1/3)*(1 + m)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
))^m*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^m)
```

3.141.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^m dx$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \left(\frac{bx^3}{a}+1\right)^m dx$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[(a + b*x^3)^m,x]
```

output
```
(x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^
3)/a)^m
```

3.141.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

3.141.4 Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m}dx$$

input
```
int((b*x^3+a)^m,x)
```

output
```
int((b*x^3+a)^m,x)
```

3.141.5 Fricas [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^{3}+a\right)^{m}\,dx$$

input
```
integrate((b*x^3+a)^m,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m, x)
```

3.141.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^3\right)^m\,dx=\frac{a^mx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-m\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**m,x)
```

output
```
a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(4/3))
```

3.141.7 Maxima [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^3+a\right)^m\,dx$$

input
```
integrate((b*x^3+a)^m,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m, x)
```

3.141.8 Giac [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^3+a\right)^m\,dx$$

input
```
integrate((b*x^3+a)^m,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m, x)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^3)^m \, dx = \frac{x\,(b\,x^3+a)^m\,{}_2\mathrm{F}_1\left(\frac{1}{3},-m;\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^m}$$

input
```
int((a + b*x^3)^m,x)
```

output
```
(x*(a + b*x^3)^m*hypergeom([1/3, -m], 4/3, -(b*x^3)/a))/((b*x^3)/a + 1)^m
```

3.142 $\int \frac{(a+bx^3)^m}{c+dx^3} \, dx$

3.142.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{c+dx^3} \, dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)
```

3.142.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{c+dx^3} \, dx =$$
$$-\frac{4acx(a+bx^3)^m \text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(-bcm\,\text{AppellF1}\left(\frac{4}{3},1-m,1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.142.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^m}{c+dx^3} \, dx$$

$$\Big\downarrow \, 937$$

$$(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \frac{\left(\frac{bx^3}{a}+1\right)^m}{dx^3+c}dx$$

$$\Big\downarrow \, 936$$

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
*(1 + (b*x^3)/a)^m)
```

3.142.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.142.4 Maple [F]

$$\int \frac{(b\,x^3+a)^m}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c),x)
```

3.142.5 Fricas [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c),x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

3.142.8 Giac [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^m}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3), x)
```

3.143 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)
```

3.143.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = -\frac{4acx(a+bx^3)^m\operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^2\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcm\operatorname{AppellF1}\left(\frac{4}{3},1-m,2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.143.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx$$

$$\Big\downarrow \text{ 937}$$

$$(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \frac{\left(\frac{bx^3}{a}+1\right)^m}{(dx^3+c)^2}dx$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
^2*(1 + (b*x^3)/a)^m)
```

3.143.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.143.4 Maple [F]

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

3.143.5 Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

3.143.8 Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

3.144 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx$

3.144.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/((1+b*x^3/a)^m)
```

3.144.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.55 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx =$$

$$-\frac{4acx(a+bx^3)^m\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^3\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcm\operatorname{AppellF1}\left(\frac{4}{3},1-m,3,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

3.144.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^m}{\left(c+dx^3\right)^3} \, dx$$

$$\Big\downarrow \text{937}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \frac{\left(\frac{bx^3}{a}+1\right)^m}{\left(dx^3+c\right)^3}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
^3*(1 + (b*x^3)/a)^m)
```

3.144.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.144.4 Maple [F]

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

3.144.5 Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

output
```
Timed out
```

3.144.7 Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

3.144.8 Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

3.145 $\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx$

3.145.1 Optimal result

Integrand size = 50, antiderivative size = 53

$$\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x(a+bx^3)^{-\frac{bc}{3bc-3ad}} (c+dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

output
```
x*(d*x^3+c)^(a*d/(-3*a*d+3*b*c))/a/c/((b*x^3+a)^(b*c/(-3*a*d+3*b*c)))
```

3.145.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x(a+bx^3)^{\frac{bc}{-3bc+3ad}} (c+dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

input
```
Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)
/(3*b*c - 3*a*d)),x]
```

output
```
(x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)
))/(a*c)
```

3.145.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}-1} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}-1} \, dx$$

$$\Big\downarrow \text{906}$$

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

input
```
Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*
c - 3*a*d)),x]
```

output
```
(x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3
*a*d)))
```

3.145.3.1 Defintions of rubi rules used

rule 906
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

3.145.4 Maple [A] (verified)

Time = 7.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
gosper	$\frac{x\left(b\,x^{3}+a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}} \left(d\,x^{3}+c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$	71

input
```
int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x,
method=_RETURNVERBOSE)
```

output
```
x/a/c*(b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*
b*c)/(a*d-b*c))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{bdx^7+(bc+ad)x^4+acx}{\left(bx^3+a\right)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}\left(dx^3+c\right)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="fricas")
```

output
```
(b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
 - a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+
3*b*c)),x)
```

output
```
Timed out
```

3.145.7 Maxima [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

3.145.8 Giac [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \,dx = \int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1} \left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1} \,dx$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \,dx = \frac{x\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}+\frac{x^4\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\,\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^7\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{{\left(d\,x^3+c\right)}^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

input
```
int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*
b*c) + 1),x)
```

output
```
(x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*
d - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d -
3*b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

3.146 $\int (a+bx^4)(c+dx^4)^4\,dx$

3.146.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^4)(c+dx^4)^4\,dx = ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9 +\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}+\frac{1}{21}bd^4x^{21}$$

output
```
a*c^4*x+1/5*c^3*(4*a*d+b*c)*x^5+2/9*c^2*d*(3*a*d+2*b*c)*x^9+2/13*c*d^2*(2*
a*d+3*b*c)*x^13+1/17*d^3*(a*d+4*b*c)*x^17+1/21*b*d^4*x^21
```

3.146.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^4\,dx = ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9 +\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}+\frac{1}{21}bd^4x^{21}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2
*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^2
1)/21
```

3.146.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)(c+dx^4)^4\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^3x^4(4ad+bc)+2c^2dx^8(3ad+2bc)+d^3x^{16}(ad+4bc)+2cd^2x^{12}(2ad+3bc)+ac^4+bd^4x^{20}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2
*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^2
1)/21
```

3.146.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.146.4 Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$a\,c^{4}x+\left(\frac{4}{5}a\,c^{3}d+\frac{1}{5}b\,c^{4}\right)x^{5}+\left(\frac{2}{3}a\,c^{2}d^{2}+\frac{4}{9}b\,c^{3}d\right)x^{9}+\left(\frac{4}{13}ac\,d^{3}+\frac{6}{13}b\,c^{2}d^{2}\right)x^{13}+\left(\frac{1}{17}a\,d^{4}+\frac{4}{17}\right.$
default	$\frac{b\,d^{4}x^{21}}{21}+\frac{\left(a\,d^{4}+4bc\,d^{3}\right)x^{17}}{17}+\frac{\left(4ac\,d^{3}+6b\,c^{2}d^{2}\right)x^{13}}{13}+\frac{\left(6a\,c^{2}d^{2}+4b\,c^{3}d\right)x^{9}}{9}+\frac{\left(4a\,c^{3}d+b\,c^{4}\right)x^{5}}{5}+a\,c^{4}x$
gosper	$a\,c^{4}x+\frac{4}{5}x^{5}a\,c^{3}d+\frac{1}{5}x^{5}b\,c^{4}+\frac{2}{3}x^{9}a\,c^{2}d^{2}+\frac{4}{9}x^{9}b\,c^{3}d+\frac{4}{13}x^{13}ac\,d^{3}+\frac{6}{13}x^{13}b\,c^{2}d^{2}+\frac{1}{17}x^{17}a\,d^{4}+$
risch	$a\,c^{4}x+\frac{4}{5}x^{5}a\,c^{3}d+\frac{1}{5}x^{5}b\,c^{4}+\frac{2}{3}x^{9}a\,c^{2}d^{2}+\frac{4}{9}x^{9}b\,c^{3}d+\frac{4}{13}x^{13}ac\,d^{3}+\frac{6}{13}x^{13}b\,c^{2}d^{2}+\frac{1}{17}x^{17}a\,d^{4}+$
parallelrisch	$a\,c^{4}x+\frac{4}{5}x^{5}a\,c^{3}d+\frac{1}{5}x^{5}b\,c^{4}+\frac{2}{3}x^{9}a\,c^{2}d^{2}+\frac{4}{9}x^{9}b\,c^{3}d+\frac{4}{13}x^{13}ac\,d^{3}+\frac{6}{13}x^{13}b\,c^{2}d^{2}+\frac{1}{17}x^{17}a\,d^{4}+$

input
```
int((b*x^4+a)*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+(4/5*a*c^3*d+1/5*b*c^4)*x^5+(2/3*a*c^2*d^2+4/9*b*c^3*d)*x^9+(4/13*
a*c*d^3+6/13*b*c^2*d^2)*x^13+(1/17*a*d^4+4/17*b*c*d^3)*x^17+1/21*b*d^4*x^2
1
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)\left(c+dx^4\right)^4\,dx=\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\left(4\,bcd^3+ad^4\right)x^{17}+\frac{2}{13}\left(3\,bc^2d^2+2\,acd^3\right)x^{13}+\frac{2}{9}\left(2\,bc^3d+3\,ac^2d^2\right)x^9+ac^4x+\frac{1}{5}\left(bc^4+4\,ac^3d\right)x^5$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

output
```
1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*
c*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4
*a*c^3*d)*x^5
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int (a+bx^4)(c+dx^4)^4\,dx = ac^4x+\frac{bd^4x^{21}}{21}+x^{17}\left(\frac{ad^4}{17}+\frac{4bcd^3}{17}\right)+x^{13}\cdot\left(\frac{4acd^3}{13}+\frac{6bc^2d^2}{13}\right)+x^9\cdot\left(\frac{2ac^2d^2}{3}+\frac{4bc^3d}{9}\right)+x^5\cdot\left(\frac{4ac^3d}{5}+\frac{bc^4}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

output
```
a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*
a*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) +
x**5*(4*a*c**3*d/5 + b*c**4/5)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)(c+dx^4)^4\,dx = \frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\,(4\,bcd^3+ad^4)x^{17}+\frac{2}{13}\,(3\,bc^2d^2+2\,acd^3)x^{13}+\frac{2}{9}\,(2\,bc^3d+3\,ac^2d^2)x^9+ac^4x+\frac{1}{5}\,(bc^4+4\,ac^3d)x^5$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

output
```
1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*
c*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4
*a*c^3*d)*x^5
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (a+bx^4)(c+dx^4)^4\,dx = \frac{1}{21}\,bd^4x^{21}+\frac{4}{17}\,bcd^3x^{17}+\frac{1}{17}\,ad^4x^{17}+\frac{6}{13}\,bc^2d^2x^{13}+\frac{4}{13}\,acd^3x^{13}+\frac{4}{9}\,bc^3dx^9+\frac{2}{3}\,ac^2d^2x^9+\frac{1}{5}\,bc^4x^5+\frac{4}{5}\,ac^3dx^5+ac^4x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

output
```
1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^1
3 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^
5 + 4/5*a*c^3*d*x^5 + a*c^4*x
```

3.146.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int (a+bx^4) (c+dx^4)^4 \, dx = x^5 \left(\frac{b c^4}{5} + \frac{4 a d c^3}{5}\right) + x^{17} \left(\frac{a d^4}{17} + \frac{4 b c d^3}{17}\right) + \frac{b d^4 x^{21}}{21} + a c^4 x + \frac{2 c^2 d x^9 (3 a d + 2 b c)}{9} + \frac{2 c d^2 x^{13} (2 a d + 3 b c)}{13}$$

input
```
int((a + b*x^4)*(c + d*x^4)^4,x)
```

output
```
x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*
d^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(
2*a*d + 3*b*c))/13
```

3.147 $\int (a+bx^4)(c+dx^4)^3 \, dx$

3.147.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^4)(c+dx^4)^3 \, dx = ac^3x + \frac{1}{5}c^2(bc+3ad)x^5 + \frac{1}{3}cd(bc+ad)x^9 + \frac{1}{13}d^2(3bc+ad)x^{13} + \frac{1}{17}bd^3x^{17}$$

output
```
a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)
*x^13+1/17*b*d^3*x^17
```

3.147.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3 \, dx = ac^3x + \frac{1}{5}c^2(bc+3ad)x^5 + \frac{1}{3}cd(bc+ad)x^9 + \frac{1}{13}d^2(3bc+ad)x^{13} + \frac{1}{17}bd^3x^{17}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*
c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

3.147.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4) (c+dx^4)^3 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^2x^4(3ad+bc)+d^2x^{12}(ad+3bc)+3cdx^8(ad+bc)+ac^3+bd^3x^{16}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*
c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

3.147.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.147.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
norman	$a\,c^{3}x+\left(\frac{3}{5}a\,c^{2}d+\frac{1}{5}c^{3}b\right)x^{5}+\left(\frac{1}{3}ac\,d^{2}+\frac{1}{3}b\,c^{2}d\right)x^{9}+\left(\frac{1}{13}a\,d^{3}+\frac{3}{13}bc\,d^{2}\right)x^{13}+\frac{b\,d^{3}x^{17}}{17}$	72
default	$\frac{b\,d^{3}x^{17}}{17}+\frac{(a\,d^{3}+3bc\,d^{2})x^{13}}{13}+\frac{(3ac\,d^{2}+3b\,c^{2}d)x^{9}}{9}+\frac{(3a\,c^{2}d+c^{3}b)x^{5}}{5}+a\,c^{3}x$	73
gosper	$a\,c^{3}x+\frac{3}{5}x^{5}a\,c^{2}d+\frac{1}{5}x^{5}c^{3}b+\frac{1}{3}x^{9}ac\,d^{2}+\frac{1}{3}x^{9}b\,c^{2}d+\frac{1}{13}x^{13}a\,d^{3}+\frac{3}{13}x^{13}bc\,d^{2}+\frac{1}{17}b\,d^{3}x^{17}$	75
risch	$a\,c^{3}x+\frac{3}{5}x^{5}a\,c^{2}d+\frac{1}{5}x^{5}c^{3}b+\frac{1}{3}x^{9}ac\,d^{2}+\frac{1}{3}x^{9}b\,c^{2}d+\frac{1}{13}x^{13}a\,d^{3}+\frac{3}{13}x^{13}bc\,d^{2}+\frac{1}{17}b\,d^{3}x^{17}$	75
parallelrisch	$a\,c^{3}x+\frac{3}{5}x^{5}a\,c^{2}d+\frac{1}{5}x^{5}c^{3}b+\frac{1}{3}x^{9}ac\,d^{2}+\frac{1}{3}x^{9}b\,c^{2}d+\frac{1}{13}x^{13}a\,d^{3}+\frac{3}{13}x^{13}bc\,d^{2}+\frac{1}{17}b\,d^{3}x^{17}$	75

input
```
int((b*x^4+a)*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*c^3*x+(3/5*a*c^2*d+1/5*c^3*b)*x^5+(1/3*a*c*d^2+1/3*b*c^2*d)*x^9+(1/13*a*
d^3+3/13*b*c*d^2)*x^13+1/17*b*d^3*x^17
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx=\frac{1}{17}\,bd^3x^{17}+\frac{1}{13}\left(3\,bcd^2+ad^3\right)x^{13}+\frac{1}{3}\left(bc^2d+acd^2\right)x^9+\frac{1}{5}\left(bc^3+3\,ac^2d\right)x^5+ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*
x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int (a+bx^4)(c+dx^4)^3\,dx = ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13}+\frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3}+\frac{bc^2d}{3}\right) + x^5\cdot\left(\frac{3ac^2d}{5}+\frac{bc^3}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

output
```
a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c
*d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3\,dx = \frac{1}{17}bd^3x^{17} + \frac{1}{13}(3\,bcd^2+ad^3)x^{13} + \frac{1}{3}(bc^2d+acd^2)x^9 + \frac{1}{5}(bc^3+3\,ac^2d)x^5 + ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*
x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^4)(c+dx^4)^3\,dx = \frac{1}{17}bd^3x^{17} + \frac{3}{13}bcd^2x^{13} + \frac{1}{13}ad^3x^{13} + \frac{1}{3}bc^2dx^9 + \frac{1}{3}acd^2x^9 + \frac{1}{5}bc^3x^5 + \frac{3}{5}ac^2dx^5 + ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 +
1/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

3.147.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^4)(c+dx^4)^3 dx = x^5\left(\frac{b c^3}{5}+\frac{3 a d c^2}{5}\right)+x^{13}\left(\frac{a d^3}{13}+\frac{3 b c d^2}{13}\right) + \frac{b d^3 x^{17}}{17}+a c^3 x+\frac{c d x^9 (a d+b c)}{3}$$

input
```
int((a + b*x^4)*(c + d*x^4)^3,x)
```

output
```
x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*
d^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

3.148 $\int (a+bx^4)(c+dx^4)^2 \, dx$

3.148.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)(c+dx^4)^2 \, dx = ac^2x + \frac{1}{5}c(bc+2ad)x^5 + \frac{1}{9}d(2bc+ad)x^9 + \frac{1}{13}bd^2x^{13}$$

output
```
a*c^2*x+1/5*c*(2*a*d+b*c)*x^5+1/9*d*(a*d+2*b*c)*x^9+1/13*b*d^2*x^13
```

3.148.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^2 \, dx = ac^2x + \frac{1}{5}c(bc+2ad)x^5 + \frac{1}{9}d(2bc+ad)x^9 + \frac{1}{13}bd^2x^{13}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)
/13
```

3.148.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)(c+dx^4)^2\,dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(dx^8(ad+2bc)+cx^4(2ad+bc)+ac^2+bd^2x^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)
/13
```

3.148.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.148.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^{2}x^{13}}{13}+\frac{\left(a\,d^{2}+2bcd\right)x^{9}}{9}+\frac{\left(2acd+b\,c^{2}\right)x^{5}}{5}+a\,c^{2}x$	49
norman	$\frac{b\,d^{2}x^{13}}{13}+\left(\frac{1}{9}a\,d^{2}+\frac{2}{9}bcd\right)x^{9}+\left(\frac{2}{5}acd+\frac{1}{5}b\,c^{2}\right)x^{5}+a\,c^{2}x$	49
gosper	$\frac{1}{13}b\,d^{2}x^{13}+\frac{1}{9}x^{9}a\,d^{2}+\frac{2}{9}x^{9}bcd+\frac{2}{5}x^{5}acd+\frac{1}{5}x^{5}b\,c^{2}+a\,c^{2}x$	51
risch	$\frac{1}{13}b\,d^{2}x^{13}+\frac{1}{9}x^{9}a\,d^{2}+\frac{2}{9}x^{9}bcd+\frac{2}{5}x^{5}acd+\frac{1}{5}x^{5}b\,c^{2}+a\,c^{2}x$	51
parallelrisch	$\frac{1}{13}b\,d^{2}x^{13}+\frac{1}{9}x^{9}a\,d^{2}+\frac{2}{9}x^{9}bcd+\frac{2}{5}x^{5}acd+\frac{1}{5}x^{5}b\,c^{2}+a\,c^{2}x$	51

input
```
int((b*x^4+a)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx=\frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\,\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\,\left(bc^2+2\,acd\right)x^5+ac^2x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 +
a*c^2*x
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx=ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\cdot\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

output
```
a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
 + b*c**2/5)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)(c+dx^4)^2 \, dx = \frac{1}{13}bd^2x^{13} + \frac{1}{9}(2bcd + ad^2)x^9 + \frac{1}{5}(bc^2 + 2acd)x^5 + ac^2x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 +
a*c^2*x
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^2 \, dx = \frac{1}{13}bd^2x^{13} + \frac{2}{9}bcdx^9 + \frac{1}{9}ad^2x^9 + \frac{1}{5}bc^2x^5 + \frac{2}{5}acdx^5 + ac^2x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*
d*x^5 + a*c^2*x
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)(c+dx^4)^2 \, dx = x^5\left(\frac{b\,c^2}{5} + \frac{2\,a\,d\,c}{5}\right) + x^9\left(\frac{a\,d^2}{9} + \frac{2\,b\,c\,d}{9}\right) + \frac{b\,d^2\,x^{13}}{13} + a\,c^2\,x$$

input
```
int((a + b*x^4)*(c + d*x^4)^2,x)
```

output
```
x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^1
3)/13 + a*c^2*x
```

3.149 $\int (a+bx^4)(c+dx^4)\,dx$

3.149.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx = acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

output
```
a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

3.149.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx = acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4),x]
```

output
```
a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

3.149.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(x^4(ad+bc)+ac+bdx^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

input
```
Int[(a + b*x^4)*(c + d*x^4),x]
```

output
```
a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

3.149.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.149.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^5}{5}+\frac{bd\,x^9}{9}$	25
norman	$\frac{bd\,x^9}{9}+\left(\frac{ad}{5}+\frac{bc}{5}\right)x^5+acx$	26
gosper	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
risch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
parallelrisch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27

input
```
int((b*x^4+a)*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^4)\left(c+dx^4\right)\,dx=\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^4)\left(c+dx^4\right)\,dx=acx+\frac{bdx^9}{9}+x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c),x)
```

output
```
a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{1}{9}bdx^9 + \frac{1}{5}(bc+ad)x^5 + acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
```

output
```
1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{1}{9}bdx^9 + \frac{1}{5}bcx^5 + \frac{1}{5}adx^5 + acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
```

output
```
1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{b\,d\,x^9}{9} + \left(\frac{a\,d}{5} + \frac{b\,c}{5}\right)x^5 + a\,c\,x$$

input
```
int((a + b*x^4)*(c + d*x^4),x)
```

output
```
x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

3.150 $\int \frac{a+bx^4}{c+dx^4}\,dx$

3.150.1 Optimal result

Integrand size = 17, antiderivative size = 223

$$\begin{aligned}\int \frac{a+bx^4}{c+dx^4}\,dx = {} & \frac{bx}{d}+\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}\\ & +\frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}\\ & -\frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}\end{aligned}$$

output
```
b*x/d-1/4*(-a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*
2^(1/2)-1/4*(-a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)
*2^(1/2)+1/8*(-a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))
/c^(3/4)/d^(5/4)*2^(1/2)-1/8*(-a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/
2)+x^2*d^(1/2))/c^(3/4)/d^(5/4)*2^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{c+dx^4}\,dx$$
$$= \frac{8bc^{3/4}\sqrt[4]{d}x+2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+\sqrt{2}(bc-ad)\,\text{lo}}{8c^{3/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^4)/(c + d*x^4),x]
```

output
```
(8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2])/(8*c^(3/4)*d^(5/4))
```

3.150.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{c+dx^4}\,dx$$
$$\Big\downarrow \text{913}$$
$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^4+c}dx}{d}$$
$$\Big\downarrow \text{755}$$
$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{d}$$
$$\Big\downarrow \text{1476}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{d}$$

$\Big\downarrow$ 1082

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$\Big\downarrow$ 217

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$\Big\downarrow$ 1479

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$\Big\downarrow$ 25

3.150. $\int \frac{a+bx^4}{c+dx^4}\,dx$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \; 1103$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

input
```
Int[(a + b*x^4)/(c + d*x^4),x]
```

output
```
(b*x)/d - ((b*c - a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2
]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(
1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d
```

3.150. $\int \frac{a+bx^4}{c+dx^4}\,dx$

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.150. $\int \frac{a+bx^4}{c+dx^4}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.150.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{4}+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^{3}}}{4d^{2}}$	42
default	$\frac{bx}{d}+\frac{(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8dc}$	120

input
```
int((b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+1/4/d^2*sum((a*d-b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

3.150.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.51

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}\log\left(cd\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)+i}{}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + I*d*(-
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c
^3*d^5))^(1/4)*log(I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - I*d*(-(b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5)
)^(1/4)*log(-I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - d*(-(b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*
log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} + \text{RootSum}\left(256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(\frac{4tcd}{ad-bc} + x\right)\right)\right)$$

input
```
integrate((b*x**4+a)/(d*x**4+c),x)
```

output
```
b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2
*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/
(a*d - b*c) + x)))
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,d}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(
2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)
*(b*c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/
4)*d^(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4
)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^2}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^
3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^
(1/4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/
4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)
*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) +
 sqrt(c/d))/(c*d^2)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{b\,x}{d}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1i}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-b\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-b\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}}\cdots\right)\cdots}{2(-c)^{3/4}d^{5/4}}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1i}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}}\cdots\right)\cdots}{2(-c)^{3/4}d^{5/4}}$$

input
```
int((a + b*x^4)/(c + d*x^4),x)
```

output
```
(b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*
d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*
(-c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i
)/(4*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - (
(16*b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*
c))/(4*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) +
 ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d -
b*c))/(4*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (a
tan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c
*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d
^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 1
6*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3
/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^
2 - 16*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*
b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c
)*1i)/(4*(-c)^(3/4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

3.151 $\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$

3.151.1 Optimal result

Integrand size = 17, antiderivative size = 245

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x}{4cd(c+dx^4)} - \frac{(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}$$

output
```
-1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/
2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/16*(3*a*d+b*c)*arctan(1+d^(1/4)*x*2^
(1/2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)-1/32*(3*a*d+b*c)*ln(-c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)+1/32*(3*a*d+b*c)*
ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{-\frac{8c^{3/4}\sqrt[4]{d}(bc-ad)x}{c+dx^4} - 2\sqrt{2}(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt{2}(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) - \sqrt{2}(bc\cdots}{32c^{7/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

output
```
((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

3.151.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(3ad+bc)\int \frac{1}{dx^4+c}dx}{4cd} - \frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \; 755$$

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}} + \frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{4cd} - \frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \; 1476$$

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 1082

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 217

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 1479

$$\frac{(3ad+bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 25

3.151. $\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx$

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{(3ad+bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

3.151. $\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$

output
```
-1/4*((b*c - a*d)*x)/(c*d*(c + d*x^4)) + ((b*c + 3*a*d)*((-(ArcTan[1 - (Sq
rt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]
*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4))
 + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/
4)*d^(1/4)))/(2*Sqrt[c])))/(4*c*d)
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.151. $\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.151.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\,\_Z^4+c\right)}\frac{(3ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{16c\,d^2}$	65
default	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{(3ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2d}$	140

input
```
int((b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(a*d-b*c)/c*x/(d*x^4+c)+1/16/c/d^2*sum((3*a*d+b*c)/_R^3*ln(x-_R),_R=
RootOf(_Z^4*d+c))
```

3.151.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.64

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$= \frac{(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4}{c^7d^5}\right.\right.}{}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*((c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 1
2*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^
5))^(1/4) + (b*c + 3*a*d)*x) - (-I*c*d^2*x^4 - I*c^2*d)*(-(b^4*c^4 + 12*a*
b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^
(1/4)*log(I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a
^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - (I*c*d^2*x^
4 + I*c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*
c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^
3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)
+ (b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*
a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*
d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*
a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4
 + c^2*d)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{x(ad-bc)}{4c^2d+4cd^2x^4}$$

$$+\mathrm{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+\right.\right.\right.$$

input
```
integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

output
```
x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
 81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**
3*d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x}{4\,(cd^2x^4+c^2d)} + \frac{\frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc+3\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\cdots\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\cdots}{32\,cd}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arc
tan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(
d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqr
t(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d
)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4))
)/(c*d)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc+3\,(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc+3\,(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc+3\,(cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc+3\,(cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{bcx-adx}{4\,(dx^4+c)cd}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)
^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^
(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3
)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32
*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.02

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)}{16(-c)^{7/4}d^{5/4}}}\right.}{8(-c)^{7/4}d^{5/4}}$$

$$+ \frac{x\,(a\,d-b\,c)}{4\,c\,d\,(d\,x^4+c)}$$

$$+ \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)}{16(-c)^{7/4}d^{5/4}}}\right.}{8(-c)^{7/4}d^{5/4}}$$

input
```
int((a + b*x^4)/(c + d*x^4)^2,x)
```

output
```
(atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c
)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(
-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) +
 ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d +
b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/
4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d
+ 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^
(7/4)*d^(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)
/(8*(-c)^(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2
))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5
/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(
-c)^(7/4)*d^(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d
^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)
) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(
12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(
-c)^(7/4)*d^(5/4))))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c
))/(4*c*d*(c + d*x^4))
```

3.151. $\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$

3.152 $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

3.152.1 Optimal result

Integrand size = 17, antiderivative size = 273

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2}+\frac{(bc+7ad)x}{32c^2d\,(c+dx^4)}-\frac{3(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}$$

output
```
-1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128
*(7*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)
+3/128*(7*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^
(1/2)-3/256*(7*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))
/c^(11/4)/d^(5/4)*2^(1/2)+3/256*(7*a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c
^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{-\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)x}{(c+dx^4)^2}+\frac{8c^{3/4}\sqrt[4]{d}(bc+7ad)x}{c+dx^4}-6\sqrt{2}(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+6\sqrt{2}(bc+7ad)\arctan\left(1\right.}{256}$$

input
```
Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

output
```
((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b
*c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)
*x + Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

3.152.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$$

$$\Big\downarrow \ 910$$

$$\frac{(7ad+bc)\int \frac{1}{(dx^4+c)^2}dx}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \ 749$$

$$\frac{(7ad+bc)\left(\frac{3\int \frac{1}{dx^4+c}dx}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \ 755$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$\big\downarrow$ 1476

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$\big\downarrow$ 1082

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$\big\downarrow$ 217

3.152. $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$\big\downarrow$ 1479

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{}{4c(c\cdots}\right)}{8cd}$$

$$\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$\big\downarrow$ 25

3.152. $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}$$

$$\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\downarrow$ 27

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd} -$$

$$\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\downarrow$ 1103

3.152. $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4}\right.}{8cd}$$

$$\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

input
```
Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

output
```
-1/8*((b*c - a*d)*x)/(c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*(x/(4*c*(c + d*x
^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(
1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))
)/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(4*c)))/(8*c*d)
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.152. $\int \frac{a+bx^4}{(c+dx^4)^3} \, dx$

3.152.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.93 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.31

method	result	si
risch	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(d\,x^4+c)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(7ad+bc)\ln\left(x-\_R\right)}{\_R^3}\right)}{128c^2d^2}$	8
default	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(d\,x^4+c)^2}+\frac{3(7ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{256c^3d}$	1

input
```
int((b*x^4+a)/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+3/128/c^2
/d^2*sum((7*a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

3.152.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.71

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$$

$$= \frac{4\left(bcd+7\,ad^2\right)x^5+3\left(c^2d^3x^8+2\,c^3d^2x^4+c^4d\right)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\log\left(3\,c^3d\right(}{}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*
a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(
b*c + 7*a*d)*x) - 3*(-I*c^2*d^3*x^8 - 2*I*c^3*d^2*x^4 - I*c^4*d)*(-(b^4*c^
4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4
)/(c^11*d^5))^(1/4)*log(3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^
2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c +
7*a*d)*x) - 3*(I*c^2*d^3*x^8 + 2*I*c^3*d^2*x^4 + I*c^4*d)*(-(b^4*c^4 + 28*
a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11
*d^5))^(1/4)*log(-3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*
d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)
*x) - 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d
+ 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)
*log(-3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3
*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c
^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.55

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{x^5\cdot(7ad^2+bcd)+x(11acd-3bc^2)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}$$
$$+\mathrm{RootSum}\left(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2268ab^3c^3d+81b^4c^4,\ \left(\right.\right.$$

input
```
integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

output
```
(x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d
**2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 19448
1*a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b*
*3*c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c)
 + x)))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx = \frac{(bcd+7\,ad^2)x^5-(3\,bc^2-11\,acd)x}{32\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)} + \frac{3\left(\frac{2\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc+7\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right.}{256\,c^2d}$$

input

```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

output

```
1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3
*d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*s
qrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
 - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*s
qrt(d))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)
*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx &= \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} \\ &+ \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} \\ &+ \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} \\ &- \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} \\ &+ \frac{bcdx^5+7\,ad^2x^5-3\,bc^2x+11\,acdx}{32\left(dx^4+c\right)^2c^2d}\end{aligned}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

output
```
3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^
3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d
)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*
d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3
/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*
c^2*x + 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.79

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{\frac{x^5\,(7\,a\,d+b\,c)}{32\,c^2}+\frac{x\,(11\,a\,d-3\,b\,c)}{32\,c\,d}}{c^2+2\,c\,d\,x^4+d^2\,x^8}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d\cdots\right.}{256\,(-c)^{15/4}\,d^{5/4}}\cdots}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\cdots\right.}{256\,(-c)^{15/4}\,d^{5/4}}\cdots}{128\,(-c)^{11/4}\,d^{5/4}}}\cdots\right.}{64\,(-c)^{11/4}\,d^{5/4}}$$

$$-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\cdots\right.}{256\,(-c)^{15/4}\,d\cdots}\cdots}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\cdots\right.}{256\,(-c)^{15/4}\,d^{5/}\cdots}\cdots}{128\,(-c)^{11/4}\,d^{5/4}}}\cdots\right.}{64\,(-c)^{11/4}\,d^{5/4}}$$

input
```
int((a + b*x^4)/(c + d*x^4)^3,x)
```

output
```
((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*
x^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/
(256*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)
))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*
c^2*d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(
256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3
*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*
c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c
)^(11/4)*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256
*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(
7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4
)*d^(5/4)) - (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4))
)*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^
2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(2
56*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*
d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^
(11/4)*d^(5/4)) - (((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4
) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(...
```

3.152. $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

3.153 $\int (a+bx^4)^2 (c+dx^4)^4 \, dx$

3.153.1 Optimal result

Integrand size = 19, antiderivative size = 154

$$\begin{aligned}\int (a+bx^4)^2 (c+dx^4)^4 \, dx = {} & a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 \\ & + \frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} \\ & + \frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} \\ & + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}\end{aligned}$$

output
```
a^2*c^4*x+2/5*a*c^3*(2*a*d+b*c)*x^5+1/9*c^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*
x^9+4/13*c*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^13+1/17*d^2*(a^2*d^2+8*a*b*c*d+
6*b^2*c^2)*x^17+2/21*b*d^3*(a*d+2*b*c)*x^21+1/25*b^2*d^4*x^25
```

3.153.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+bx^4)^2 (c+dx^4)^4 \, dx = {} & a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 \\ & + \frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} \\ & + \frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} \\ & + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}\end{aligned}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

output
```
a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*
a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(
6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/2
1 + (b^2*d^4*x^25)/25
```

3.153.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int (d^2x^{16}(a^2d^2+8abcd+6b^2c^2)+4cdx^{12}(a^2d^2+3abcd+b^2c^2)+c^2x^8(6a^2d^2+8abcd+b^2c^2)+a^2c^4+2ac^3x^4(2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{17}d^2x^{17}(a^2d^2+8abcd+6b^2c^2)+\frac{4}{13}cdx^{13}(a^2d^2+3abcd+b^2c^2)+\frac{1}{9}c^2x^9(6a^2d^2+8abcd+b^2c^2)+ a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

output
```
a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*
a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(
6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/2
1 + (b^2*d^4*x^25)/25
```

3.153.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.153.4 Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
norman	$\frac{b^2 d^4 x^{25}}{25}+\left(\frac{2}{21} a b\, d^4+\frac{4}{21} b^2 c\, d^3\right) x^{21}+\left(\frac{1}{17} a^2 d^4+\frac{8}{17} a b c\, d^3+\frac{6}{17} b^2 c^2 d^2\right) x^{17}+\left(\frac{4}{13} a^2 c\, d^3+\frac{12}{13} a b\, c^2$
default	$\frac{b^2 d^4 x^{25}}{25}+\frac{\left(2 a b\, d^4+4 b^2 c\, d^3\right) x^{21}}{21}+\frac{\left(a^2 d^4+8 a b c\, d^3+6 b^2 c^2 d^2\right) x^{17}}{17}+\frac{\left(4 a^2 c\, d^3+12 a b\, c^2 d^2+4 b^2 c^3 d\right) x^{13}}{13}+\frac{\left(6 a^2 c^2 d^2+8 a b\, c\right.}{9}$
gosper	$\frac{1}{25} b^2 d^4 x^{25}+\frac{2}{21} x^{21} a b\, d^4+\frac{4}{21} x^{21} b^2 c\, d^3+\frac{1}{17} x^{17} a^2 d^4+\frac{8}{17} x^{17} a b c\, d^3+\frac{6}{17} x^{17} b^2 c^2 d^2+\frac{4}{13} x^{13} a^2 c\, d^3$
risch	$\frac{1}{25} b^2 d^4 x^{25}+\frac{2}{21} x^{21} a b\, d^4+\frac{4}{21} x^{21} b^2 c\, d^3+\frac{1}{17} x^{17} a^2 d^4+\frac{8}{17} x^{17} a b c\, d^3+\frac{6}{17} x^{17} b^2 c^2 d^2+\frac{4}{13} x^{13} a^2 c\, d^3$
parallelrisch	$\frac{1}{25} b^2 d^4 x^{25}+\frac{2}{21} x^{21} a b\, d^4+\frac{4}{21} x^{21} b^2 c\, d^3+\frac{1}{17} x^{17} a^2 d^4+\frac{8}{17} x^{17} a b c\, d^3+\frac{6}{17} x^{17} b^2 c^2 d^2+\frac{4}{13} x^{13} a^2 c\, d^3$

input
```
int((b*x^4+a)^2*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/25*b^2*d^4*x^25+(2/21*a*b*d^4+4/21*b^2*c*d^3)*x^21+(1/17*a^2*d^4+8/17*a*
b*c*d^3+6/17*b^2*c^2*d^2)*x^17+(4/13*a^2*c*d^3+12/13*a*b*c^2*d^2+4/13*b^2*
c^3*d)*x^13+(2/3*a^2*c^2*d^2+8/9*a*b*c^3*d+1/9*b^2*c^4)*x^9+a^2*c^4*x+(4/5
*a^2*c^3*d+2/5*a*b*c^4)*x^5
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = \frac{1}{25} b^2 d^4 x^{25} + \frac{2}{21} (2 b^2 c d^3 + a b d^4) x^{21} + \frac{1}{17} (6 b^2 c^2 d^2 + 8 a b c d^3 + a^2 d^4) x^{17} + \frac{4}{13} (b^2 c^3 d + 3 a b c^2 d^2 + a^2 c d^3) x^{13} + \frac{1}{9} (b^2 c^4 + 8 a b c^3 d + 6 a^2 c^2 d^2) x^9 + a^2 c^4 x + \frac{2}{5} (a b c^4 + 2 a^2 c^3 d) x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

output
```
1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^
2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*
d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x +
2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = a^2 c^4 x + \frac{b^2 d^4 x^{25}}{25} + x^{21} \cdot \left(\frac{2abd^4}{21} + \frac{4b^2 c d^3}{21}\right) + x^{17} \left(\frac{a^2 d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2 c^2 d^2}{17}\right) + x^{13} \cdot \left(\frac{4a^2 c d^3}{13} + \frac{12abc^2 d^2}{13} + \frac{4b^2 c^3 d}{13}\right) + x^9 \cdot \left(\frac{2a^2 c^2 d^2}{3} + \frac{8abc^3 d}{9} + \frac{b^2 c^4}{9}\right) + x^5 \cdot \left(\frac{4a^2 c^3 d}{5} + \frac{2abc^4}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

output
```
a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21
) + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(
4*a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*
c**2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*
b*c**4/5)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = \frac{1}{25} b^2d^4x^{25} + \frac{2}{21} (2b^2cd^3 + abd^4)x^{21} + \frac{1}{17} (6b^2c^2d^2 + 8abcd^3 + a^2d^4)x^{17} + \frac{4}{13} (b^2c^3d + 3abc^2d^2 + a^2cd^3)x^{13} + \frac{1}{9} (b^2c^4 + 8abc^3d + 6a^2c^2d^2)x^9 + a^2c^4x + \frac{2}{5} (abc^4 + 2a^2c^3d)x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

output
```
1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^
2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*
d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x +
2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = \frac{1}{25} b^2d^4x^{25} + \frac{4}{21} b^2cd^3x^{21} + \frac{2}{21} abd^4x^{21} + \frac{6}{17} b^2c^2d^2x^{17} + \frac{8}{17} abcd^3x^{17} + \frac{1}{17} a^2d^4x^{17} + \frac{4}{13} b^2c^3dx^{13} + \frac{12}{13} abc^2d^2x^{13} + \frac{4}{13} a^2cd^3x^{13} + \frac{1}{9} b^2c^4x^9 + \frac{8}{9} abc^3dx^9 + \frac{2}{3} a^2c^2d^2x^9 + \frac{2}{5} abc^4x^5 + \frac{4}{5} a^2c^3dx^5 + a^2c^4x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

output
```
1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2
*d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13
+ 12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b
*c^3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a
^2*c^4*x
```

3.153.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = x^9 \left(\frac{2 a^2 c^2 d^2}{3}+\frac{8 a b c^3 d}{9}+\frac{b^2 c^4}{9}\right) + x^{17} \left(\frac{a^2 d^4}{17}+\frac{8 a b c d^3}{17}+\frac{6 b^2 c^2 d^2}{17}\right) + a^2 c^4 x + \frac{b^2 d^4 x^{25}}{25} + \frac{2 a c^3 x^5 (2 a d+b c)}{5} + \frac{2 b d^3 x^{21} (a d+2 b c)}{21} + \frac{4 c d x^{13} (a^2 d^2+3 a b c d+b^2 c^2)}{13}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

output
```
x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/
17 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/2
5 + (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c
*d*x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

3.154 $\int (a+bx^4)^2 (c+dx^4)^3 \, dx$

3.154.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9 + \frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}$$

output
```
a^2*c^3*x+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^9+1/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^13+1/17*b*d^2*(2*a*d+3*b*c)*x^1
7+1/21*b^2*d^3*x^21
```

3.154.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9 + \frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*
b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

3.154.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^{12}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^8\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^4(3ad+2bc)+bd^2x^{16}(2ad+3bc)+b$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*
b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

3.154.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.154.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x+\left(\frac{3}{5}a^2c^2d+\frac{2}{5}ab\,c^3\right)x^5+\left(\frac{1}{3}c\,a^2d^2+\frac{2}{3}ab\,c^2d+\frac{1}{9}b^2c^3\right)x^9+\left(\frac{1}{13}a^2d^3+\frac{6}{13}abc\,d^2+\frac{3}{13}b^2c\right.$
default	$\frac{b^2d^3x^{21}}{21}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{17}}{17}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{13}}{13}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^9}{9}+\frac{(3a^2c^2d+2ab\,c^3)x^5}{5}+$
gosper	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}abc\,d^2$
risch	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}abc\,d^2$
parallelrisch	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}abc\,d^2$

input
```
int((b*x^4+a)^2*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x+(3/5*a^2*c^2*d+2/5*a*b*c^3)*x^5+(1/3*c*a^2*d^2+2/3*a*b*c^2*d+1/9
*b^2*c^3)*x^9+(1/13*a^2*d^3+6/13*a*b*c*d^2+3/13*b^2*c^2*d)*x^13+(2/17*a*b*
d^3+3/17*b^2*c*d^2)*x^17+1/21*b^2*d^3*x^21
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx=&\ \frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}\\&+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}\\&+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9\\&+a^2c^3x+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5\end{aligned}$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{21}}{21} + x^{17} \cdot \left(\frac{2abd^3}{17} + \frac{3b^2cd^2}{17}\right) + x^{13}\left(\frac{a^2d^3}{13} + \frac{6abcd^2}{13} + \frac{3b^2c^2d}{13}\right) + x^9\left(\frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9}\right) + x^5 \cdot \left(\frac{3a^2c^2d}{5} + \frac{2abc^3}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

output
```
a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17
) + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2
*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*
c**3/5)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = \frac{1}{21} b^2d^3x^{21} + \frac{1}{17} (3 b^2cd^2 + 2 abd^3)x^{17} + \frac{1}{13} (3 b^2c^2d + 6 abcd^2 + a^2d^3)x^{13} + \frac{1}{9} (b^2c^3 + 6 abc^2d + 3 a^2cd^2)x^9 + a^2c^3x + \frac{1}{5} (2 abc^3 + 3 a^2c^2d)x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = \frac{1}{21} b^2 d^3 x^{21} + \frac{3}{17} b^2 c d^2 x^{17} + \frac{2}{17} a b d^3 x^{17} + \frac{3}{13} b^2 c^2 d x^{13} + \frac{6}{13} a b c d^2 x^{13} + \frac{1}{13} a^2 d^3 x^{13} + \frac{1}{9} b^2 c^3 x^9 + \frac{2}{3} a b c^2 d x^9 + \frac{1}{3} a^2 c d^2 x^9 + \frac{2}{5} a b c^3 x^5 + \frac{3}{5} a^2 c^2 d x^5 + a^2 c^3 x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2
*d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*
a*b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 +
a^2*c^3*x
```

3.154.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = x^9 \left(\frac{a^2 c d^2}{3} + \frac{2 a b c^2 d}{3} + \frac{b^2 c^3}{9}\right) + x^{13} \left(\frac{a^2 d^3}{13} + \frac{6 a b c d^2}{13} + \frac{3 b^2 c^2 d}{13}\right) + a^2 c^3 x + \frac{b^2 d^3 x^{21}}{21} + \frac{a c^2 x^5 (3 a d + 2 b c)}{5} + \frac{b d^2 x^{17} (2 a d + 3 b c)}{17}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

output
```
x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
 (3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a
*c^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

3.154. $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

3.155 $\int (a+bx^4)^2 (c+dx^4)^2 \, dx$

3.155.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = a^2c^2x + \frac{2}{5}ac(bc+ad)x^5 + \frac{1}{9}(b^2c^2+4abcd+a^2d^2)\,x^9 + \frac{2}{13}bd(bc+ad)x^{13} + \frac{1}{17}b^2d^2x^{17}$$

output
```
a^2*c^2*x+2/5*a*c*(a*d+b*c)*x^5+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2/13*b
*d*(a*d+b*c)*x^13+1/17*b^2*d^2*x^17
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = a^2c^2x + \frac{2}{5}ac(bc+ad)x^5 + \frac{1}{9}(b^2c^2+4abcd+a^2d^2)\,x^9 + \frac{2}{13}bd(bc+ad)x^{13} + \frac{1}{17}b^2d^2x^{17}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

3.155.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^8\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^{12}(ad+bc)+2acx^4(ad+bc)+b^2d^2x^{16}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

3.155.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.155.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$a^2c^2x + \left(\frac{2}{5}a^2cd + \frac{2}{5}b\,c^2a\right)x^5 + \left(\frac{1}{9}a^2d^2 + \frac{4}{9}abcd + \frac{1}{9}b^2c^2\right)x^9 + \left(\frac{2}{13}ab\,d^2 + \frac{2}{13}b^2cd\right)x^{13} + \frac{b^2d^2x^1}{17}$
default	$\frac{b^2d^2x^{17}}{17} + \frac{(2ab\,d^2+2b^2cd)x^{13}}{13} + \frac{(a^2d^2+4abcd+b^2c^2)x^9}{9} + \frac{(2a^2cd+2b\,c^2a)x^5}{5} + a^2c^2x$
gosper	$a^2c^2x + \frac{2}{5}x^5a^2cd + \frac{2}{5}x^5b\,c^2a + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{13}x^{13}ab\,d^2 + \frac{2}{13}x^{13}b^2cd + \frac{1}{17}b$
risch	$a^2c^2x + \frac{2}{5}x^5a^2cd + \frac{2}{5}x^5b\,c^2a + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{13}x^{13}ab\,d^2 + \frac{2}{13}x^{13}b^2cd + \frac{1}{17}b$
parallelrisch	$a^2c^2x + \frac{2}{5}x^5a^2cd + \frac{2}{5}x^5b\,c^2a + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{13}x^{13}ab\,d^2 + \frac{2}{13}x^{13}b^2cd + \frac{1}{17}b$

input
```
int((b*x^4+a)^2*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^2*x+(2/5*a^2*c*d+2/5*b*c^2*a)*x^5+(1/9*a^2*d^2+4/9*a*b*c*d+1/9*b^2*c
^2)*x^9+(2/13*a*b*d^2+2/13*b^2*c*d)*x^13+1/17*b^2*d^2*x^17
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a + bx^4\right)^2 \left(c + dx^4\right)^2 \, dx = \frac{1}{17}b^2d^2x^{17} + \frac{2}{13}\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13} \cdot \left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5 \cdot \left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
 x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a
*b*c**2/5)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,b^2cdx^{13} + \frac{2}{13}\,abd^2x^{13} + \frac{1}{9}\,b^2c^2x^9 + \frac{4}{9}\,abcdx^9 + \frac{1}{9}\,a^2d^2x^9 + \frac{2}{5}\,abc^2x^5 + \frac{2}{5}\,a^2cdx^5 + a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^
9 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5
+ a^2*c^2*x
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = x^9 \left(\frac{a^2 d^2}{9}+\frac{4 a b c d}{9}+\frac{b^2 c^2}{9}\right)+a^2 c^2 x+\frac{b^2 d^2 x^{17}}{17} + \frac{2 a c x^5 (a d+b c)}{5}+\frac{2 b d x^{13} (a d+b c)}{13}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

output
```
x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^1
7)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

3.156 $\int (a+bx^4)^2 (c+dx^4)\, dx$

3.156.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)^2 (c+dx^4)\, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

output
```
a^2*c*x+1/5*a*(a*d+2*b*c)*x^5+1/9*b*(2*a*d+b*c)*x^9+1/13*b^2*d*x^13
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)\, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)
/13
```

3.156.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right) \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(a^2c+bx^8(2ad+bc)+ax^4(ad+2bc)+b^2dx^{12}\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)
/13
```

3.156.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.156.4 Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^{13}}{13}+\frac{(2abd+b^2c)x^9}{9}+\frac{(a^2d+2abc)x^5}{5}+a^2cx$	49
norman	$\frac{b^2 d\,x^{13}}{13}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{5}a^2d+\frac{2}{5}abc\right)x^5+a^2cx$	49
gosper	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51
risch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51
parallelrisch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51

input
```
int((b*x^4+a)^2*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)\,dx=\frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,(b^2c+2\,abd)x^9+\frac{1}{5}\,(2\,abc+a^2d)x^5+a^2cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 +
a^2*c*x
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)\,dx=a^2cx+\frac{b^2dx^{13}}{13}+x^9\cdot\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c),x)
```

output
```
a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5
+ 2*a*b*c/5)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\, dx = \frac{1}{13}\, b^2 d x^{13} + \frac{1}{9}\left(b^2 c + 2\, abd\right)x^9 + \frac{1}{5}\left(2\, abc + a^2 d\right)x^5 + a^2 cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

output
```
1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 +
a^2*c*x
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\, dx = \frac{1}{13}\, b^2 d x^{13} + \frac{1}{9}\, b^2 c x^9 + \frac{2}{9}\, abdx^9 + \frac{2}{5}\, abcx^5 + \frac{1}{5}\, a^2 dx^5 + a^2 cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

output
```
1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*
d*x^5 + a^2*c*x
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\, dx = x^5 \left(\frac{d\, a^2}{5} + \frac{2\, b\, c\, a}{5}\right) + x^9 \left(\frac{c\, b^2}{9} + \frac{2\, a\, d\, b}{9}\right) + \frac{b^2\, d\, x^{13}}{13} + a^2\, c\, x$$

input
```
int((a + b*x^4)^2*(c + d*x^4),x)
```

output
```
x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^1
3)/13 + a^2*c*x
```

3.157 $\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$

3.157.1 Optimal result

Integrand size = 19, antiderivative size = 253

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*arctan(-1+d^(1/4)*x*2
^(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+d^(1/4)*
x*2^(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-c^(1/4)*d^
(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/8*(-a*d+b*c
)^2*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1
/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$
$$= \frac{-40bc^{3/4}\sqrt[4]{d}(bc-2ad)x+8b^2c^{3/4}d^{5/4}x^5-10\sqrt{2}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+10\sqrt{2}(bc-ad)^2\text{arcta}}{}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

output
```
(-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(
b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b
*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^
(3/4)*d^(9/4))
```

3.157.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$
$$\Big\downarrow \text{ 915}$$
$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^4)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (
Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b
*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqr
t[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

3.157.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.157.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

method	result
risch	$\frac{b^2x^5}{5d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^4+c)}\frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^3}}{4d^3}$
default	$\frac{b(\frac{1}{5}bdx^5+2adx-bcx)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)(\frac{c}{d})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{c}{d})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{c}{d})^{\frac{1}{4}}}-1\right)\right.}{8d^2c}$

input
```
int((b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*c*x+1/4/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

3.157.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1092, normalized size of antiderivative = 4.32

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$
$$= \frac{4\,b^2dx^5+5\,d^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^3d^9}\right)^{\frac{1}{4}}\log\left(cd^2\left(-\right.\right.}{}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/20*(4*b^2*d*x^5 + 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*d^2*(-(b^8*c^
8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^3*d^9))^(1/4)*log(I*c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*x) - 5*I*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-I*c*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^...
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2x^5}{5d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right)$$
$$+\mathrm{RootSum}\left(256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6\right.$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c),x)
```

output
```
b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9
+ a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3
*d**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**
6*d**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*
d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2dx^5 - 5\,(b^2c - 2\,abd)x}{5\,d^2} + \frac{\frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2-2\,abc\cdots}{\cdots}}{8\,d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/
4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(
1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqr
t(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4
)))/d^2
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$
$$= \frac{\sqrt{2}\left((cd^3)^{\frac14} b^2c^2 - 2\,(cd^3)^{\frac14} abcd + (cd^3)^{\frac14} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\,cd^3}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac14} b^2c^2 - 2\,(cd^3)^{\frac14} abcd + (cd^3)^{\frac14} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\,cd^3}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac14} b^2c^2 - 2\,(cd^3)^{\frac14} abcd + (cd^3)^{\frac14} a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3}$$
$$- \frac{\sqrt{2}\left((cd^3)^{\frac14} b^2c^2 - 2\,(cd^3)^{\frac14} abcd + (cd^3)^{\frac14} a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3}$$
$$+ \frac{b^2d^4x^5 - 5\,b^2cd^3x + 10\,abd^4x}{5\,d^5}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c
*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c
*d + (c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(
c*d^3) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
+ 1/5*(b^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

3.157.9 Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.27

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2x^5}{5d} - x\left(\frac{b^2c}{d^2}-\frac{2ab}{d}\right)$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d}-\frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)}{4(-c)^{3/4}d^{9/4}}\right)1i}{(-c)^{3/4}d^{9/4}}+\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3b\cdots}{\cdots}\right.}{\cdots}}{\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d}-\frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)}{4(-c)^{3/4}d^{9/4}}\right)}{(-c)^{3/4}d^{9/4}}-\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3b\cdots}{\cdots}\right.}{\cdots}}\right)\cdots}{2(-c)^{3/4}d^{9/4}}$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d}-\frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)1i}{4(-c)^{3/4}d^{9/4}}\right)}{(-c)^{3/4}d^{9/4}}+\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a\cdots}{\cdots}\right.}{\cdots}}{\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d}-\frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)1i}{4(-c)^{3/4}d^{9/4}}\right)1i}{(-c)^{3/4}d^{9/4}}-\frac{(ad-bc)^2\left(\frac{x(a^4d^4-4a\cdots}{\cdots}\right.}{\cdots}}\right)\cdots}{2(-c)^{3/4}d^{9/4}}$$

input
```
int((a + b*x^4)^2/(c + d*x^4),x)
```

output
```
(b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d
 - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/
4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)
^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i
)/((-c)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9
/4)) - ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)
^2*1i)/(2*(-c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^
2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))
/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3
+ 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b
^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*...
```

3.158 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$

3.158.1 Optimal result

Integrand size = 19, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx &= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2(c+dx^4)}+\frac{(bc-ad)(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}\\ &-\frac{(bc-ad)(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}\\ &+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}\\ &-\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}\end{aligned}$$

output
```
b^2*x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)
*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)-1/16*(-a*d+b
*c)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1
/2)+1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^
2*d^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)-1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/4
)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$= \frac{32b^2\sqrt[4]{d}x + \frac{8\sqrt[4]{d}(bc-ad)^2x}{c(c+dx^4)} + \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}}{32d^{9/4}}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

output
```
(32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[
2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1
/4)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d -
3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4)
 - (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

3.158.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$\Big\downarrow \ 915$$
$$\int \left(\frac{b^2}{d^2} - \frac{-a^2d^2+2bdx^4(bc-ad)+b^2c^2}{d^2(c+dx^4)^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{(bc-ad)(3ad+5bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{x(bc-ad)^2}{4cd^2(c+dx^4)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(
9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
- ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

3.158.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.158.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.96 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

method	result
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{4c\,d^2(d\,x^4+c)}+\frac{\sum_{\_R=\operatorname{RootOf}(d\_Z^4+c)}\frac{(3a^2d^2+2abcd-5b^2c^2)\ln(x-\_R)}{\_R^3}}{16d^3c}$
default	$\frac{b^2x}{d^2}+\frac{\frac{(a^2d^2-2abcd+b^2c^2)x}{4c(d\,x^4+c)}+\frac{(3a^2d^2+2abcd-5b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}}{d^2}$

input
```
int((b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^4+c)+1/16/d^3/c*sum
((3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

3.158.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1210, normalized size of antiderivative = 4.16

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*(16*b^2*c*d*x^5 + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c
^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/
(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b
^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)
 - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (-I*c*d^3*x^4 - I*c^2*d^2)*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^
3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(I*c^2*d^2*(-(625*b^8*c^8 -
1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 +
81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (I
*c*d^3*x^4 + I*c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^
6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 -
 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(
-I*c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*
c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*
a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a...
```

3.158.6 Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{4c^2d^2+4cd^3x^4}$$
$$+\text{RootSum}\Big(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+646a^4b^4c^4d^4+1640a^3b^5c$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

output
```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**
3*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7
 - 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*
d**4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c*
*7*d + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a
*b*c*d - 5*b**2*c**2) + x)))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(cd^3x^4+c^2d^2)} + \frac{b^2x}{d^2} - \frac{\frac{2\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(5\,b^2c}{}}{32\,cd^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 -
1/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*
sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1
/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d
^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4
)) - sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$= \frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}$$
$$+ \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(dx^4+c)cd^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*
d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^
3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^
2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^
2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a
^2*d^2*x)/((d*x^4 + c)*c*d^2)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 1254, normalized size of antiderivative = 4.31

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4))
+ (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*
d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3
- 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*
a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*
c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((
x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b
*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2
*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(
16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2
 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b
*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a
*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d +
5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 -
26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d -
 b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) +
(((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12...
```

3.159 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx$

3.159.1 Optimal result

Integrand size = 19, antiderivative size = 349

$$\begin{aligned}\int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx = & -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} \\ & - \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ & + \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ & - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ & + \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}\end{aligned}$$

output
```
-1/8*(-a*d+b*c)*x*(b*x^4+a)/c/d/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+5*b*c)*
x/c^2/d^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(-1+d^(1/
4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d
+5*b^2*c^2)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1
/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2
)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*
c^2)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^
(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$
$$= \frac{\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)^2x}{(c+dx^4)^2} - \frac{8c^{3/4}\sqrt[4]{d}(9b^2c^2-2abcd-7a^2d^2)x}{c+dx^4} - 2\sqrt{2}(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

output
```
((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(
9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 +
 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[
2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/
4)*d^(9/4))
```

3.159.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {930, 910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$
$$\Big\downarrow \text{930}$$
$$\frac{\int \frac{b(5bc+3ad)x^4+a(bc+7ad)}{(dx^4+c)^2}dx}{8cd} - \frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$
$$\Big\downarrow \text{910}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\int\frac{1}{dx^4+c}dx-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$$\Big\downarrow 755$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$$\Big\downarrow 1476$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$$\Big\downarrow 1082$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2}{c}\cdots\right.}{4\cdots}}{8cd}\cdots\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$$\Big\downarrow 217$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

3.159. $\int\frac{(a+bx^4)^2}{(c+dx^4)^3}dx$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{8cd}$$

$$\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{8cd}$$

$$\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+\cdots\right)}{4(c\cdots}}{8cd}$$

$$\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{1103}$$

3.159. $\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{8cd}$$

$$\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

output
```
-1/8*((b*c - a*d)*x*(a + b*x^4))/(c*d*(c + d*x^4)^2) + (-1/4*((2*a*b + (5*
b^2*c)/d - (7*a^2*d)/c)*x)/(c + d*x^4) + ((6*a*b + (5*b^2*c)/d + (21*a^2*d
)/c)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))
) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*
Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(S
qrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/4)/(8*c*d)
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.159. $\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$

3.159.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128c^2d^3}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{c}{d}\right)}\right.\right.}{256c^3d^2}$

input
```
int((b*x^4+a)^2/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d
-5*b^2*c^2)/c/d^2*x)/(d*x^4+c)^2+1/128/c^2/d^3*sum((21*a^2*d^2+6*a*b*c*d+5
*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

3.159.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
-1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - (c^2*d^4*x^8 + 2*c
^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^
3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^1
1*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^
6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^
3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/
(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (-I*c^2*d^4*
x^8 - 2*I*c^3*d^3*x^4 - I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 159
00*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 1769
04*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*
a^8*d^8)/(c^11*d^9))^(1/4)*log(I*c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d
 + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
+ 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x)
 + (I*c^2*d^4*x^8 + 2*I*c^3*d^3*x^4 + I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b
^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*
c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c
*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-I*c^3*d^2*(-(625*b^8*c^8 + 3
000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 11280...
```

3.159.6 Sympy [A] (verification not implemented)

Time = 85.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \frac{x^5\cdot(7a^2d^3+2abcd^2-9b^2c^2d)+x(11a^2cd^2-6abc^2d-5b^2c^3)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8}$$
$$+\mathrm{RootSum}\Big(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7bcd^7+280476a^6b^2c^2d^6+176904a^5b^3c^3d^5+112$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

output
```
(x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6
*a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d*
*4*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*
a**7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 +
112806*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c
**6*d**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c*
*3*d**2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3)x^5+(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2)x}{32\,(c^2d^4x^8+2\,c^3d^3x^4+c^4d^2)} + \frac{\frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(5\,b}{}}{256\,c^2d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
-1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^
2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*
sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)
*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)
*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(
c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*l
og(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) -
sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(
1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \frac{\sqrt{2}\left(5\,(cd^3)^{\frac14}\,b^2c^2+6\,(cd^3)^{\frac14}\,abcd+21\,(cd^3)^{\frac14}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{128\,c^3d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac14}\,b^2c^2+6\,(cd^3)^{\frac14}\,abcd+21\,(cd^3)^{\frac14}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{128\,c^3d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac14}\,b^2c^2+6\,(cd^3)^{\frac14}\,abcd+21\,(cd^3)^{\frac14}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac14}\,b^2c^2+6\,(cd^3)^{\frac14}\,abcd+21\,(cd^3)^{\frac14}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3} - \frac{9\,b^2c^2dx^5-2\,abcd^2x^5-7\,a^2d^3x^5+5\,b^2c^3x+6\,abc^2dx-11\,a^2cd^2x}{32\,(dx^4+c)^2c^2d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*
a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d
)^(1/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 +
 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*
x^5 - 7*a^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^
4 + c)^2*c^2*d^2)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 1401, normalized size of antiderivative = 4.01

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

output
```
- ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
 9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((
((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*
c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2
*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2
 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c
)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 6
0*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6
*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a
*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)
) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*
(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3
 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(
256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)
)))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (a
tan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6
*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^...
```

3.160 $\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$

3.160.1 Optimal result

Integrand size = 19, antiderivative size = 332

$$\begin{aligned}\int \frac{(c+dx^4)^4}{a+bx^4}\,dx =& \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} \\ &+ \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b} \\ &- \frac{(bc-ad)^4\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} \\ &- \frac{(bc-ad)^4\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ &+ \frac{(bc-ad)^4\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}\end{aligned}$$

output
```
d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^13/b+1/4*(-
a*d+b*c)^4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)+1
/4*(-a*d+b*c)^4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/
2)-1/8*(-a*d+b*c)^4*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(
3/4)/b^(17/4)*2^(1/2)+1/8*(-a*d+b*c)^4*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*b^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$= \frac{4680\sqrt[4]{b}d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+936b^{5/4}d^2(6b^2c^2-4abcd+a^2d^2)\,x^5+520b^{9/4}d^3(4bc-ac}{}$$

input
```
Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

output
```
(4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x +
936*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4
*b*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*
d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (58
5*Sqrt[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/a^(3/4))/(4680*b^(17/4))
```

3.160.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d(2bc-ad)\,(a^2d^2-2abcd+2b^2c^2)}{b^4}+\frac{d^2x^4\,(a^2d^2-4abcd+6b^2c^2)}{b^3}+\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d}{b^4\,(a+bx^4)}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

input
```
Int[(c + d*x^4)^4/(a + b*x^4),x]
```

output
```
(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^
2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) +
(d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) +
((b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(3/4)*b^(17/4))
```

3.160.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.160.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.61

method	result
risch	$\frac{d^4x^{13}}{13b} - \frac{d^4x^9a}{9b^2} + \frac{4d^3x^9c}{9b} - \frac{4d^3ac\,x^5}{5b^2} + \frac{6d^2c^2x^5}{5b} + \frac{d^4a^2x^5}{5b^3} - \frac{d^4a^3x}{b^4} + \frac{4d^3a^2cx}{b^3} - \frac{6d^2a\,c^2x}{b^2} + \frac{4d\,c^3x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(}}{}$
default	$-\frac{d\left(-\frac{b^3d^3x^{13}}{13}+\frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^9}{9}+\frac{\left(2(ad-2bc)b^2cd-bd\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^5}{5}+(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4} + \frac{(a^4}{}$

input
```
int((d*x^4+c)^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*d^4*x^13/b-1/9*d^4/b^2*x^9*a+4/9*d^3/b*x^9*c-4/5*d^3/b^2*a*c*x^5+6/5*
d^2/b*c^2*x^5+1/5*d^4/b^3*a^2*x^5-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^
2*a*c^2*x+4*d/b*c^3*x+1/4/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2
-4*a*b^3*c^3*d+b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.160.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 2190, normalized size of antiderivative = 6.60

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*
c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*b^4*(-(b^16*c^16 - 16*a*b^1
5*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c
^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*
c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^
6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^
3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))
^(1/4)*log(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 -
 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 +
 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 -
11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 +
1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 1
6*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 585*I*b^4*(-(b^16*c^16
 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820
*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 114
40*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*
a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*
a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/
(a^3*b^17))^(1/4)*log(I*a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2...
```

3.160.6 Sympy [A] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$
$$= x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right)+x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)$$
$$+\operatorname{RootSum}\Bigg(256t^4a^3b^{17}+a^{16}d^{16}-16a^{15}bcd^{15}+120a^{14}b^2c^2d^{14}-560a^{13}b^3c^3d^{13}+1820a^{12}b^4c^4d^{12}-4368$$
$$+\frac{d^4x^{13}}{13b}$$

input
```
integrate((d*x**4+c)**4/(b*x**4+a),x)
```

output
```
x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*
c*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/
b**3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a
**16*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b*
*3*c**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 +
8008*a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c
**8*d**8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a
**5*b**11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d
**3 + 120*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda
(_t, _t*log(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d*
*2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$
$$= \frac{45\,b^3d^4x^{13} + 65\,(4\,b^3cd^3 - ab^2d^4)x^9 + 117\,(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4)x^5 + 585\,(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a}{585\,b^4}$$
$$+ \frac{\frac{2\sqrt{2}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\,\mathrm{ar}}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2
*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2
 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3
*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
 sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
1/4)))/b^4
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(261) = 522$.

Time = 0.29 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.86

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^4c^4 - 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 + (ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^4c^4 - 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 + (ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\right.}{2\left(\frac{a}{b}\right.}\right.}{4\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^4c^4 - 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 + (ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right.\right.}{8\,ab^5}$$

$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^4c^4 - 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 + (ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right.\right.}{8\,ab^5}$$

$$+ \frac{45\,b^{12}d^4x^{13} + 260\,b^{12}cd^3x^9 - 65\,ab^{11}d^4x^9 + 702\,b^{12}c^2d^2x^5 - 468\,ab^{11}cd^3x^5 + 117\,a^2b^{10}d^4x^5 + 2340\,b^{12}c^3d}{585\,b^{13}}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^
3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5)
+ 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*
b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a
^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5
) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(
a*b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)
*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(
2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*
a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^1
3 + 260*b^12*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*
b^11*c*d^3*x^5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^
2*d^2*x + 2340*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

3.160.9 Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 1822, normalized size of antiderivative = 5.49

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^4/(a + b*x^4),x)
```

output
```
x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b
) - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*
d^3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*
c*d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)
/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^
4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a
)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8
*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^
3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(
-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a
*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^...
```

3.161 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

3.161.1 Optimal result

Integrand size = 19, antiderivative size = 288

$$\begin{aligned}\int \frac{(c+dx^4)^3}{a+bx^4}\,dx &= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b} \\ &\quad -\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} \\ &\quad -\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} \\ &\quad +\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}\end{aligned}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/5*d^2*(-a*d+3*b*c)*x^5/b^2+1/9*d^3
*x^9/b+1/4*(-a*d+b*c)^3*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(13
/4)*2^(1/2)+1/4*(-a*d+b*c)^3*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b
^(13/4)*2^(1/2)-1/8*(-a*d+b*c)^3*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+1/8*(-a*d+b*c)^3*ln(a^(1/4)*b^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$
$$= \frac{360a^{3/4}\sqrt[4]{b}d(3b^2c^2-3abcd+a^2d^2)\,x-72a^{3/4}b^{5/4}d^2(-3bc+ad)x^5+40a^{3/4}b^{9/4}d^3x^9-90\sqrt{2}(bc-ad)^3\,\mathrm{arc}}{}$$

input
```
Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

output
```
(360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^
(5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*
c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d
)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*
d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*
b^(13/4))
```

3.161.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$
$$\Big\downarrow \; 915$$
$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3(a+bx^4)}+\frac{d^2x^4(3bc-ad)}{b^2}+\frac{d^3x^8}{b}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

input
```
Int[(c + d*x^4)^3/(a + b*x^4),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b
^2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)
) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(4*Sqrt[2]*a^(3/4)*b^(13/4))
```

3.161.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.161.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.45

method	result
risch	$\frac{d^3x^9}{9b}-\frac{d^3a\,x^5}{5b^2}+\frac{3d^2c\,x^5}{5b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^4}$
default	$\frac{d\left(\frac{1}{9}b^2d^2x^9-\frac{1}{5}ab\,d^2x^5+\frac{3}{5}b^2cd\,x^5+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}\right.\right.}{8b^3a}$

input
```
int((d*x^4+c)^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^3*x^9/b-1/5*d^3/b^2*a*x^5+3/5*d^2/b*c*x^5+d^3/b^3*a^2*x-3*d^2/b^2*a*
c*x+3*d/b*c^2*x+1/4/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.161.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1642, normalized size of antiderivative = 5.70

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 45*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d
^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d +
 3*a^2*b*c*d^2 - a^3*d^3)*x) - 45*I*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a^3*b^13))^(1/4)*log(I*a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x) + 45*I*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*...
```

3.161.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-220a^9b^3c^3d^9+495a^8b^4c^4d^8-792a^7b^5c^5d\right.$$
$$+\frac{d^3x^9}{9b}$$

input
```
integrate((d*x**4+c)**3/(b*x**4+a),x)
```

output
```
x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/
b**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11
*b*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8
*b**4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*
a**5*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 +
66*a**2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t
*log(-4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c*
*3) + x))) + d**3*x**9/(9*b)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \frac{5\,b^2d^3x^9+9\,(3\,b^2cd^2-abd^3)x^5+45\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{45\,b^3} + \frac{\frac{2\,\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{8}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt
(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sq
rt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(219) = 438.

Time = 0.29 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$+ \frac{5\,b^8d^3x^9 + 27\,b^8cd^2x^5 - 9\,ab^7d^3x^5 + 135\,b^8c^2dx - 135\,ab^7cd^2x + 45\,a^2b^6d^3x}{45\,b^9}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c
^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a*b^4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 -
3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*
a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^
8*d^3*x^9 + 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b
^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^9
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1433, normalized size of antiderivative = 4.98

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^3/(a + b*x^4),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
 - (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2
*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(
(-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a
^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3
+ ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*
d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((
((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*
d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(1
3/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*
b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)
^(3/4)*b^(13/4))))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x
*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3
 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b...
```

3.162 $\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$

3.162.1 Optimal result

Integrand size = 19, antiderivative size = 253

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/5*d^2*x^5/b+1/4*(-a*d+b*c)^2*arctan(-1+b^(1/4)*x*2^
(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+b^(1/4)*x
*2^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(-a*d+b*c)
^2*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/
2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{-40a^{3/4}\sqrt[4]{b}d(-2bc+ad)x+8a^{3/4}b^{5/4}d^2x^5-10\sqrt{2}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}(bc-ad)^2\text{arc}}{}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

output
```
(-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*S
qrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(
b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a
^(3/4)*b^(9/4))
```

3.162.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^4)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

3.162.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.162.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

method	result
risch	$\frac{d^2x^5}{5b} - \frac{d^2ax}{b^2} + \frac{2dcx}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)} \frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^3}$
default	$-\frac{d\left(-\frac{1}{5}bd\,x^5+adx-2bcx\right)}{b^2} + \frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b^2a}$

input
```
int((d*x^4+c)^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*c*x+1/4/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.162.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.32

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$$
$$= \frac{4\,bd^2x^5 + 5\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\right.\right.}{}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/20*(4*b*d^2*x^5 + 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*b^2*(-(b^8*c^
8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(a^3*b^9))^(1/4)*log(I*a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*x) - 5*I*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-I*a*b^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^...
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3b^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6\right.$$
$$+\frac{d^2x^5}{5b}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a
**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b
**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7
*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d
+ b**2*c**2) + x))) + d**2*x**5/(5*b)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{bd^2x^5+5\,(2\,bcd-ad^2)x}{5\,b^2} + \frac{\frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(b^2c^2-2\,abc}{}}{8\,b^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(
1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4
)))/b^2
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2-2\,(ab^3)^{\frac{1}{4}}abcd+(ab^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2-2\,(ab^3)^{\frac{1}{4}}abcd+(ab^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2-2\,(ab^3)^{\frac{1}{4}}abcd+(ab^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2-2\,(ab^3)^{\frac{1}{4}}abcd+(ab^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} + \frac{b^4d^2x^5+10\,b^4cdx-5\,ab^3d^2x}{5\,b^5}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*
b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c
*d + (a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a
*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
+ 1/5*(b^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

3.162.9 Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.27

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{d^2\,x^5}{5\,b} - x\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right)$$

$$+ \frac{\operatorname{atan}\left(\dfrac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{(-a)^{3/4}\,b^{9/4}} + \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\right.}{}\right.}{}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}} - \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\right.}{}\right.}{}}\right)}{2\,(-a)^{3/4}\,b^{9/4}}$$

$$+ \frac{\operatorname{atan}\left(\dfrac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)1\mathrm{i}}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}} + \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a\right.}{}\right.}{}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)1\mathrm{i}}{4\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{(-a)^{3/4}\,b^{9/4}} - \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a\right.}{}\right.}{}}\right)}{2\,(-a)^{3/4}\,b^{9/4}}$$

input
```
int((c + d*x^4)^2/(a + b*x^4),x)
```

output
```
(d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b
 - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/
4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)
^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i
)/((-a)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9
/4)) - ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)
^2*1i)/(2*(-a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^
2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))
/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2
+ 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b
^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^...
```

3.163 $\int \frac{c+dx^4}{a+bx^4} dx$

3.163.1 Optimal result

Integrand size = 17, antiderivative size = 223

$$\int \frac{c+dx^4}{a+bx^4} dx = \frac{dx}{b} - \frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
d*x/b+1/4*(-a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*
2^(1/2)+1/4*(-a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)
*2^(1/2)-1/8*(-a*d+b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
/a^(3/4)/b^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{a+bx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{b}dx - 2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}(bc-ad)\,\mathrm{lo}}{8a^{3/4}b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4),x]
```

output
```
(8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(8*a^(3/4)*b^(5/4))
```

3.163.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{a+bx^4}\,dx$$
$$\Big\downarrow \; 913$$
$$\frac{(bc-ad)\int \frac{1}{bx^4+a}dx}{b}+\frac{dx}{b}$$
$$\Big\downarrow \; 755$$
$$\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$
$$\Big\downarrow \; 1476$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\Big\downarrow$ 1082

$$\frac{(bc-ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\Big\downarrow$ 217

$$\frac{(bc-ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\Big\downarrow$ 1479

$$\frac{(bc-ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\Big\downarrow$ 25

3.163. $\int \frac{c+dx^4}{a+bx^4}\,dx$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\big\downarrow$ 27

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$\big\downarrow$ 1103

$$\frac{(bc-ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

input
```
Int[(c + d*x^4)/(a + b*x^4),x]
```

output
```
(d*x)/b + ((b*c - a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2
]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b
```

3.163. $\int \frac{c+dx^4}{a+bx^4}\,dx$

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.163.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.94 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$	42
default	$\frac{dx}{b}+\frac{(-ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ba}$	120

input
```
int((d*x^4+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+1/4/b^2*sum((-a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.163.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.51

$$\int \frac{c+dx^4}{a+bx^4}\,dx =$$

$$-\frac{b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)+}{}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) + I*b*(
-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
a^3*b^5))^(1/4)*log(I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - I*b*(-(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5
))^(1/4)*log(-I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - b*(-(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)
*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{c+dx^4}{a+bx^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3b^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(-\frac{4tab}{ad-bc}+x\right)\right)\right)$$
$$+\frac{dx}{b}$$

input
```
integrate((d*x**4+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c*
*2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d -
b*c) + x))) + d*x/b
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{a+bx^4}\,dx=\frac{dx}{b}$$
$$+\frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(bc-ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8b}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*(b*c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,bc - (ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,bc - (ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,bc - (ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,bc - (ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^
3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/
4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a*b^2)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{d\,x}{b} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}+\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-b}{4(-a)^{3/4}b^{5/4}}\right.}{4(-a)^{3/4}b^{5/4}}}{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}-\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-b}{4(-a)^{3/4}b^{5/4}}\right.}{4(-a)^{3/4}b^{5/4}}}\right.}{2(-a)^{3/4}b^{5/4}} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}+\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-}{4(-a)^{3/4}b^{5/4}}\right.}{4(-a)^{3/4}b^{5/4}}}{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}-\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-}{4(-a)^{3/4}b^{5/4}}\right.}{4(-a)^{3/4}b^{5/4}}}\right.}{2(-a)^{3/4}b^{5/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4),x)
```

output
```
(d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^
2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*
(-a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i
)/(4*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - (
(16*a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*
c))/(4*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) +
 ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d -
b*c))/(4*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (a
tan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b
^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b
^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 1
6*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3
/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*
d - 16*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*
a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c
)*1i)/(4*(-a)^(3/4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

3.164 $\int \frac{1}{(a+bx^4)(c+dx^4)} dx$

3.164.1 Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)} dx = & -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} \\ & + \frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} \\ & - \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} \\ & + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} \\ & + \frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} \\ & - \frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\end{aligned}$$

output
```
1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2
)+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/
2)-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(
1/2)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^
(1/2)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/
4)/(-a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(
1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4
)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a}{}$$

input
```
Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^
(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

3.164.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {917, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{917}$$

$$\frac{b\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{755}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1479$$

3.164. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 27}$$

3.164. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

3.164. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

output
```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.164.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a}+\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}c^{3}d^{4}-4c^{4}d^{3}a^{3}b+6c^{5}d^{2}a^{2}b^{2}-4c^{6}da\,b^{3}+b^{4}c^{7}\right)\_Z^{4}+d^{3}\right)}\_R\ln\left(\left(\left(-a^{7}d^{7}+4a^{6}bc\,d^{6}-6a^{5}b^{2}c^{2}d^{5}+3a^{4}b^{3}c^{3}d^{4}+3a^{3}b^{4}c^{4}\right.\right.\right.\right.$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/(a*d-b*c)*(c/d)^(1/4)/c
*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/
d)^(1/4)*x-1))
```

3.164.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.61

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*
b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^
7*d^4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4
))^(1/4)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*
c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^
(1/4)*log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2
*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*l
og(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^
2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*
x - (I*b*c^2 - I*a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*...
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\sqrt{\cdots}\right.}{a^{\frac{3}{4}}}}{8(bc-ad)}$$
$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x\cdots\right.}{\cdots}}{8(bc-ad)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*s
qrt(2)*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sq
rt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d)
- 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d
) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2
)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^
2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1
/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
^2 - sqrt(2)*a*c*d)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 6153, normalized size of antiderivative = 13.70

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*
a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c
^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3
/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*
b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4
*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2
*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 -
4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a
^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6
+ 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 10
24*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1
024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3
/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^
2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 -
20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 2
0480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x
*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^...
```

3.164. $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

3.165 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

3.165.1 Optimal result

Integrand size = 19, antiderivative size = 513

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = &-\frac{dx}{4c(bc-ad)(c+dx^4)}-\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}\\
&+\frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}\\
&+\frac{d^{3/4}(7bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\\
&-\frac{d^{3/4}(7bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\\
&-\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}\\
&+\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}\\
&+\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}\\
&-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)
/a^(1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctan(
-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-
3*a*d+7*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1
/2)-1/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)
/(-a*d+b*c)^2*2^(1/2)+1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/32*d^(3/4)*(-3*a*d+7*b*c)*ln(-c^(
1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1
/32*d^(3/4)*(-3*a*d+7*b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2
))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$$

$$= \frac{8a^{3/4}c^{3/4}d(-bc+ad)x - 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1+\right.}{}$$

input
```
Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

output
```
(8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x
^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c
 + d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3
/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] +
 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c
+ d*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*
a^(3/4)*d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d
*x^4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*
c^(7/4)*(b*c - a*d)^2*(c + d*x^4))
```

3.165.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {931, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^4+4bc-3ad}{(bx^4+a)(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{4b^2c\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d(7bc-3ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 755$$

$$\frac{\frac{4b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{4b^2c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\int \frac{\sqrt{c}\ldots}{\ldots}\right)}{bc-ad}}{4c(bc-ad)}$$

$$\frac{dx}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{4b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{\cdots}{-\left(1\cdots\right.}\cdots}{\cdots}\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 217

$$\frac{\frac{4b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\cdots\right.}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 1479

$$\frac{\frac{4b^2c\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{-\frac{\int-\cdots}{\cdots}\cdots}{\cdots}\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 25

3.165. $\int\frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

$$\frac{\frac{4b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\int \frac{\sqrt{\cdots}}{\sqrt[4]{d}\left(\cdots\right)}\cdots\right)}{\cdots}}{4c(bc-ad)}\cdots\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\cdots}{2\sqrt{\cdots}}\cdots\right)}{\cdots}}{4c(bc-ad)}\cdots\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{4b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\arctan\left(\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots}{\cdots}\right)}{\cdots}}{4c(bc-ad)}\cdots\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

3.165. $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b^2*c*((-(ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(7*b*c - 3*a*d)*((-(ArcTan[1 - (Sqr
t[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*
d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4))
+ Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4
)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*c*(b*c - a*d))
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.165.4 Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.51

method	result
default	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)^2a}+\frac{d\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\frac{(3ad-7bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+d/(a*d-b*c)^2*(1/4*(a*d-b*
c)/c*x/(d*x^4+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)
^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*a
rctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)))
```

3.165.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.02 (sec) , antiderivative size = 2955, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+bx^4)\,(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6
*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^
6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a
*c^2*d)*log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*
d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^
9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d
 + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2
 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b
^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 +
b*c^3 - a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5
*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^
5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*
a^2*b*c*d + a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^
6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 +
 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(-I*(b*c^2*d - a*c
*d^2)*x^4 - I*b*c^3 + I*a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^
7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 5
6*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)
*(I*a*b^2*c^2 - 2*I*a^2*b*c*d + I*a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4
*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d...
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

output
```
Timed out
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = -\frac{dx}{4\left((bc^2d-acd^2)x^4+bc^3-ac^2d\right)} + \frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}\right.}{}}{8\left(b^2c^2-2\,abcd+a^2d^2\right)} - \frac{\frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(7\,bcd-3\,ad^2)\log\left(\sqrt{d}x^2+\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/3
2*(2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)
*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c
^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)
```

3.165. $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

3.165.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} + \frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} + \frac{(ab^3)^{\frac{1}{4}}\, b \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{(ab^3)^{\frac{1}{4}}\, b \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} + \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{dx}{4\,(dx^4+c)(bc^2-acd)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a
*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^
(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 -
2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d +
 sqrt(2)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2
*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(
c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/
4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(
7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4)
 + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2
) + 1/16*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a
^2*c^2*d^2) - 1/4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 21975, normalized size of antiderivative = 42.84

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

output

```
2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288
*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458
7520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6
 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65
536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 367001
6*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 +
 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10
)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16
 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2
 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
+ 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*
d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^
12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6
*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*
d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*
b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13
*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28...
```

3.165. $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

3.166 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx$

3.166.1 Optimal result

Integrand size = 19, antiderivative size = 407

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx = \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4} + \frac{d^4(5bc-2ad)x^9}{9b^3} + \frac{d^5x^{13}}{13b^2} + \frac{(bc-ad)^5x}{4ab^5(a+bx^4)} - \frac{(bc-ad)^4(3bc+17ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} + \frac{(bc-ad)^4(3bc+17ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} - \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} + \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}$$

output
```
d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/5*d^3*(3
*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^5/b^4+1/9*d^4*(-2*a*d+5*b*c)*x^9/b^3+1/1
3*d^5*x^13/b^2+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^4+a)+1/16*(-a*d+b*c)^4*(17*a*
d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21/4)*2^(1/2)+1/1
6*(-a*d+b*c)^4*(17*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/
b^(21/4)*2^(1/2)-1/32*(-a*d+b*c)^4*(17*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1/2)+1/32*(-a*d+b*c)^4*(17*
a*d+3*b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21
/4)*2^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$$

$$= \frac{18720\sqrt[4]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+3744b^{5/4}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5+2080b^9}{}$$

input
```
Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

output
```
(18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d
^3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*
b^(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(
b*c - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*
a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c
 - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4
) - (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c
+ 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))
/(18720*b^(21/4))
```

3.166.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{b^4}+\frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{5bdx^4(bc-ad)^4+(4ad+}{b^5(a+bx^4)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{d^3x^5(3a^2d^2-10abcd+10b^2c^2)}{5b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{4ab^5(a+bx^4)}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{d^5x^{13}}{13b^2}$$

input
```
Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a
*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b
*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c
 - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(21/4))
```

3.166.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.166.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.75

method	result
risch	$\frac{d^5x^{13}}{13b^2}-\frac{2d^5a\,x^9}{9b^3}+\frac{5d^4c\,x^9}{9b^2}+\frac{3d^5a^2x^5}{5b^4}-\frac{2d^4ac\,x^5}{b^3}+\frac{2d^3c^2x^5}{b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5}{}$
default	$-\frac{d^2\left(-\frac{1}{13}b^3d^3x^{13}+\frac{2}{9}a\,b^2d^3x^9-\frac{5}{9}b^3c\,d^2x^9-\frac{3}{5}a^2b\,d^3x^5+2a\,b^2c\,d^2x^5-2b^3c^2d\,x^5+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+$

input
```
int((d*x^4+c)^5/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*d^5*x^13/b^2-2/9*d^5/b^3*a*x^9+5/9*d^4/b^2*c*x^9+3/5*d^5/b^4*a^2*x^5-
2*d^4/b^3*a*c*x^5+2*d^3/b^2*c^2*x^5-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-20*
d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/4*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2
*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^4+a)+1/16/b^6/
a*sum((17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d^3-50*a^2*b^3*c^3*d^2+5*a
*b^4*c^4*d+3*b^5*c^5)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.166.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 2884, normalized size of antiderivative = 7.09

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 2
08*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a
*b^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 +
 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*
b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264
*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7
 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*
b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12
 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a
^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^1
7 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(
a^7*b^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^
2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 132
64*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d
^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^1
0*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^
12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144
*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d
^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)
/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + ...
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \frac{(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)x}{4\,(ab^6x^4+a^2b^5)}$$
$$+\frac{45\,b^3d^5x^{13}+65\,(5\,b^3cd^4-2\,ab^2d^5)x^9+117\,(10\,b^3c^2d^3-10\,ab^2cd^4+3\,a^2bd^5)x^5+585\,(10\,b^3c^3d^2-20\,ab}{585\,b^5}$$
$$+\frac{\frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3}{}}{}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 +
 65*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4
 + 3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*
d^4 - 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d -
 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d
 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
 sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*
d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^5)
```

3.166.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(334) = 668$.

Time = 0.29 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.96

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$$

$$= \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^5c^5 + 5\,(ab^3)^{\frac14}\,ab^4c^4d - 50\,(ab^3)^{\frac14}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac14}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac14}\,a^4bcd^4 + 17\,(ab^3)}{16\,a^2b^6}$$

$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^5c^5 + 5\,(ab^3)^{\frac14}\,ab^4c^4d - 50\,(ab^3)^{\frac14}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac14}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac14}\,a^4bcd^4 + 17\,(a}{16\,a^2b^6}$$

$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^5c^5 + 5\,(ab^3)^{\frac14}\,ab^4c^4d - 50\,(ab^3)^{\frac14}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac14}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac14}\,a^4bcd^4 + 17\,(a}{32\,a^2b^6}$$

$$- \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^5c^5 + 5\,(ab^3)^{\frac14}\,ab^4c^4d - 50\,(ab^3)^{\frac14}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac14}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac14}\,a^4bcd^4 + 17\,(a}{32\,a^2b^6}$$

$$+ \frac{b^5c^5x - 5\,ab^4c^4dx + 10\,a^2b^3c^3d^2x - 10\,a^3b^2c^2d^3x + 5\,a^4bcd^4x - a^5d^5x}{4\,(bx^4+a)ab^5}$$

$$+ \frac{45\,b^{24}d^5x^{13} + 325\,b^{24}cd^4x^9 - 130\,ab^{23}d^5x^9 + 1170\,b^{24}c^2d^3x^5 - 1170\,ab^{23}cd^4x^5 + 351\,a^2b^{22}d^5x^5 + 5850\,b^{2}}{585\,b^{26}}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(
a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^
3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/
4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^
2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(
a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4
)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^
2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3
)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c
^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 +
 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^2*b^6) + 1/4*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3
*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*
(45*b^24*d^5*x^13 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^
2*d^3*x^5 - 1170*a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d
^2*x - 11700*a*b^23*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*
x)/b^26
```

3.166.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 2490, normalized size of antiderivative = 6.12

$$\int \frac{\left(c+dx^4\right)^5}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

output
```
x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2
*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b
^2) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b
^3 - (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x
^13)/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^
2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((
((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^
3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 -
13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b
*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5
*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*
c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-
a)^(7/4)*b^(21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d
^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14
770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*
b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*
c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a
*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a
*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^
10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - ...
```

3.166. $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

3.167 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

3.167.1 Optimal result

Integrand size = 19, antiderivative size = 357

$$\begin{aligned}\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx &= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}\\ &\quad+\frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)}-\frac{(bc-ad)^3(3bc+13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}\\ &\quad+\frac{(bc-ad)^3(3bc+13ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}\\ &\quad-\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}\\ &\quad+\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}\end{aligned}$$

output
```
d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9
*d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+
3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)+1/16*
(-a*d+b*c)^3*(13*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^
(17/4)*2^(1/2)-1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+1/32*(-a*d+b*c)^3*(13*a*
d+3*b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4
)*2^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$= \frac{1440\sqrt[4]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+576b^{5/4}d^3(2bc-ad)x^5+160b^{9/4}d^4x^9+\frac{360\sqrt[4]{b}(bc-ad)^4x}{a(a+bx^4)}+\frac{90\sqrt{2}(-bc+a}{}}{}$$

input
```
Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

output
```
(1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*
(2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a
*(a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c +
13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(
b*c) + a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

3.167.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{ 915}$$

$$\int \left(\frac{d^2(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{4bdx^4(bc-ad)^3+(bc-ad)^3(3ad+bc)}{b^4(a+bx^4)^2}+\frac{2d^3x^4(2bc-ad)}{b^3}+\frac{d^4x^8}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{4ab^4(a+bx^4)}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{d^4x^9}{9b^2}$$

input
```
Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

output
```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5
)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) -
((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3
*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
(16*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4)
)
```

3.167.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.167.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.61

method	result
risch	$\frac{d^4x^9}{9b^2} - \frac{2d^4a\,x^5}{5b^3} + \frac{4d^3c\,x^5}{5b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{4a\,b^4(b\,x^4+a)} - \frac{\sum_{\_R=\text{RootO}}}{}$
default	$\frac{d^2\left(\frac{1}{9}b^2d^2x^9-\frac{2}{5}ab\,d^2x^5+\frac{4}{5}b^2cd\,x^5+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{4a\left(b\,x^4+a\right)}+\frac{\left(13a^4d^4-36a^3bc\,d^3\right.}{}}{}$

input
```
int((d*x^4+c)^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9/b^2-2/5*d^4/b^3*a*x^5+4/5*d^3/b^2*c*x^5+3*d^4/b^4*a^2*x-8*d^3/
b^3*a*c*x+6*d^2/b^2*c^2*x+1/4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/a*x/b^4/(b*x^4+a)-1/16/b^5/a*sum((13*a^4*d^4-36*a^3*b*
c*d^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(
_Z^4*b+a))
```

3.167.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 2315, normalized size of antiderivative = 6.48

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*
(30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 45*(a*b^5*x^4 +
 a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8
304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5
- 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d
^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b
^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577
784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17)
)^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^
14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11
*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486
160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d
^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/
(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a
^3*b*c*d^3 - 13*a^4*d^4)*x) - 45*(I*a*b^5*x^4 + I*a^2*b^4)*(-(81*b^16*c^16
 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 4
5724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^
6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*
d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*...
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c + dx^4)^4}{(a + bx^4)^2}\, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{4\,(ab^5x^4 + a^2b^4)} + \frac{5\,b^2d^4x^9 + 18\,(2\,b^2cd^3 - abd^4)x^5 + 45\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{45\,b^4} + \frac{\frac{2\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3}{\sqrt{a}\sqrt{\sqrt{}}}}{}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^
4)*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqr
t(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13
*a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 +
 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30
*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*
b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(286) = 572$.

Time = 0.29 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$
$$= \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4 + 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d - 30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 + 36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 - 13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\frac{\sqrt{2}(}{}\right.}{16\,a^2b^5}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4 + 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d - 30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 + 36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 - 13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\right.}{16\,a^2b^5}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4 + 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d - 30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 + 36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 - 13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2 + \right.}{32\,a^2b^5}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4 + 4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d - 30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2 + 36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3 - 13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2 - \right.}{32\,a^2b^5}$$
$$+ \frac{b^4c^4x - 4\,ab^3c^3dx + 6\,a^2b^2c^2d^2x - 4\,a^3bcd^3x + a^4d^4x}{4\,(bx^4+a)ab^4}$$
$$+ \frac{5\,b^{16}d^4x^9 + 36\,b^{16}cd^3x^5 - 18\,ab^{15}d^4x^5 + 270\,b^{16}c^2d^2x - 360\,ab^{15}cd^3x + 135\,a^2b^{14}d^4x}{45\,b^{18}}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(
a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(
1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*
c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 -
13*(a*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^
(1/4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^
3*b*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/
4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b
*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a^2*b^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x -
 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 +
 36*b^16*c*d^3*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c
*d^3*x + 135*a^2*b^14*d^4*x)/b^18
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2043, normalized size of antiderivative = 5.72

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

output
```
x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b
^2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x
*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/
(4*a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b
^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*
d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5
) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c
^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d -
b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 +
 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d
^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^
7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*
a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7
/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4))
)/((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^
3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 2
4*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3
*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^
3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(
-a)^(7/4)*b^(17/4)) - (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d...
```

3.167. $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

3.168 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$

3.168.1 Optimal result

Integrand size = 19, antiderivative size = 317

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3(a+bx^4)} - \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a
)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/32*(-a*d+b*c)^2*(3*a*d+b*c)*ln(-a^(
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+3/32*
(-a*d+b*c)^2*(3*a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
/a^(7/4)/b^(13/4)*2^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$

$$= \frac{160\sqrt[4]{b}d^2(3bc-2ad)x+32b^{5/4}d^3x^5+\frac{40\sqrt[4]{b}(bc-ad)^3x}{a(a+bx^4)}-\frac{30\sqrt{2}(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{30\sqrt{2}(bc-ad)^2(bc\cdots}{\cdots}}{1\cdots}$$

input
```
Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

output
```
(160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
 - a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*
(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[
2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

3.168.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^4(bc-ad)^2+(bc-ad)^2(2ad+bc)}{b^3(a+bx^4)^2}+\frac{d^3x^4}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3(a+bx^4)}+\frac{d^3x^5}{5b^2}$$

input
```
Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b
^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
- (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^
(7/4)*b^(13/4))
```

3.168.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.168.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.48

method	result
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4a\,b^3(b\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\frac{(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3)\ln(\cdots}{\_R^3}\cdots\right.}{16b^4a}$
default	$-\frac{d^2\left(-\frac{1}{5}bd\,x^5+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{4a\left(b\,x^4+a\right)}+\frac{3\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}\right.\right.}{32a^2}\cdots}{b^3}$

input
```
int((d*x^4+c)^3/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*d^3*x^5/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^4+a)+3/16/b^4/a*sum((3*a^3*d^3-5*a^2*b*c*
d^2+a*b^2*c^2*d+b^3*c^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.168.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1741, normalized size of antiderivative = 5.49

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 15*(a*b^4*
x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*
a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*
c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^
9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))
^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2
 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^
6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*
c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*
b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) -
15*(-I*a*b^4*x^4 - I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932
*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^
12)/(a^7*b^13))^(1/4)*log(3*I*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*
a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7
*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^
8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81
*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 +
 3*a^3*d^3)*x) - 15*(I*a*b^4*x^4 + I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c...
```

3.168.6 Sympy [A] (verification not implemented)

Time = 86.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{4a^2b^3+4ab^4x^4}$$
$$+\mathrm{RootSum}\Big(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}-156492a^9b^3c^3d^9+84159a$$
$$+\frac{d^3x^5}{5b^2}$$

input
```
integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t
**4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b
**2*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 +
26568*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c*
*7*d**5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2
*b**10*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*lo
g(16*_t*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b*
*3*c**3) + x))) + d**3*x**5/(5*b**2)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{4\,(ab^4x^4+a^2b^3)}+\frac{bd^3x^5+5\,(3\,bcd^2-2\,ad^3)x}{5\,b^3}$$
$$+\frac{3\left(\frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}\cdots\right.}{2\sqrt{\sqrt{a}\sqrt{\cdots}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right.}{\cdots}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2
*b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*
(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a
^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2
*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^
2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(1/4)))/(a*b^3)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$
$$= \frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5(ab^3)^{\frac14}a^2bcd^2+3(ab^3)^{\frac14}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^4}$$
$$+\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5(ab^3)^{\frac14}a^2bcd^2+3(ab^3)^{\frac14}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^4}$$
$$+\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5(ab^3)^{\frac14}a^2bcd^2+3(ab^3)^{\frac14}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}$$
$$-\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5(ab^3)^{\frac14}a^2bcd^2+3(ab^3)^{\frac14}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}$$
$$+\frac{b^3c^3x-3ab^2c^2dx+3a^2bcd^2x-a^3d^3x}{4(bx^4+a)ab^3}+\frac{b^8d^3x^5+15b^8cd^2x-10ab^7d^3x}{5b^{10}}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3
)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*
c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b
^3)^(1/4)*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
+ 1/4*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4
+ a)*a*b^3) + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

3.168.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 1616, normalized size of antiderivative = 5.10

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^3/(a + b*x^4)^2,x)
```

output
```
(d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a
*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 -
 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))
/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 1
2*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7
/4)*b^(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 -
9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d
 - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d
^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4
)))*3i)/(16*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9
*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^
2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3
*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(
16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a
*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d
^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (
3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d -
60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*
d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d...
```

3.169 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$

3.169.1 Optimal result

Integrand size = 19, antiderivative size = 291

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}-\frac{(bc-ad)(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}$$
$$+\frac{(bc-ad)(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}$$
$$-\frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$
$$+\frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$

output
```
d^2*x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)
*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b
*c)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1
/2)-1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*b^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(a^(1/4
)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$=\frac{32\sqrt[4]{b}d^2x+\frac{8\sqrt[4]{b}(bc-ad)^2x}{a(a+bx^4)}+\frac{2\sqrt{2}(-3b^2c^2-2abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt{2}(3b^2c^2+2abcd-5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}}{32b^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

output
```
(32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[
2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/
4) + (Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

3.169.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{915}$$
$$\int \left(\frac{-a^2d^2+2bdx^4(bc-ad)+b^2c^2}{b^2(a+bx^4)^2}+\frac{d^2}{b^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2(a+bx^4)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*
c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(
9/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
+ ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

3.169.3.1 Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.169.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

method	result
risch	$\frac{d^2 x}{b^2}+\frac{(a^2 d^2-2abcd+b^2 c^2) x}{4a\, b^2 (b\, x^4+a)}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\, \_Z^4+a\right)} \frac{\left(5a^2 d^2-2abcd-3b^2 c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16 b^3 a}$
default	$\frac{d^2 x}{b^2}-\frac{-\frac{\left(a^2 d^2-2abcd+b^2 c^2\right) x}{4a\left(b\, x^4+a\right)}+\frac{\left(5a^2 d^2-2abcd-3b^2 c^2\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right.}{32a^2}}{b^2}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^4+a)-1/16/b^3/a*sum
((5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.169.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1210, normalized size of antiderivative = 4.16

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^2}\, dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*a*b*d^2*x^5 - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7
*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/
(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6
*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)
 - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (I*a*b^3*x^4 + I*a^2*b^2)*(-(8
1*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(I*a^2*b^2*(-(81*b^8*c^8 + 21
6*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^
4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (-I
*a*b^3*x^4 - I*a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*
d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 9
00*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(
-I*a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3
*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^
2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*
a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b...
```

3.169.6 Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{4a^2b^2+4ab^3x^4}$$
$$+\text{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b^4c^4d^4-984a^3\right.$$
$$+\frac{d^2x}{b^2}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + Root
Sum(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*
b**2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*
a**3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**
8*c**8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b*
*2*c**2) + x))) + d**2*x/b**2
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(ab^3x^4+a^2b^2)}+\frac{d^2x}{b^2} + \frac{\frac{2\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3\,b^2c^{2}\cdots}{}}{32\,ab^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 +
1/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d
^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4
)) - sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$= \frac{d^2x}{b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$+ \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(bx^4+a)ab^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*
d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^
3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a
^2*d^2*x)/((b*x^4 + a)*a*b^2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1254, normalized size of antiderivative = 4.31

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4))
+ (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2
*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*
(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(
25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*
d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2
+ 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a
*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2
 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b
*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(
16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*
b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b)
 + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))
/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d +
3*b*c)*1i)/(8*(-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((
x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b
*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d
^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) +
((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c...
```

3.170 $\int \frac{c+dx^4}{(a+bx^4)^2} dx$

3.170.1 Optimal result

Integrand size = 17, antiderivative size = 245

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{(bc-ad)x}{4ab(a+bx^4)} - \frac{(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
1/4*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2
)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(
1/2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-1/32*(a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*(a*d+3*b*c)*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}\sqrt[4]{b}(-bc+ad)x}{a+bx^4} - 2\sqrt{2}(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}(3}{32a^{7/4}b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

3.170.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \; 910$$
$$\frac{(ad+3bc)\int \frac{1}{bx^4+a}dx}{4ab} + \frac{x(bc-ad)}{4ab\,(a+bx^4)}$$
$$\Big\downarrow \; 755$$
$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab} + \frac{x(bc-ad)}{4ab\,(a+bx^4)}$$
$$\Big\downarrow \; 1476$$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{(ad+3bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 25$$

3.170. $\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

3.170. $\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$

output
```
((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) + ((3*b*c + a*d)*((-(ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a])))/(4*a*b)
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.170.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.27

method	result	size
risch	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(ad+3bc)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$	65
default	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{(ad+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2b}$	140

input
```
int((d*x^4+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a*d-b*c)/b/a*x/(b*x^4+a)+1/16/a/b^2*sum((a*d+3*b*c)/_R^3*ln(x-_R),_R
=RootOf(_Z^4*b+a))
```

3.170.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.64

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{(ab^2x^4+a^2b)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4}{a^7b^5}\right.\right.}{}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*((a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4
+ 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^
5))^(1/4) + (3*b*c + a*d)*x) - (-I*a*b^2*x^4 - I*a^2*b)*(-(81*b^4*c^4 + 10
8*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^
(1/4)*log(I*a^2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) - (I*a*b^2*x^
4 + I*a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-I*a^2*b*(-(81*b^4*c^4 + 108*a*b^
3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)
+ (3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*
b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4
 + a^2*b)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{x(-ad+bc)}{4a^2b+4ab^2x^4} + \operatorname{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4, \left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}+\right.\right.\right.$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

output
```
x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5
+ a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*
d + 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx = \frac{(bc-ad)x}{4\left(ab^2x^4+a^2b\right)} + \frac{\frac{2\sqrt{2}(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3\,bc+ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{\ldots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)
*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))
/(a*b)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} + \frac{bcx-adx}{4\,(bx^4+a)ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^
3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3
)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32
*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.02

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}+\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)}{16(-a)^{7/4}b^{5/4}}}{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)1i}{16(-a)^{7/4}b^{5/4}}-\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)}{16(-a)^{7/4}b^{5/4}}}\right)}{8(-a)^{7/4}b^{5/4}}$$
$$-\frac{x(ad-bc)}{4ab(bx^4+a)}$$
$$+\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)1i}{16(-a)^{7/4}b^{5/4}}+\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(}{16(-a)^{7/4}b^{5/4}}}{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}-\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(}{16(-a)^{7/4}b^{5/4}}}\right)}{8(-a)^{7/4}b^{5/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^2,x)
```

output
```
(atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c
)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(
-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) +
 ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*
b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/
4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2
+ 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^
(7/4)*b^(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)
/(8*(-a)^(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d
))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5
/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(
-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c
^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)
) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(
12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(
-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c
))/(4*a*b*(a + b*x^4))
```

3.170. $\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$

3.171 $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

3.171.1 Optimal result

Integrand size = 19, antiderivative size = 513

$$\begin{aligned}\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx &= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} \\
&\quad + \frac{b^{3/4}(3bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} \\
&\quad - \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} \\
&\quad - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&\quad + \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&\quad - \frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} \\
&\quad + \frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}\end{aligned}$$

output
```
1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctan(-1+b^(1/
4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/16*b^(3/4)*(-7*a*d+3*
b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/4*
d^(7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)+
1/4*d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/
2)-1/32*b^(3/4)*(-7*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/32*b^(3/4)*(-7*a*d+3*b*c)*ln(a^(1/4
)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*
d^(7/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b
*c)^2*2^(1/2)+1/8*d^(7/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2)
)/c^(3/4)/(-a*d+b*c)^2*2^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$$
$$= \frac{8a^{3/4}bc^{3/4}(bc-ad)x-2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad)}{}$$

input
```
Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

output
```
(8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*
a*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/
4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3
/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*
Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4
)*(b*c - a*d)^2*(a + b*x^4))
```

3.171.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {931, 25, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{4a(a+bx^4)(bc-ad)}-\frac{\int -\frac{3bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)}dx}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{4ad^2\int \frac{1}{dx^4+c}dx}{bc-ad}+\frac{b(3bc-7ad)\int \frac{1}{bx^4+a}dx}{bc-ad}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{4ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\cdots\right.}{bc-ad}}{4a(bc-ad)}$$

$$\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{4ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\cdots\right.}{\cdots}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 217

$$\frac{\frac{4ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\cdots\right.}{bc-ad}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 1479

$$\frac{\frac{4ad^2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\cdots\right.}{\cdots}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 25

3.171. $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx$

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \frac{\sqrt{\cdots}}{\sqrt[4]{b}(\cdots)}\cdots}{\cdots}\cdots\right)}{\cdots}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\cdots}{2\sqrt{\cdots}}\cdots\right)}{\cdots}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{4ad^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\frac{\arctan\left(\sqrt{\cdots}\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots}{\cdots}\right)}{\cdots}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

3.171. $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx$

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*(3*b*c - 7*a*d)*((-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (4*a*d^2*((-(ArcTan[1 - (Sqrt[2
]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c
] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d
^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*a*(b*c - a*d))
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.171.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.51

method	result
default	$-\frac{b\left(\frac{(ad-bc)x}{4a\left(b\,x^4+a\right)}+\frac{(7ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)}{(ad-bc)^2}+\frac{d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)^2*b*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(7*a*d-3*b*c)/a^2*(a/b)
^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)))+1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(
c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))
+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

3.171.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.46 (sec) , antiderivative size = 2955, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/16*(4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5
*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c -
 a^3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2
- 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^
2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d +
 a^2*c*d^2)) - 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 5
6*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c
^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 +
a^2*b*c - a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 +
28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^
9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*
a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(-I*(a*b^2*c - a^2
*b*d)*x^4 - I*a^2*b*c + I*a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^1
0*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^
5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)
*(I*b^2*c^3 - 2*I*a*b*c^2*d + I*a^2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*
c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 -...
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

output
```
Timed out
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \frac{\left(\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,(ab^2c^2-2\,a^2bcd+a^3d^2)}$$

$$+ \frac{bx}{4\,((ab^2c-a^2bd)x^4+a^2bc-a^3d)}$$

$$+ \frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}} - \frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{}\right.}{}}{8\,(b^2c^2-2\,abcd+a^2d^2)}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^
3*d^2) + 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt
(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sq
rt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - s
qrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)
```

3.171. $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

3.171.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)}\,dx = \frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} - \frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{\left(3(ab^3)^{\frac{1}{4}}bc-7(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{\left(3(ab^3)^{\frac{1}{4}}bc-7(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{\left(3(ab^3)^{\frac{1}{4}}bc-7(ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} - \frac{\left(3(ab^3)^{\frac{1}{4}}bc-7(ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{bx}{4(bx^4+a)(abc-a^2d)}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c
*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/
(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^
(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*
sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqr
t(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + s
qrt(2)*a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2
 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(
a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(
3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2
) - 1/16*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(
2)*a^4*d^2) + 1/4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

3.171.9 Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 21975, normalized size of antiderivative = 42.84

$$\int \frac{1}{(a+bx^4)^2\,(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

output
```
2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*
b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458
7520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^
6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16
- (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^
9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(
81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2
- 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^
7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^
4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a
^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^
3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^
8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*
c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^
13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 409
6*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 25
3952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 +
 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^
12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d...
```

3.171. $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

3.172 $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2} \, dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 596

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{b^{7/4}(3bc-11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} - \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}$$

output
```
1/4*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^4+c)+1/4*b*x/a/(-a*d+b*c)/(b*x^4+a
)/(d*x^4+c)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/
4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(
1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*a*d+
11*b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+
1/16*d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(
-a*d+b*c)^3*2^(1/2)-1/32*b^(7/4)*(-11*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*b^(7/4)*(-11*a
*d+3*b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+
b*c)^3*2^(1/2)-1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+
c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*d^(7/4)*(-3*a*d+11*
b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^
3*2^(1/2)
```

3.172. $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

3.172.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = \frac{1}{32}\left(\frac{8b^2x}{a(bc-ad)^2(a+bx^4)}+\frac{8d^2x}{c(bc-ad)^2(c+dx^4)}\right.$$
$$+\frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^3}$$
$$+\frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(-bc+ad)^3}$$
$$+\frac{2\sqrt{2}d^{7/4}(-11bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3}$$
$$+\frac{2\sqrt{2}d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3}$$
$$+\frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(bc-ad)^3}$$
$$+\frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(-bc+ad)^3}$$
$$+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(-bc+ad)^3}$$
$$\left.+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

output
```
((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*d)^2*(c +
 d*x^4)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) +
(2*Sqrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4
)*(-3*b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(a^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/4)*(-(b*c) + a*d)^3) +
(Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(c^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(b*c
 - a*d)^3))/32
```

3.172.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {931, 25, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)} - \frac{\int -\frac{7bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)^2}dx}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{7bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)^2}dx}{4a(bc-ad)} + \frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{4\left(3bd(bc+ad)x^4+3b^2c^2+3a^2d^2-8abcd\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{4c(bc-ad)} + \frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)} + \frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3bd(bc+ad)x^4+3b^2c^2+3a^2d^2-8abcd}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1020

$$\frac{\frac{\frac{b^2c(3bc-11ad)\int\frac{1}{bx^4+a}dx}{bc-ad}+\frac{ad^2(11bc-3ad)\int\frac{1}{dx^4+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 755

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+$$
$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1476

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}\right.}{bc-ad}}{c(bc-ad)}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1082

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\right.}{}}{c(bc-ad)}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 217

3.172. $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx$

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right)}{bc-ad}}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1479

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad\cdots}{\cdots}}{c(bc-ad)}\cdots}{4a(bc-\cdots}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\int\cdots\right)}{\cdots}}{c(bc-ad)}\cdots}{4a(bc-ad\cdots}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

3.172. $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx$

$$\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\cdots}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\cdots\right)}{c(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\cdots\right)}{c(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^4)) + ((b^2*c*(3*b*c - 11*a*d)*((-(ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(
1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (a*d^2*(11*b*c - 3*a*d)*((-(ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt
[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/
4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^
(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(c*(b*c - a*d)))/(4*a*(b*c - a*
d))
```

3.172. $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.172.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.50

method	result
default	$\frac{b^2\left(\frac{(ad-bc)x}{4a\left(b\,x^4+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(11*a*d-3*b*c)/a^2*(a/b)
^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)))+d^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3
*a*d-11*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)))
```

3.172.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 165.51 (sec) , antiderivative size = 5234, normalized size of antiderivative = 8.78

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

output
```
Timed out
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$$
$$= \frac{\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac14}\right.}{a^{\frac34}b^{\frac14}}}{32\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)}$$
$$+\frac{(b^2cd+abd^2)x^5+(b^2c^2+a^2d^2)x}{4\,((ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^8+a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)}$$
$$+\frac{\frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(11\,bcd^2-3\,ad^3)\log\left(\right.}{c^{\frac34}}}{32\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1
/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt
(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a)
)/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*
c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2
 + a^2*d^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b
*c^2*d^2 + a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arcta
n(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sq
rt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(
2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sq
rt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3
*c*d^3)
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(462) = 924$.

Time = 0.45 (sec) , antiderivative size = 967, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^
3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1
/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*s
qrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c
*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 -
sqrt(2)*a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3
*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d
^3) + 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^
2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2
*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - s
qrt(2)*a^5*d^3) + 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*
b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d
^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4...
```

3.172.9 Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 37266, normalized size of antiderivative = 62.53

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
5*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d +
b*c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 6553
6*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 144
17920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14
*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^
8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 7
86432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*
b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7
*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 324403
20*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^1
0 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d
^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*
b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11
)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^
8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 ...
```

3.172. $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\, dx$

3.173 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4} dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx =& -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d}\\ &+\frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21d^3\sqrt{a-bx^4}}\\ &-\frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}\\ &-\frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/7*b*x*(-b*x^4+a)^(3/2)/d-1/21*b*(-13*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/d^2+1
/21*a^(1/4)*b^(3/4)*(47*a^2*d^2-56*a*b*c*d+21*b^2*c^2)*EllipticF(b^(1/4)*x
/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^
3*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^
4/a)^(1/2)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*Ellipti
cPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)
/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)
```

3.173.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.90

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{x\left(5b(-a+bx^4)\,(7bc-16ad+3bdx^4)-\frac{b(21b^2c^2-56abcd+47a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx}{a}\right.}{c}\right.}{1\ldots}$$

input
```
Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

output
```
(x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*
b*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*
x^4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Ap
pellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*Appell
F1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/
2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/
a, (d*x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

3.173.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {933, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\int \frac{\sqrt{a-bx^4}\,(a(bc-7ad)-b(7bc-13ad)x^4)}{c-dx^4}dx}{7d}$$

$$\Big\downarrow \text{1025}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\int \frac{a\left(7b^2c^2-16abdc+21a^2d^2\right)-b\left(21b^2c^2-56abdc+47a^2d^2\right)x^4}{\sqrt{a-bx^4}\,(c-dx^4)}dx}{3d}}{7d}$$

$$\Big\downarrow \text{1021}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{b(47a^2d^2-56abcd+21b^2c^2)\int\frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}}{3d}}{7d}$$

$$\Big\downarrow \text{765}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}}{3d}}{7d}$$

$$\Big\downarrow \text{762}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}}{3d}}{7d}$$

$$\Big\downarrow \text{925}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\int\frac{\sqrt{c}}{(\sqrt{c}-\sqrt{d}x^2)\sqrt{a-bx^4}}dx}{2c}+\frac{\int\frac{}{(\sqrt{d}x}\cdots}{}\right.}{d}}{3d}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\int\frac{1}{(\sqrt{c}-\sqrt{d}x^2)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int\frac{}{(\sqrt{d}x}\cdots}{}\right.}{d}}{3d}}{7d}$$

$$\Big\downarrow \text{1543}$$

3.173. $\int\frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}} \ldots\right)}{d}}{3d}}{7d}$$

$\downarrow$ 1542

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-21(bc-ad)^3\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},a \ldots\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} \ldots\right)}{3d}}{7d}$$

input
```
Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

output
```
(b*x*(a - b*x^4)^(3/2))/(7*d) - ((b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(3
*d) - ((a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) -
 (21*(b*c - a*d)^3*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqr
t[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a
 - b*x^4])))/d)/(3*d))/(7*d)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.173. $\int \frac{\left(a-bx^4\right)^{5/2}}{c-dx^4}\,dx$

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.173. $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

3.173.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.41 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.09

method	result
risch	$\frac{bx(-3bd\,x^4+16ad-7bc)\sqrt{-b\,x^4+a}}{21d^2}+\frac{\frac{b(47a^2d^2-56abcd+21b^2c^2)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}+\frac{(-21a^3d^3+63a^2bc\,d^2-63a\,b^2c^2d+}{}}{}$
default	$-\frac{b^2x^5\sqrt{-b\,x^4+a}}{7d}-\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b(3a^2d^2-3abcd+b^2c^2)}{d^3}+\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}$
elliptic	$-\frac{b^2x^5\sqrt{-b\,x^4+a}}{7d}-\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b(3a^2d^2-3abcd+b^2c^2)}{d^3}+\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}$

input
```
int((-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/21*b*x*(-3*b*d*x^4+16*a*d-7*b*c)*(-b*x^4+a)^(1/2)/d^2+1/21/d^2*(b*(47*a^
2*d^2-56*a*b*c*d+21*b^2*c^2)/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^
(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1
/a^(1/2)*b^(1/2))^(1/2),I)+1/8*(-21*a^3*d^3+63*a^2*b*c*d^2-63*a*b^2*c^2*d+
21*b^3*c^3)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_
alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^
(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(
1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2
)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d-c))
```

3.173.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.173.6 Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = -\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

input
```
integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

output
```
-Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqr
t(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-
c + d*x**4), x)
```

3.173.7 Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

3.173.8 Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \int \frac{(a-b\,x^4)^{5/2}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(5/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(5/2)/(c - d*x^4), x)
```

3.174 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 277

$$
\begin{aligned}
\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx &= \frac{bx\sqrt{a-bx^4}}{3d} \\
&- \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3d^2\sqrt{a-bx^4}} \\
&+ \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}} \\
&+ \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}
\end{aligned}
$$

output
```
1/3*b*x*(-b*x^4+a)^(1/2)/d-1/3*a^(1/4)*b^(3/4)*(-5*a*d+3*b*c)*EllipticF(b^
(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*
d+b*c)^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*
(1-b*x^4/a)^(1/2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+b*c)^2*
EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a
)^(1/2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)
```

3.174.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.23

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = -\frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1-\frac{bx^4}{a}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(3a^2d-abdx^4+b^2x^4(-c+dx^4)\right)\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4(a-bx\cdots}{(-c+dx^4)\left(5ac\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\mathrm{App}\cdots}\right.}{15d\sqrt{a-bx^4}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
-1/15*(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4
*(-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*
(a - b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^
4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(d*Sqrt[a - b*x^4])
```

3.174.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {933, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{3d}$$

$$\Big\downarrow \text{1021}$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{b(3bc-5ad)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$$\Big\downarrow \; 765$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$$\Big\downarrow \; 762$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$$\Big\downarrow \; 925$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{3d}$$

$$\Big\downarrow \; 1543$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}}{3d}$$

$$\Big\downarrow \; 1542$$

3.174. $\int \frac{\left(a-bx^4\right)^{3/2}}{c-dx^4}\,dx$

$$\frac{bx\sqrt{a-bx^4}}{3d} - \frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}} - \frac{3(bc-ad)^2\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\cdots}{\cdots}\right)}{d}}{3d}$$

input
```
Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
(b*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (
b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4])
- (3*(b*c - a*d)^2*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqr
t[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a
 - b*x^4])))/d)/(3*d)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.174.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.26 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.10

method	result
risch	$\frac{bx\sqrt{-bx^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}+\frac{\left(-3a^2d^2+6abcd-3b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2bx^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\cdots}\right)}{\sqrt{\frac{ad-bc}{d}}\cdots}\cdots}{\cdots}\right.}{\cdots}}{3d}\cdots$
default	$\frac{bx\sqrt{-bx^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{-\cdots}{2\cdots}\right)}{\cdots}\cdots\right.}{\cdots}}{\cdots}$
elliptic	$\frac{bx\sqrt{-bx^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{-\cdots}{2\cdots}\right)}{\cdots}\cdots\right.}{\cdots}}{\cdots}$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x*(-b*x^4+a)^(1/2)/d+1/3/d*(b*(5*a*d-3*b*c)/d/(1/a^(1/2)*b^(1/2))^(1
/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)
^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/8*(-3*a^2*d^2+6*a*b*c*d-
3*b^2*c^2)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1
/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)
/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*d-c)))
```

3.174. $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

3.174.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.174.6 Sympy [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = -\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

input
```
integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
 b*x**4)/(-c + d*x**4), x)
```

3.174.7 Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

3.174.8 Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(3/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(3/2)/(c - d*x^4), x)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{c-dx^4} \, dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4} \, dx = \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right), -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}, \arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right), -1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}, \arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right), -1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

output
```
a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d/(-b*x^4
+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(
1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)-1/2
*a^(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c
^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)
```

3.175.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx =$$
$$-\frac{5acx\sqrt{a-bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{Appe}\right.\right.}$$

input
```
Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

output
```
(-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c
])/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
+ 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Ap
pellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

3.175.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {922, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}dx$$
$$\Big\downarrow \text{922}$$
$$\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$
$$\Big\downarrow \text{765}$$
$$\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$
$$\Big\downarrow \text{762}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$

$\big\downarrow$ 925

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\int\frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int\frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}$$

$\big\downarrow$ 27

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}$$

$\big\downarrow$ 1543

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}$$

$\big\downarrow$ 1542

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)}{d}$$

input
```
Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

3.175. $\int\frac{\sqrt{a-bx^4}}{c-dx^4}\,dx$

output
```
(a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)]
, -1])/(d*Sqrt[a - b*x^4]) - ((b*c - a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*El
lipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.175.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08

method	result
default	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}} - \frac{\sum_{\_\alpha=\text{RootOf}(d\_Z^4-c)} \frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2bx^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-bx^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}} - \frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2 \ldots}{\ldots}}}{\sqrt{\ldots}} \ldots\right)}{\_\alpha^3}}{8d^2}$
elliptic	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}} - \frac{\sum_{\_\alpha=\text{RootOf}(d\_Z^4-c)} \frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2bx^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-bx^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}} - \frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2 \ldots}{\ldots}}}{\sqrt{\ldots}} \ldots\right)}{\_\alpha^3}}{8d^2}$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-
1/8/d^2*sum((a*d-b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_
alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^
(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(
1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2
)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d-c))
```

3.175.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.175.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = -\int \frac{\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

input
```
integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

3.175.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

3.175.8 Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx=\int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx=\int \frac{\sqrt{a-b\,x^4}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(1/2)/(c - d*x^4), x)
```

3.176 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

output
```
1/2*a^(1/4)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),
I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*EllipticPi(b^(
1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4
)/c/(-b*x^4+a)^(1/2)
```

3.176.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = -\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\,(-c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{A}\right.\right.}$$

input
```
Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^
4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appell
F1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

3.176.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}dx$$

$$\Big\downarrow \text{925}$$

$$\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

input
```
Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

output
```
(a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[
c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a
^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]),
 ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.176.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^4+a}}}{\_\alpha^3}}{8d}$	183
elliptic	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^4+a}}}{\_\alpha^3}}{8d}$	183

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b
*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1
/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/
2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)
*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

3.176.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.176.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = -\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

3.176.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

3.176.8 Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx=\int \frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)}\,dx$$

input
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)
```

3.177 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}}$$

output
```
1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1/2*b^(3/4)*EllipticF(b^(1/4)*x/a^(1
/4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d
*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4
/a)^(1/2)/b^(1/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*EllipticPi(b
^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1
/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

3.177.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{5acx\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(-2bc+2ad+bdx^4)+bdx^4\sqrt{1-\frac{b}{}}\right.}{10ac(-bc+ad)\sqrt{a}}$$

input
```
Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c +
2*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4,
1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqr
t[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c
 + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

3.177.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {931, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-bdx^4+bc-2ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 765

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 762

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 925

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 1543

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$\downarrow$ 1542

3.177. $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}dx$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}} - 2ad\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticP}}{2\sqrt[4]{}}\right.}{2a(bc-ad)}$$
$$\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^
4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] - 2*a*d*
((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt
[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (
a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/(2*a*(
b*c - a*d))
```

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.177.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.07

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-}{}\right.}{}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-}{}\right.}{}$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x/a/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(1/a^(1/2)*b^(
1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/(a*d-b*c
)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(
1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^
3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+
a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c
*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z
^4*d-c))
```

3.177.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.177.6 Sympy [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = -\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*
sqrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

3.177.7 Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

3.177.8 Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)), x)
```

3.178 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 334

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}}$$
$$+ \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}}$$
$$+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}}$$
$$+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}}$$

output
```
1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(3/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b
*c)^2/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-11*a*d+5*b*c)*EllipticF(b^(1/4)*x/a^
(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/
4)*d^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1
-b*x^4/a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*d^2*El
lipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^
(1/2)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

3.178.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.72 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac(12a^3d^2+a^2bd(-24c+dx^4)+5b^3\right.}{}\right.}{}$$

input
```
Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9
/4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*
x^4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^
8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4
)*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/
a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/
(60*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

3.178.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {931, 1024, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-5bdx^4+5bc-6ad}{(a-bx^4)^{3/2}(c-dx^4)}dx}{6a(bc-ad)} + \frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{-bd(5bc-11ad)x^4+5b^2c^2+12a^2d^2-11abcd}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1021$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+b(5bc-11ad)\int \frac{1}{\sqrt{a-bx^4}}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+\frac{b\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1543$$

3.178. $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx$

$$\frac{\frac{12a^2d^2\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{\cdots}{2\cdots}}{6a(bc-ad)}$$
$$+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{12a^2d^2\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\cdots}{\sqrt{a-\cdots}}}{2a(bc-ad)}\cdots}{6a(bc-ad)}$$
$$+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + ((b*(5*b*c - 11*a*d)*x)/(2*a*(
b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + 1
2*a^2*d^2*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b
*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b
]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]
)))/(2*a*(b*c - a*d)))/(6*a*(b*c - a*d))
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

3.178. $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$

3.178.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.08

method	result
default	$-\frac{x\sqrt{-b\,x^4+a}}{6ab(ad-bc)\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_\right.}\right.}{}$
elliptic	$-\frac{x\sqrt{-b\,x^4+a}}{6ab(ad-bc)\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_\right.}\right.}{}$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x/a/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b*x/a^2*(11*a*d-5*b
*c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2
/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^
(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8
*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1
/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)
/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*d-c))
```

3.178.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.178.6 Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx =$$
$$-\int \frac{1}{-a^2c\sqrt{a-bx^4}+a^2dx^4\sqrt{a-bx^4}+2abcx^4\sqrt{a-bx^4}-2abdx^8\sqrt{a-bx^4}-b^2cx^8\sqrt{a-bx^4}+b^2dx^{12}\sqrt{a$$

input
```
integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a**2*c*sqrt(a - b*x**4) + a**2*d*x**4*sqrt(a - b*x**4) + 2*a
*b*c*x**4*sqrt(a - b*x**4) - 2*a*b*d*x**8*sqrt(a - b*x**4) - b**2*c*x**8*s
qrt(a - b*x**4) + b**2*d*x**12*sqrt(a - b*x**4)), x)
```

3.178.7 Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

3.178.8 Giac [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)), x)
```

3.179 $\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx$

3.179.1 Optimal result

Integrand size = 21, antiderivative size = 926

$$
\begin{aligned}
\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx &= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} \\
&- \frac{(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} \\
&- \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
-1/4*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+
a)^(1/2))/(-c)^(3/4)/d^(7/4)-1/4*(a*d-b*c)^(3/2)*arctan(x*(a*d-b*c)^(1/2)/
(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/d^(7/4)+1/3*b*x*(b*x^4+a)^(
1/2)/d-1/6*b^(3/4)*(-5*a*d+3*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/d^2/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2
)*(-c)^(1/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^
(1/4)/d^2/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
Pi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2
))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))
*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+
b*c)^2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*...
```

3.179.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.37

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(-5ac\left(3a^2d+abdx^4+b^2x^4(c+dx^4)\right)\mathrm{AppellF1}\left(\frac{1}{4}\right.\right.}{(c+dx^4)\left(-5ac\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2}\right.\right.}\right.}{15d}$$

input
```
Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

output
```
(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^
4*(c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*
b*x^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/
a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)
/c)])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(1
5*d*Sqrt[a + b*x^4])
```

3.179.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 1004, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {933, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int -\frac{b(3bc-5ad)x^4+a(bc-3ad)}{\sqrt{bx^4+a}(dx^4+c)}dx}{3d}+\frac{bx\sqrt{a+bx^4}}{3d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\int \frac{b(3bc-5ad)x^4+a(bc-3ad)}{\sqrt{bx^4+a}(dx^4+c)}dx}{3d}$$

$$\Big\downarrow \ 1021$$

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b(3bc-5ad)\int \frac{1}{\sqrt{bx^4+a}}dx}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{d}}{3d}$$

$$\Big\downarrow \ 761$$

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-5ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{d}}{3d}$$

$\big\downarrow$ 925

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-5ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{3(bc-ad)^2\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}}{3d}$$

$\big\downarrow$ 1541

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-5ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{3(bc-ad)^2\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{\cdots}\right.}{\cdots}}{2c}\cdots\right.}{\cdots}}{3d}$$

$\big\downarrow$ 27

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-5ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{3(bc-ad)^2\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{\cdots}\right.}{\cdots}}{2c}\cdots\right.}{\cdots}}{3d}$$

$\big\downarrow$ 761

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-5ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{3(bc-ad)^2\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{Ell}\cdots}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\big\downarrow$ 2221

3.179. $\displaystyle\int\frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx$

$$\frac{bx\sqrt{bx^4+a}}{3d}-\frac{\frac{b^{3/4}(3bc-5ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}-\frac{3(bc-ad)^2\left(\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{Ell}}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}\right.}{}}{}$$

$\downarrow$ 2223

$$\frac{bx\sqrt{bx^4+a}}{3d}-\frac{\frac{b^{3/4}(3bc-5ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}-\frac{3(bc-ad)^2\left(\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{Ellipti}}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}\right.}{}}{}$$

input
```
Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

3.179. $\int \frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx$

output
```
(b*x*Sqrt[a + b*x^4])/(3*d) - ((b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4]) - (3*(b*c - a*d)^2*((
(b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[
-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt[
d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x
^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d])
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c
]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a
 + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*
Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*S
qrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1/
4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(
1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] -
(Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2...
```

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.179. $\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx$

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.179. $\int \frac{(a+bx^4)^{3/2}}{c+dx^4} dx$

3.179.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.34

method	result
risch	$\frac{bx\sqrt{bx^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}+\frac{\left(3a^2d^2-6abcd+3b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}-\frac{\operatorname{arctanh}\left(\frac{2bx^2\_\alpha^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\sqrt{\cdots}}\right)}{\sqrt{\frac{ad-bc}{d}}\cdots}\cdots\right)}{\cdots}}{3d}\cdots$
default	$\frac{bx\sqrt{bx^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{bx\sqrt{bx^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int((b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x*(b*x^4+a)^(1/2)/d+1/3/d*(b*(5*a*d-3*b*c)/d/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*(3*a^2*d^2-6*a*b*c*d
+3*b^2*c^2)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(
1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*d+c)))
```

3.179.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.179.6 Sympy [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{3}{2}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

3.179.7 Maxima [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

3.179.8 Giac [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx=\int \frac{(b\,x^4+a)^{3/2}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(3/2)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(3/2)/(c + d*x^4), x)
```

3.180 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

3.180.1 Optimal result

Integrand size = 21, antiderivative size = 881

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx \\
&= \frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} - \frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} \\
&+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} \\
&- \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d(bc+ad)\sqrt{a+bx^4}} \\
&- \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}d(bc+ad)\sqrt{a+bx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/4*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))*(-a*d+b*
c)^(1/2)/(-c)^(3/4)/d^(3/4)-1/4*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4
)/(b*x^4+a)^(1/2))*(a*d-b*c)^(1/2)/(-c)^(3/4)/d^(3/4)+1/2*b^(3/4)*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((
b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(b*x^4+a)^(1/2)-1/4*b^(1
/4)*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)-1
/8*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(
-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)
/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1
/2)-1/8*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(
1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*
2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*...
```

3.180.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{5acx\sqrt{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,\text{Ap}\right.\right.}$$

input
```
Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

output
```
(5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^
4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x
^4)/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

3.180.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {922, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$

$$\Big\downarrow \text{922}$$

$$\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \text{761}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \text{925}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}$$

$$\Big\downarrow \text{1541}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\sqrt{a}\sqrt{\cdots}\right)}{\cdots}}{2c}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-\cdots}\right.}{\cdots}}{2c}\cdots\right)}{d}$$

$$\Big\downarrow \; 761$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\cdots\right)}{d}$$

$$\Big\downarrow \; 2221$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}+\frac{\sqrt[4]{b}\left(\cdots\right.}{\cdots}\right)}{\cdots}$$

$$\Big\downarrow \; 2223$$

3.180. $\int\frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}-$$

$$(bc-ad)\left(\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\cdots}}{\cdots}\right.}{2c}\right.$$

input
```
Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

output
```
(b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*
x^4]) - ((b*c - a*d)*(((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*
x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(S
qrt[b] - (Sqrt[a]*Sqrt[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/
4)*d^(1/4)*Sqrt[a + b*x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqr
t[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(
Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4
*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt
[d])*Sqrt[d]*(((-c)^(1/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqr
t[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[a] - (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c]
+ Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^...
```

3.180. $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.180.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.31

method	result
default	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\cdots}}{\cdots}\right)}{\_\alpha^3}}{8d^2}$
elliptic	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\cdots}}{\cdots}\right)}{\_\alpha^3}}{8d^2}$

input
```
int((b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(
2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*
b^(1/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I
*a^(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2))),_alpha=RootOf(_Z^4*d+c))
```

3.180.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.180.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

output
```
Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

3.180.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

3.180.8 Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{b\,x^4+a}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

3.181 $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx$

3.181.1 Optimal result

Integrand size = 21, antiderivative size = 742

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx &= -\frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
-1/4*d^(1/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))
/(-c)^(3/4)/(-a*d+b*c)^(1/2)-1/4*d^(1/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/
4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(1/2)+1/8*(cos(2*arctan(b
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/
a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(
1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/a^(1/4)/b^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(cos(2*arctan(b^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arct
an(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)
/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-
c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
/4)/b^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)
*d^(1/2)/(-c)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(a*
d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(a*d+b*c)/(b*...
```

3.181.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\,(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a
+ b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

3.181.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}dx$$

$$\Big\downarrow \text{925}$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}+$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\frac{\sqrt[4]{-c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\arctan\left(\right.}{2\sqrt[4]{d}\sqrt{bc-c}}\right.}{}}{2c}$$

$$\Big\downarrow \; 2223$$

$$\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4-}{\left(\sqrt{b}x^2+\right.}}}{}\right.}{}}{2c}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\sqrt[4]{-c}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\arctan\left(\right.}{2\sqrt[4]{d}\sqrt{bc-c}}\right.}{}}{2c}$$

input
```
Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

3.181. $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx$

output
```
((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt
[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt
[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d]
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-
c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[
a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]
*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*
Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1
/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] -
 (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(
4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c)
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.181.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.26

method	result	size
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$	191
elliptic	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$	191

input
```
int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha^2*b*x
^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)
*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/
2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*d+c))
```

3.181.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.181.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

output
```
Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

3.181.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

3.181.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)
```

3.182 $\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx$

3.182.1 Optimal result

Integrand size = 21, antiderivative size = 913

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx &= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} \\
&+ \frac{d^{5/4}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}} \\
&+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} \\
&- \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{a+bx^4}} \\
&- \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c\left(b^2c^2-a^2d^2\right)\sqrt{a+bx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
1/4*d^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/
(-c)^(3/4)/(-a*d+b*c)^(3/2)-1/4*d^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4
)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(3/2)+1/2*b*x/a/(-a*d+b*c)
/(b*x^4+a)^(1/2)+1/4*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/a^(5/4)/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)
/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)
-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1
/4)/c/(-a^2*d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b
^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(
1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(
1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/
b^(1/4)/c/(-a^2*d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+
a^(1/2)*d^(1/2)/(-c)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a...
```

3.182.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.36

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \frac{x\left(-\frac{bdx^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(2ad-b(2c+dx^4)\right)\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\right.}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\right.\right.}\right.}{10a(-bc+}$$

input
```
Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

output
```
(x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/
2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1
/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2,
2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*
x^4)/a), -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

3.182.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}-\frac{\int -\frac{bdx^4+bc-2ad}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^4+bc-2ad}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx-2ad\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 925

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int\frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 1541

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}\ldots}{\sqrt{a}\left(1-\ldots\right)\ldots}}{ad+bc}}{2c}\ldots\right.}{2a(bc-ad)}$$
$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\ldots}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\ldots}}{ad+bc}}{2c}\ldots\right.}{2a(bc-ad)}$$
$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 761

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\ldots\right.\right.}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\ldots}{2c}\ldots\right.}{\ldots}$$
$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 2221

3.182. $\int\frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\ldots}}{\ldots}}{2c}\right.}{\ldots}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 2223

$$\frac{bx}{2a(bc-ad)\sqrt{bx^4+a}}+$$

$$\frac{\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{bx^4+a}}-2ad\left(\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.\right.}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}\right.}{\ldots}$$

input
```
Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

3.182. $\int\frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}dx$

output
```
(b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) - 2*a*d*(((b^(1/4)*c*(Sqrt[
b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqr
t[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt[d])/Sqrt[-c])*Arc
Tanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(d^(1/4)*S
qrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[
b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c
 + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) -
 ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1/4)*(Sqrt[b]*Sqrt[
-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt
[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] - (Sqrt[b]*Sqrt[-c]
)/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[...
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.182. $\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx$

3.182.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.34

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{(x^4+\frac{a}{b})b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{(x^4+\frac{a}{b})b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x/a/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*sum(1/(a*d-b
*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/
(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^
3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alph
a^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*d+c))
```

3.182.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.182.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{2}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

3.182.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

3.182.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx=\int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx=\int \frac{1}{(b\,x^4+a)^{3/2}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)), x)
```

3.183 $\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx$

3.183.1 Optimal result

Integrand size = 21, antiderivative size = 976

$$
\begin{aligned}
&\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx = \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} \\
&- \frac{d^{9/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}} - \frac{d^{9/4}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{5/2}} \\
&+ \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
1/6*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/2)-1/4*d^(9/4)*arctan(x*(-a*d+b*c)^(1/2)
/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(5/2)-1/4*d^(9/
4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)
/(a*d-b*c)^(5/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^(1/2)
+1/24*b^(3/4)*(-11*a*d+5*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(9/4)/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan
(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^
(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^
(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin
(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/
a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(
1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/a^(1/4)/b^(1/4)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^
2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^...
```

3.183.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.44

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx =$$

$$-\frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(12a^3d^2+5b^3cx^4(2c+dx^4)-a^2bd(24c+dx^4)+ab^2(12c^2-15cdx^4-11\cdots\right.}{(a+bx^4)(c+dx^4)\left(-5ac\cdots\right.}\right.}{60\cdots}$$

input
```
Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

output
```
-1/60*(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c
*x^4*(2*c + d*x^4) - a^2*b*d*(24*c + d*x^4) + a*b^2*(12*c^2 - 15*c*d*x^4 -
 11*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b
*x^4*(c + d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*A
ppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4,
3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a
*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*A
ppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4,
3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(a^2*(b*c - a*d)^2*Sqrt[a +
b*x^4])
```

3.183.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 1071, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {931, 25, 1024, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{6a(a+bx^4)^{3/2}(bc-ad)}-\frac{\int -\frac{5bdx^4+5bc-6ad}{(bx^4+a)^{3/2}(dx^4+c)}dx}{6a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^4+5bc-6ad}{(bx^4+a)^{3/2}(dx^4+c)}dx}{6a(bc-ad)}+\frac{bx}{6a(a+bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(5bc-11ad)}{2a\sqrt{a+bx^4}(bc-ad)}-\frac{\int -\frac{bd(5bc-11ad)x^4+5b^2c^2+12a^2d^2-11abcd}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a(a+bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{bd(5bc-11ad)x^4+5b^2c^2+12a^2d^2-11abcd}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a+bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx+b(5bc-11ad)\int \frac{1}{\sqrt{bx^4+a}}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a+bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5bc-11ad)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a+bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5bc-11ad)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(bc-ad)}+\frac{bx(5\ldots}{2a\sqrt{a-\ldots}}}{6a(bc-ad)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-\ldots}\right.}{\ldots}}{2c}\ldots\right.}{2a(bc-ad)}\ldots}{6a(bc-ad)}\ldots$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

3.183. $\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}dx$

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\cdots}{ad+bc}}{2c}\cdots\right)\cdots}{2a(bc-ad)}\cdots}{6a(bc-ad)}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$\downarrow$ 761

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}\cdots\right.}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(bc-ad)}$$

$\downarrow$ 2221

$$\frac{bx}{6a(bc-ad)\left(bx^4+a\right)^{3/2}}+$$

$$\frac{\frac{b(5bc-11ad)x}{2a(bc-ad)\sqrt{bx^4+a}}+\frac{12a^2\left(\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\sqrt[4]{-c}\left(\sqrt{b}\sqrt{-\cdots}\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$\downarrow$ 2223

3.183. $\int\frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx$

$$\frac{bx}{6a(bc-ad)\left(bx^4+a\right)^{3/2}}+\frac{\frac{b(5bc-11ad)x}{2a(bc-ad)\sqrt{bx^4+a}}+\frac{12a^2\left(\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

output
```
(b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + ((b*(5*b*c - 11*a*d)*x)/(2*a*(
b*c - a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) + 12*a^2*d^2*(((b^(1/4
)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt[d])/Sqr
t[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/
(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-
1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[
d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^
4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d]
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a +
 b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1/4)*(Sqr
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^
(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] - (Sqrt[b
]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] ...
```

3.183. $\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.183. $\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx$

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.183.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.38

method	result
default	$-\frac{x\sqrt{b\,x^{4}+a}}{6ab(ad-bc)\left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{bx(11ad-5bc)}{12a^{2}(ad-bc)^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^{2}(ad-bc)^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+ \quad d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\right.}\right.$
elliptic	$-\frac{x\sqrt{b\,x^{4}+a}}{6ab(ad-bc)\left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{bx(11ad-5bc)}{12a^{2}(ad-bc)^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^{2}(ad-bc)^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+ \quad d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\right.}\right.$

input
```
int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x/a/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b*x/a^2*(11*a*d-5*b*
c)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
8*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(
1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*d+c))
```

3.183.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.183.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)} dx = \int \frac{1}{(a+bx^4)^{\frac{5}{2}}(c+dx^4)} dx$$

input
```
integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

3.183.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)} dx = \int \frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)} dx$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

3.183.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)} dx = \int \frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)} dx$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/2}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(5/2)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(5/2)*(c + d*x^4)), x)
```

3.184 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 426

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{84cd^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}$$

output
```
1/28*b*(-7*a*d+11*b*c)*x*(-b*x^4+a)^(3/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a
)^(5/2)/c/d/(-d*x^4+c)-1/84*b*(21*a^2*d^2-122*a*b*c*d+77*b^2*c^2)*x*(-b*x^
4+a)^(1/2)/c/d^3+1/84*a^(1/4)*b^(3/4)*(21*a^3*d^3+349*a^2*b*c*d^2-553*a*b^
2*c^2*d+231*b^3*c^3)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^
4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(1
/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4
)/c^2/d^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*Ellipti
cPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)
/b^(1/4)/c^2/d^4/(-b*x^4+a)^(1/2)
```

3.184.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.12

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\,x^5\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx\left(-84a^4d^3+\right.\right.}{}}{}$$

input
```
Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

output
```
-1/420*(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x
^5*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] +
(5*c*(5*a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b
^3*c*d*x^4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*
x^8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4
)*(-63*a^2*b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-7
7*c^2 + 44*c*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4
)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/
((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x
^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^3*Sqrt[a - b*x^4])
```

3.184.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {930, 25, 1025, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}dx$$

$$\Big\downarrow \text{930}$$

$$-\frac{\int -\frac{\left(a-bx^4\right)^{3/2}\left(a(bc+3ad)-b(11bc-7ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(a-bx^4\right)^{3/2}\left(a(bc+3ad)-b(11bc-7ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\int \frac{\sqrt{a-bx^4}\left(a\left(11b^2c^2-14abdc-21a^2d^2\right)-b\left(77b^2c^2-122abdc+21a^2d^2\right)x^4\right)}{c-dx^4}dx}{7d}}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\int \frac{a\left(77b^3c^3-155ab^2dc^2+63a^2bd^2c+63a^3d^3\right)-b\left(231b^3c^3-553ab^2dc^2+349a^2bd^2c+21a \cdots\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{3d} \cdots}{7d}}{4cd} \cdots \frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{b\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{21(bc-ad)^3(3ad+11bc)\int \cdots}{d}}{3d} \cdots}{7d}}{4cd} \cdots \frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3(3a}{}}{3d}}{7d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{d\sqrt{a-bx^4}}}{3d}}{7d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{d\sqrt{a-bx^4}}}{7d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{d\sqrt{a-bx^4}}}{7d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1543$$

3.184. $\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}\,dx$

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d} - \frac{\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{3d} - \frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{d\sqrt{a-bx^4}}}{7d}}{4cd}$$

$$\frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d} - \frac{\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{3d} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{d\sqrt{a-bx^4}}}{4cd}$$

$$\frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

input
```
Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a - b*x^4)^(5/2))/(c*d*(c - d*x^4)) + ((b*(11*b*c - 7
*a*d)*x*(a - b*x^4)^(3/2))/(7*d) - ((b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*
d^2)*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a*b^2
*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSi
n[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - (21*(b*c - a*d)^3*(11*b
*c + 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(
Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a -
 b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^
4])))/d)/(3*d))/(7*d))/(4*c*d)
```

3.184. $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

3.184.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.184. $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.184.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.94 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.27

method	result
default	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+3b\right.}{d^4}\right.}{}$
elliptic	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+3b\right.}{d^4}\right.}{}$
risch	$\frac{b^2x(-3bd\,x^4+23ad-14bc)\sqrt{-b\,x^4+a}}{21d^3}+\frac{\frac{b^2\left(103a^2d^2-154abcd+63b^2c^2\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{21b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2\right.}{}}{}$

input
```
int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

3.184. $\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}\,dx$

output
```
1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)
/(-d*x^4+c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5
/7*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^
4+1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*b/c+1/3*(-2*b^3/d^
3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)
/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a
^2*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/
2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1
/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2
)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b
^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a
^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

3.184.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

output
```
Timed out
```

3.184.7 Maxima [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

3.184.8 Giac [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(a-b\,x^4)^{7/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

3.185 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} dx$

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 365

$$\begin{aligned}\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx &= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} \\ &- \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12cd^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}\end{aligned}$$

output
```
-1/4*(-a*d+b*c)*x*(-b*x^4+a)^(3/2)/c/d/(-d*x^4+c)+1/12*b*(-3*a*d+7*b*c)*x*
(-b*x^4+a)^(1/2)/c/d^2-1/12*a^(1/4)*b^(3/4)*(-3*a^2*d^2-26*a*b*c*d+21*b^2*
c^2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^3/(-b*x^4+a)^(1/
2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^
(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4
+a)^(1/2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1
/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(
-b*x^4+a)^(1/2)
```

3.185.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.58 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(-21b^2c^2+26abcd+3a^2d^2)\,x^5\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx\left(12a^3d^2+2ab^2cdx^4-3a^2bd^2x^4+b^3\right.\right.}{}}{}$$

input
```
Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

output
```
-1/60*(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*Ap
pellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2
 + 2*a*b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF
1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d
+ 3*a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*
x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
))/((-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^2*Sqrt[a - b*x^4])
```

3.185.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {930, 25, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx$$

$$\Big\downarrow \text{930}$$

$$-\frac{\int -\frac{\sqrt{a-bx^4}\,(a(bc+3ad)-b(7bc-3ad)x^4)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\,(c-dx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{a-bx^4}\left(a(bc+3ad)-b(7bc-3ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\int \frac{a\left(7b^2c^2-6abdc-9a^2d^2\right)-b\left(21b^2c^2-26abdc-3a^2d^2\right)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1021$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{b\left(-3a^2d^2-26abcd+21b^2c^2\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)^2(3ad+7bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\right.}{d}}{3d}}{4cd}$$
$$\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

3.185. $\int \frac{\left(a-bx^4\right)^{5/2}}{\left(c-dx^4\right)^2}dx$

$\Big\downarrow$ 27

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\cdots\right.}{d}}{3d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 1543

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1\cdots}}}{2\sqrt{c}\sqrt{a-bx^4}}\cdots\right.}{d}}{3d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-3(bc-ad)^2(3ad+7bc)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\cdots\right.}{2\sqrt[4]{b}c\cdots}\cdots\right.}{3d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

input
```
Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a - b*x^4)^(3/2))/(c*d*(c - d*x^4)) + ((b*(7*b*c - 3*
a*d)*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d
 - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(d*Sqrt[a - b*x^4]) - (3*(b*c - a*d)^2*(7*b*c + 3*a*d)*((a^(1/4)*Sqrt[
1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(
b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1
- (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d)/(3*d))/(4*c*d)
```

3.185. $\int \frac{\left(a-bx^4\right)^{5/2}}{\left(c-dx^4\right)^2}\,dx$

3.185.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.185. $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} \, dx$

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.185.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.16 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-bx^4+a}}{4d^2c(-dx^4+c)}+\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)b}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}$
elliptic	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-bx^4+a}}{4d^2c(-dx^4+c)}+\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)b}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}$
risch	$\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\frac{2b^2(4ad-3bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}-9b\left(a^2d^2-2abcd+b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}-\frac{\operatorname{arctanh}\left(\frac{-2bx^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\cdots}\right)}{\sqrt{\frac{ad-b\cdots}{d}}}\cdots\right)}{\cdots}$

input
```
int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/d^2/c*(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/3*b^
2/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^3*b/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1
/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a
^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^
2*d+7*b^3*c^3)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2
*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^
(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/
2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*d-c))
```

3.185. $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx$

3.185.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.185.6 Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{5}{2}}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

output
```
Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

3.185.7 Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

3.185.8 Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-b\,x^4)^{5/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(5/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(5/2)/(c - d*x^4)^2, x)
```

3.186 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx$

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}}$$

output
```
-1/4*(-a*d+b*c)*x*(-b*x^4+a)^(1/2)/c/d/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(a*d
+3*b*c)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^2/(-b*x^4+a)^
(1/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/
2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)
^(1/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/
2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)
^(1/2)
```

3.186.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.11

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \frac{x\left(-b(3bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d+b^2c\right.}{}\right.}{20c^2d}$$

input
```
Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

output
```
(x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a
*b*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a
*d)*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

3.186.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {930, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}dx$$

$$\Big\downarrow \text{930}$$

$$-\frac{\int -\frac{a(bc+3ad)-b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(bc+3ad)-b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{b(ad+3bc)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1543}$$

3.186. $\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}}{4cd}$$

$$\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\cdots}{\cdots}\right)}{d}}{4cd}$$

$$\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

input
```
Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*Sqrt[a - b*x^4])/(c*d*(c - d*x^4)) + ((a^(1/4)*b^(3/4)
*(3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
-1])/(d*Sqrt[a - b*x^4]) - (3*(b*c - a*d)*(b*c + a*d)*((a^(1/4)*Sqrt[1 - (
b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*
x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d)/(4*c*d)
```

3.186.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.186. $\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx$

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.186. $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx$

3.186.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.03 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.06

method	result
default	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(-\frac{\mathrm{arctanh}}{}\right.}{}\right.}{}$
elliptic	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(-\frac{\mathrm{arctanh}}{}\right.}{}\right.}{}$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+(b^2/d^2+1/4*b/d^2*(a*d-b*
c)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/
2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I
)-3/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arct
anh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/
(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^
2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))
^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*
b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

3.186.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.186.6 Sympy [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{3}{2}}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

output
```
Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

3.186.7 Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

3.186.8 Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(3/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(3/2)/(c - d*x^4)^2, x)
```

3.187 $\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx$

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx = \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

output
```
1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*
x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*
c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x
^4/a)^(1/2)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)*Ellipt
icPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2
)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)
```

3.187.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \frac{x\left(-\frac{5(a-bx^4)}{c}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}(c-dx^4)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{75a^2\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\right.}{5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{bx}{a}\right.\right.}\right.}{20\sqrt{a-bx^4}\,(-c+dx^4)}$$

input
```
Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

output
```
(x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5
/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1
, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
 (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a -
b*x^4]*(-c + d*x^4))
```

3.187.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {929, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx$$

$$\Big\downarrow \text{ 929}$$

$$\frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}-\frac{\int -\frac{3a-bx^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3a-bx^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c}+\frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \text{ 1542}$$

3.187. $\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\right.}{2\sqrt[4]{b}c\sqrt{}}\right.}{d}}{4c}$$

$$\frac{x\sqrt{a-bx^4}}{4c\,(c-dx^4)}$$

input
```
Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

output
```
(x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + ((a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - ((b*
c - 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(S
qrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a -
b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4
])))/d)/(4*c)
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.187.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.06

method	result
default	$\frac{x\sqrt{-b\,x^{4}+a}}{4c\left(-d\,x^{4}+c\right)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}}{\cdots}\right.}{\cdots}}{32c\,d^{2}}$
elliptic	$\frac{x\sqrt{-b\,x^{4}+a}}{4c\left(-d\,x^{4}+c\right)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}}{\cdots}\right.}{\cdots}}{32c\,d^{2}}$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4/c/d*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1
-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)
*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((3*a*d-b*c)/_alph
a^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*
d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(
1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2
)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/
a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c)
)
```

3.187.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.187.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{a-bx^4}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

3.187.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

3.187.8 Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx=\int \frac{\sqrt{a-b\,x^4}}{{\left(c-d\,x^4\right)}^2}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(1/2)/(c - d*x^4)^2, x)
```

3.188 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2} dx$

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 310

$$\begin{aligned}
&\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx \\
&= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} \\
&\quad + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}} \\
&\quad + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}}
\end{aligned}$$

output
```
-1/4*d*x*(-b*x^4+a)^(1/2)/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^(1/4)*b^(3/4)*Elli
pticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)
+1/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/
b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/
2)+1/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)
/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1
/2)
```

3.188.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx$$
$$= \frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(4bc-4ad+bdx^4)+bdx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\right.\right.}{20c^2(bc-ad)\sqrt{a-bx^4}\,(-c+dx^4)\left(5ac\,\text{Ap}\right.}$$

input
```
Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

output
```
(5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt
[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)
/c])*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF
1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b
*x^4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
 + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*App
ellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

3.188.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {931, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx$$
$$\Big\downarrow \text{931}$$
$$-\frac{\int -\frac{bdx^4+4bc-3ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^4+4bc-3ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-b\int \frac{1}{\sqrt{a-bx^4}}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{(5bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(5bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1543}$$

3.188. $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2}\,dx$

$$\frac{(5bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-\right.}{\sqrt{a-bx^4}}}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 1542$$

$$\frac{(5bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)-\frac{\sqrt[4]{a}b^{3}}{}}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

output
```
-1/4*(d*x*Sqrt[a - b*x^4])/(c*(b*c - a*d)*(c - d*x^4)) + (-((a^(1/4)*b^(3/
4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a
- b*x^4]) + (5*b*c - 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sq
rt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^
(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)
*c*Sqrt[a - b*x^4])))/(4*c*(b*c - a*d))
```

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.188. $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2}\,dx$

3.188.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.48 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.04

method	result
default	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{32c}$
elliptic	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{32c}$

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d/c/(a*d-b*c)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/4*b/c/(a*d-b*c)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*sum
((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-
2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)
*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(
1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*d-c))
```

3.188.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.188.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{a-bx^4}\,(-c+dx^4)^2}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(1/(sqrt(a - b*x**4)*(-c + d*x**4)**2), x)
```

3.188.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

3.188.8 Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{a-b\,x^4}\,(c-d\,x^4)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2), x)
```

3.189 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2} dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2} dx = \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}}$$

output
```
1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)
/(-d*x^4+c)/(-b*x^4+a)^(1/2)+1/4*b^(3/4)*(a*d+2*b*c)*EllipticF(b^(1/4)*x/a
^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^
(1/4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)
/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/
8*a^(1/4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1
/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

3.189.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \frac{x\left(-bd(2bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{c\left(25ac\left(4a^2d^2+2b\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

output
```
(x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
 (b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) -
a*b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10
*x^4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4,
 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^
4)/a, (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)
/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4
)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2
*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

3.189.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {931, 25, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$-\frac{\int -\frac{5bdx^4+4bc-3ad}{(a-bx^4)^{3/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^4+4bc-3ad}{(a-bx^4)^{3/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 1024

$$\frac{\frac{\int \frac{2\left(-bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{2a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{-bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 1021

$$\frac{\frac{b(ad+2bc)\int \frac{1}{\sqrt{a-bx^4}}dx-3ad(3bc-ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 765

$$\frac{\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-3ad(3bc-ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 762

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

$\big\downarrow$ 925

3.189. $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\int\frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int\frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1543

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1542

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{Elliptic}}{2}\right)}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

3.189. $\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)^2}\,dx$

output
```
-1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + ((b*(2*b*c + a*d)
*x)/(a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(2*b*c + a*d)*Sqrt
[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4
] - 3*a*d*(3*b*c - a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4
)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4])))/(a*(b*c - a*d)))/(4*c*(b*c - a*d))
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.189.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.75 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

method	result
default	$\frac{d^2x\sqrt{-b\,x^4+a}}{4c(ad-bc)^2(-d\,x^4+c)}+\frac{b^2x}{2a(ad-bc)^2\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\left(\frac{bd}{4(ad-bc)^2c}+\frac{b^2}{2a(ad-bc)^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-$ 3 $\_\alpha=$
elliptic	$\frac{d^2x\sqrt{-b\,x^4+a}}{4c(ad-bc)^2(-d\,x^4+c)}+\frac{b^2x}{2a(ad-bc)^2\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\left(\frac{bd}{4(ad-bc)^2c}+\frac{b^2}{2a(ad-bc)^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-$ 3 $\_\alpha=$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2/c/(a*d-b*c)^2*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/2*b^2*x/a/(a*d-b*c)^
2/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d/(a*d-b*c)^2/c+1/2*b^2/a/(a*d-b*c)^2)/(1/a^
(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*su
m((a*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(
-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2
)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^
(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d-c))
```

3.189. $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

3.189.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

output
```
Timed out
```

3.189.7 Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

3.189.8 Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2), x)
```

3.190 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 439

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx &= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2(a-bx^4)^{3/2}} \\ &+ \frac{b(5b^2c^2-17abcd-3a^2d^2)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)(a-bx^4)^{3/2}(c-dx^4)} \\ &+ \frac{b^{3/4}(5b^2c^2-17abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(3/2)-1/4*d*x/c/(-a*d+b
*c)/(-b*x^4+a)^(3/2)/(-d*x^4+c)+1/12*b*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*x
/a^2/c/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-3*a^2*d^2-17*a*b*c*d+5
*b^2*c^2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/c/(-a*d
+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1/4
)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/
c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*Elliptic
Pi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/
b^(1/4)/c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)
```

3.190.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \frac{x\left(\frac{bd(-5b^2c^2+17abcd+3a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2}+5\left(\frac{5b^3c}{a^2}-\frac{17b^2d}{a}-\cdots\right.\right.}{\cdots}$$

input
```
Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

output
```
(x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*App
ellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^
2 - (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*
a*d^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 1
7*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*
x^4)/a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x
^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))))/(60*(b
*c - a*d)^3*Sqrt[a - b*x^4])
```

3.190.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {931, 25, 1024, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$-\frac{\int -\frac{9bdx^4+4bc-3ad}{(a-bx^4)^{5/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9bdx^4+4bc-3ad}{(a-bx^4)^{5/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\int \frac{2\left(-5bd(2bc+3ad)x^4+10b^2c^2+9a^2d^2-24abcd\right)}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{6a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-5bd(2bc+3ad)x^4+10b^2c^2+9a^2d^2-24abcd}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{\int \frac{2\left(-bd\left(5b^2c^2-17abdc-3a^2d^2\right)x^4+5b^3c^3-9a^3d^3+36a^2bcd^2-17ab^2c^2d\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{2a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-bd\left(5b^2c^2-17abdc-3a^2d^2\right)x^4+5b^3c^3-9a^3d^3+36a^2bcd^2-17ab^2c^2d}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1021

$$\frac{\frac{\frac{b\left(-3a^2d^2-17abcd+5b^2c^2\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 765

$$\frac{\frac{\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 762

$$\frac{\frac{\frac{3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{}{3a(}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 925

3.190. $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

$$\frac{3a^2d^2(13bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)} + \ldots$$

$$3a(bc-ad)$$

$$4c(bc-ad)$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{3a^2d^2(13bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)} + \ldots$$

$$3a(bc-ad)$$

$$4c(bc-ad)$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1543

$$\frac{3a^2d^2(13bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\text{arcsin}\ldots\right.}{\sqrt{a-bx^4}}}{a(bc-ad)}$$

$$3a(bc-ad)$$

$$4c(bc-ad)$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}+3a^2d^2(13bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\ldots\right.}{a(bc-ad)}$$

$$3a(bc-ad)$$

$$4c(bc-ad)$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

3.190. $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx$

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + ((b*(2*b*c + 3*
a*d)*x)/(3*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + ((b*(5*b^2*c^2 - 17*a*b*c*d
- 3*a^2*d^2)*x)/(a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(5*b^2
*c^2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + 3*a^2*d^2*(13*b*c - 3*a*d)*((a^(1/4
)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), A
rcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*
Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin
[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/(a*(b*c - a*d)
))/(3*a*(b*c - a*d)))/(4*c*(b*c - a*d))
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.190. $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

3.190.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.43 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.10

method	result
default	$-\frac{b\,d^3 x\sqrt{-b\,x^4+a}}{4(ad-bc)c(a^2d^2-2abcd+b^2c^2)(bd\,x^4-bc)}+\frac{x\sqrt{-b\,x^4+a}}{6(ad-bc)^2 a\left(x^4-\frac{a}{b}\right)^2}+\frac{b^2x(17ad-5bc)}{12a^2(ad-bc)^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^2}{4(ad-bc)c\left(a^2d^2-2abcd+\right.}\right.}{}$
elliptic	$-\frac{b\,d^3 x\sqrt{-b\,x^4+a}}{4(ad-bc)c(a^2d^2-2abcd+b^2c^2)(bd\,x^4-bc)}+\frac{x\sqrt{-b\,x^4+a}}{6(ad-bc)^2 a\left(x^4-\frac{a}{b}\right)^2}+\frac{b^2x(17ad-5bc)}{12a^2(ad-bc)^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^2}{4(ad-bc)c\left(a^2d^2-2abcd+\right.}\right.}{}$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*d^3/(a*d-b*c)/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(b*d
*x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)^2+1/12*b^2*x/a^2*
(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d^2/(a*d-b*c)/c/(a^
2*d^2-2*a*b*c*d+b^2*c^2)+1/12*b^2/a^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32*d/c*sum
((3*a*d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2
*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1
/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/
2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),
a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2)
)^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

3.190.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.190.6 Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-bx^4)^{\frac{5}{2}}(-c+dx^4)^2}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)
```

3.190.7 Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

3.190.8 Giac [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)
```

3.191 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

3.191.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

output
```
1/4*arctan(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c*2^
(1/2)+1/4*arctanh(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/
4)/c*2^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

output
```
(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]] + ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

3.191.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {920, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$$

$$\Big\downarrow \text{ 920}$$

$$\frac{\int \frac{1}{1-\frac{4abx^4}{\left(bx^4+a\right)^2}}d\frac{x}{\sqrt{bx^4+a}}}{c}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}}d\frac{x}{\sqrt{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}+1}d\frac{x}{\sqrt{bx^4+a}}}{c}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}}d\frac{x}{\sqrt{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{c}$$

input
```
Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

output
```
(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a
^(1/4)*b^(1/4)))/c
```

3.191.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 920
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[a/c
  Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

3.191.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\left(-2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x\left(ab\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{-\sqrt{2}\,\left(ab\right)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}{\sqrt{2}\,\left(ab\right)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}\right)\right)}{8c\left(ab\right)^{\frac{1}{4}}}$	89
default	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x\left(ab\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+\left(ab\right)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-\left(ab\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c\left(ab\right)^{\frac{1}{4}}}$	94
elliptic	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x\left(ab\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+\left(ab\right)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-\left(ab\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c\left(ab\right)^{\frac{1}{4}}}$	94

input
```
int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(-2*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4))+ln((-2^(
1/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2))/(2^(1/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2)
)))/c/(a*b)^(1/4)
```

3.191.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 437, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

output
```
1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b
*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(
a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4
))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/
4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) +
x^2))/(b*x^4 - a)) - 1/4*I*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^
(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) - 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4)
)^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a)) +
1/4*I*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(1
/(a*b*c^4))^(3/4) + 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4
 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a))
```

3.191.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = -\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

input
```
integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

output
```
-Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

3.191.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

3.191.8 Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

3.192 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$

3.192.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

output
```
1/2*arctan(b^(1/4)*x*(a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^(1/2))/a^(1/
4)/b^(1/4)/c+1/2*arctanh(b^(1/4)*x*(a^(1/2)-x^2*b^(1/2))/a^(1/4)/(-b*x^4+a
)^(1/2))/a^(1/4)/b^(1/4)/c
```

3.192.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(\arctan\left(\frac{(1+i)\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a-bx^4}}\right)-i\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-bx^4}}{\sqrt[4]{a}\sqrt[4]{b}x}\right)\right)}{\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

output
```
((1/4 - I/4)*(ArcTan[((1 + I)*a^(1/4)*b^(1/4)*x)/Sqrt[a - b*x^4]] - I*ArcT
an[((1/2 + I/2)*Sqrt[a - b*x^4])/(a^(1/4)*b^(1/4)*x)]))/(a^(1/4)*b^(1/4)*c
)
```

3.192.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {921}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4} \, dx$$

$$\Big\downarrow \ 921$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

output
```
ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a
^(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*S
qrt[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

3.192.3.1 Defintions of rubi rules used

rule 921
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

3.192.4 Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\ln\left(\frac{\frac{-b\,x^{4}+a}{2x^{2}}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}}{x}+\sqrt{ab}}{\frac{-b\,x^{4}+a}{2x^{2}}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141
elliptic	$-\frac{\ln\left(\frac{\frac{-b\,x^{4}+a}{2x^{2}}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}}{x}+\sqrt{ab}}{\frac{-b\,x^{4}+a}{2x^{2}}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141
pseudoelliptic	$-\frac{\ln\left(\frac{-b\,x^{4}+2x^{2}\sqrt{ab}-2(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}\,x+a}{-b\,x^{4}+2(ab)^{\frac{1}{4}}\sqrt{-b\,x^{4}+a}\,x+2x^{2}\sqrt{ab}+a}\right)+2\arctan\left(\frac{x(ab)^{\frac{1}{4}}+\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{-x(ab)^{\frac{1}{4}}+\sqrt{-b\,x^{4}+a}}{x(ab)^{\frac{1}{4}}}\right)}{8(ab)^{\frac{1}{4}}c}$	149

input
```
int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c/(a*b)^(1/4)*(ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+
(a*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a*b)^(1/2
)))+2*arctan((-b*x^4+a)^(1/2)/x/(a*b)^(1/4)+1)+2*arctan((-b*x^4+a)^(1/2)/x
/(a*b)^(1/4)-1))
```

3.192.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{}}{bx^4+a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}-\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{}}{bx^4+a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-}{bx^4+a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-}{bx^4+a}\right.$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

output
```
-1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/
(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/
4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*(
1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^
4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*
x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) - 1/4*I*(1/4)^
(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))
^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) + 2*I*(1/4)^(1/4)*a*c*x
*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*I*(1/4)^(
1/4)*(-1/(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))
^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*I*(1/4)^(1/4)*a*c*x
*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a))
```

3.192.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

input
```
integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

output
```
Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

3.192.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

3.192.8 Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
```

output
```
int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

3.193 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

3.193.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}$$

output
```
1/4*b*x*(b*x^4+a)^(3/4)/d-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x
^4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/d^2-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctanh(b^(1/4)*x/(b
*x^4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)
/(b*x^4+a)^(1/4))/c^(3/4)/d^2
```

3.193.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{2bdx(a+bx^4)^{3/4} - b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{(2+2i)(bc-ad)^{7/4}\arctan\left(\frac{\frac{(1-i)\sqrt[4]{b \cdots}}{\sqrt[4]{c}\sqrt[4]{\cdots}}}{\cdots}\right)}{c^{\cdots}}}{\cdots}$$

input
```
Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

output
```
(2*b*d*x*(a + b*x^4)^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTan[(((1 - I)*(b*c - a*
d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(
1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTanh
[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTanh[(((
1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/
4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4))/(8*d^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {933, 25, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\frac{b(4bc-7ad)x^4+a(bc-4ad)}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{4d} + \frac{bx(a+bx^4)^{3/4}}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\int \frac{b(4bc-7ad)x^4+a(bc-4ad)}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{4d}$$

$$\Big\downarrow \; 1026$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \; 770$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \; 756$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \; 216$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \; 219$$

$$\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \; 902$$

3.193. $\int \frac{\left(a+bx^4\right)^{7/4}}{c+dx^4}dx$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}}{4d}$$

$\downarrow$ 756

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4}}}{2\sqrt{c}}\right.}{d}}{4d}$$

$\downarrow$ 218

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right.}{2c^{3/4}\sqrt[4]{bc-ad}}\right.}{d}}{4d}$$

$\downarrow$ 221

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right.}{2c^{3/4}\sqrt[4]{bc-ad}}\right.}{d}}{4d}$$

input
```
Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

3.193. $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

output
```
(b*x*(a + b*x^4)^(3/4))/(4*d) - ((b*(4*b*c - 7*a*d)*(ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b
^(1/4))))/d - (4*(b*c - a*d)^2*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/d)/(4*d)
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.193. $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.193.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(167) = 334$.

Time = 8.93 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{2}\,(ad-bc)^{2} \ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)}{2}+\sqrt{2}\,(ad-bc)^{2} \arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-}{}$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-1/2*2^(1/2)*(a*d-b*c)^2*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^
4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+2^(1/2)*(a*
d-b*c)^2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)
/c)^(1/4)/x)-2^(1/2)*(a*d-b*c)^2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*
x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-7/2*((a*d-b*c)/c)^(1/4)*c*((-1/2*a*d*
b^(3/4)+2/7*b^(7/4)*c)*ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a
)^(1/4)))+(a*d*b^(3/4)-4/7*b^(7/4)*c)*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-
2/7*(b*x^4+a)^(3/4)*x*b*d))/((a*d-b*c)/c)^(1/4)/d^2/c
```

3.193.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 1962, normalized size of antiderivative = 9.30

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(3/4)*b*x + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 +
7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*
b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^
5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7
*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^
4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*
c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*
d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) + 4*I*d*((b^7*c^7 - 7*a*b^6*c^6*d + 2
1*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c
^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((I*c^2*d^6*x*((b^7*
c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4)
- (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a
^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*I*d*((b^7*c^7 - 7*a*b^6*c^
6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21...
```

3.193.6 Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{7}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

3.193.7 Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

3.193.8 Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{7/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(7/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

3.194 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

3.194.1 Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

output
```
1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arcta
n((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d+1/2*b^(3/4)*arctan
h(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctanh((-a*d+b*c)^(1/
4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d
```

3.194.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx =$$

$$-\frac{-2b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\frac{(1+i)\left((-1+i)b^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+(bc-ad)^{3/4}\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}{2x}\right.\cdots\right.}{c^{3/4}}}{4d}$$

input
```
Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

output
```
-1/4*(-2*b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((1 + I)*((-1 +
I)*b^(3/4)*c^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (b*c - a*d)^(3/
4)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (b*c - a*d)
^(3/4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)
) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/c^(3/4
))/d
```

3.194.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {916, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \text{ 916}$$

$$\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow 770$$

$$\frac{b\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow 756$$

$$\frac{b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow 216$$

$$\frac{b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow 219$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow 902$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}$$

$$\Big\downarrow 756$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow 218$$

3.194. $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}dx$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

output
```
(b*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x
)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/d - ((b*c - a*d)*(ArcTan[((b*c - a*d)^(
1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTa
nh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*
d)^(1/4))))/d
```

3.194.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(133) = 266$.

Time = 5.58 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.95

method	result
pseudoelliptic	$-\frac{2\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} c\left(2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}\,x}\right)-\ln\left(\frac{-b^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)\right)b^{\frac{3}{4}}+\sqrt{2}\,(ad-bc)\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{ac}}\right.\right.}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}dc}$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*d-b*c)/c)^(1/4)*(2*((a*d-b*c)/c)^(1/4)*c*(2*arctan(1/b^(1/4)/x*(b
*x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))
))*b^(3/4)+2^(1/2)*(a*d-b*c)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^
4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan(((
(a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arc
tan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)
))/d/c
```

3.194.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.24

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx =$$

$$-\frac{1}{4}\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,abc}{x}\right.$$

$$+\frac{1}{4}\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(-\frac{c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,ab}{x}\right.$$

$$+\frac{1}{4}i\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{i\,c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,ab}{x}\right.$$

$$-\frac{1}{4}i\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{-i\,c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2}{x}\right.$$

$$+\frac{1}{4}\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)-\frac{1}{4}\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(-\frac{d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

$$-\frac{1}{4}i\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{i\,d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

$$+\frac{1}{4}i\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{-i\,d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

3.194. $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

output
```
-1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)
*log((c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*
d^4))^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(
-(c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4)
)^(3/4) - (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*I*((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((I
*c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) - 1/4*I*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((-I
*c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/
d^4)^(1/4)*log((d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b
^3/d^4)^(1/4)*log(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x) - 1/
4*I*(b^3/d^4)^(1/4)*log((I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/
x) + 1/4*I*(b^3/d^4)^(1/4)*log((-I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/
4)*b^2)/x)
```

3.194.6 Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx=\int \frac{(a+bx^4)^{\frac{3}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

3.194.7 Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

3.194.8 Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx=\int \frac{(b\,x^4+a)^{3/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(3/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

3.195 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx$

3.195.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

output
```
1/2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^
(1/4)+1/2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*
d+b*c)^(1/4)
```

3.195.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$
$$=\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+\operatorname{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)\right)}{c^{3/4}\sqrt[4]{bc-ad}}$$

input
```
Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

output
```
((1/4 + I/4)*(ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)
^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] +
ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1
 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/(c^(3/4)*(b*c
 - a*d)^(1/4))
```

3.195.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

$$\Big\downarrow \; 902$$

$$\int \frac{1}{c-\frac{x^4(bc-ad)}{a+bx^4}}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 756$$

$$\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}$$

$$\Big\downarrow \; 218$$

$$\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

$$\Big\downarrow \; 221$$

$$\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

input
```
Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

output
```
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c
- a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]
/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

3.195.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

3.195.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(81) = 162$.

Time = 4.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)+2\arctan\left(\frac{\left(\frac{ad-b}{c}\right.}{}\right.\right.}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c}$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*d-b*c)/c)^(1/4)*2^(1/2)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)
*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/((((a*d-b*c)/c)^(1/4)*(
b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arcta
n((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2
*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4
)/x))/c
```

3.195.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.195.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

3.195.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

3.195.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

3.196 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} \, dx$

3.196.1 Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} \, dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/4)-1/2*d*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(
b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)-1/2*d*arctanh((-a*d+b*c)^(1/4)*x/
c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)
```

3.196.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \frac{1}{4}\left(\frac{4bx}{(abc-a^2d)\sqrt[4]{a+bx^4}} - \frac{(1+i)d\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}} - \frac{(1+i)d\operatorname{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}}\right)$$

input
```
Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

output
```
((4*b*x)/((a*b*c - a^2*d)*(a + b*x^4)^(1/4)) - ((1 + I)*d*ArcTan[(((1 - I)
*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a
+ b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d)^(5/4)) - (
(1 + I)*d*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1
/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(
3/4)*(b*c - a*d)^(5/4)))/4
```

3.196.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {907, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{902}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}$$

$$\Big\downarrow \text{756}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}$$

$$\Big\downarrow \text{221}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*(ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*
c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4
))))/(b*c - a*d)
```

3.196.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.196.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(108) = 216$.

Time = 4.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.43

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)ad\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x+\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)ad\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\ln\left(\frac{-\left(\frac{ad-}{c}\right.}{\left(\frac{ad-}{c}\right.}\right.}{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}(ad-bc)ca}$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(1/4)*(arctan((((a*d-b*c)/c)^(1/4)*x-2^(
1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)*a*d*2^(1/2)*(b*x^4+a)^(1/4)-a
rctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/
x)*a*d*2^(1/2)*(b*x^4+a)^(1/4)-1/2*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4
)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*
(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))*a*d*2^
(1/2)*(b*x^4+a)^(1/4)-4*b*x*c*((a*d-b*c)/c)^(1/4))/(a*d-b*c)/c/a
```

3.196.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.196.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/4}\,(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{4}}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

3.196.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

3.196.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx=\int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

3.197 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx$

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 180

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}$$

output
```
1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^(5/4)+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)
^2/(b*x^4+a)^(1/4)+1/2*d^2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/
4))/c^(3/4)/(-a*d+b*c)^(9/4)+1/2*d^2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(9/4)
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 12.12 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.45

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{-585c^4(bc-ad)x^4(a+bx^4)^2 - 936c^3d(bc-ad)x^8(a+bx^4)^2 - 416c^2d^2(bc-}{}$$

input
```
Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

output
```
(-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b
*x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4
)^3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925
*c^5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, (
(b*c - a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hyperge
ometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c
- a*d)^3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b
*x^4))] + 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*
c - a*d)*x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric
2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)
^3*x^12*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a
 + b*x^4))] + 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {
1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*H
ypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4)
)])/(325*c^4*(b*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

3.197.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {931, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bdx^4+4bc-5ad}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4bdx^4+4bc-5ad}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}+\frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int -\frac{5a^2d^2}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{5ad^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$\big\downarrow$ 902

$$\frac{\frac{5ad^2\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$\big\downarrow$ 756

$$\frac{\frac{5ad^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$\big\downarrow$ 218

$$\frac{\frac{5ad^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$\big\downarrow$ 221

3.197. $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}dx$

$$\frac{\frac{5ad^2\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(4*b*c - 9*a*d)*x)/(a*(b*c
 - a*d)*(a + b*x^4)^(1/4)) + (5*a*d^2*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/
4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d
)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(b
*c - a*d))/(5*a*(b*c - a*d))
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(148) = 296$.

Time = 4.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.83

method	result
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2} d^{2} \left(\ln\left(\frac{-\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{a d -b c}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{a d -b c}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)-2\arctan\left(\frac{\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} x}\right)+2\,\mathrm{arc}}{8\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(a d -b c\right)^{2}}$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((b*x^4+a)^(5/4)*a^2*d^2*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^
(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x
^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((
((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*ar
ctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x
))*2^(1/2)+16*x*((a*d-b*c)/c)^(1/4)*b*c*(a^2*d-1/2*b*(-9/5*d*x^4+c)*a-2/5*
b^2*c*x^4))/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(5/4)/(a*d-b*c)^2/c/a^2
```

3.197.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.197.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

3.197.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

3.197.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

3.198 $\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

3.198.1 Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)\,x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{d^3\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}$$

output
```
1/9*b*x/a/(-a*d+b*c)/(b*x^4+a)^(9/4)+1/45*b*(-17*a*d+8*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^4+a)^(5/4)+1/45*b*(113*a^2*d^2-100*a*b*c*d+32*b^2*c^2)*x/a^3/(-a
*d+b*c)^3/(b*x^4+a)^(1/4)-1/2*d^3*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4
+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)-1/2*d^3*arctanh((-a*d+b*c)^(1/4)*x/c^
(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)
```

3.198.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.50 (sec) , antiderivative size = 1172, normalized size of antiderivative = 5.03

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx =$$
$$-\frac{-16575c^5(bc-ad)^2x^8(a+bx^4)^2 - 39780c^4d(bc-ad)^2x^{12}(a+bx^4)^2 - 35360c^3d^2(bc-ad)^2x^{16}(a+bx^4)^2}{}$$

input
```
Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

output
```
-1/11475*(-16575*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 - 39780*c^4*d*(b*c -
a*d)^2*x^12*(a + b*x^4)^2 - 35360*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^2
 - 10880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 - 29835*c^6*(b*c - a*d)*
x^4*(a + b*x^4)^3 - 71604*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 - 63648*c^4*
d^2*(b*c - a*d)*x^12*(a + b*x^4)^3 - 19584*c^3*d^3*(b*c - a*d)*x^16*(a + b
*x^4)^3 - 149175*c^7*(a + b*x^4)^4 - 358020*c^6*d*x^4*(a + b*x^4)^4 - 3182
40*c^5*d^2*x^8*(a + b*x^4)^4 - 97920*c^4*d^3*x^12*(a + b*x^4)^4 + 149175*c
^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a +
b*x^4))] + 358020*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, (
(b*c - a*d)*x^4)/(c*(a + b*x^4))] + 318240*c^5*d^2*x^8*(a + b*x^4)^4*Hyper
geometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 97920*c^4*d
^3*x^12*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*
(a + b*x^4))] + 13620*c^3*(b*c - a*d)^4*x^16*Hypergeometric2F1[2, 17/4, 21
/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 36900*c^2*d*(b*c - a*d)^4*x^20*Hy
pergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 33840*
c*d^2*(b*c - a*d)^4*x^24*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4
)/(c*(a + b*x^4))] + 10560*d^3*(b*c - a*d)^4*x^28*Hypergeometric2F1[2, 17/
4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 6480*c^3*(b*c - a*d)^4*x^16*
HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4
))] + 18720*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 17/4}, {1...
```

3.198.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {931, 25, 1024, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}-\frac{\int -\frac{8bdx^4+8bc-9ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{9a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{8bdx^4+8bc-9ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bd(8bc-17ad)x^4+32b^2c^2+45a^2d^2-68abcd}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{4bd(8bc-17ad)x^4+32b^2c^2+45a^2d^2-68abcd}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\frac{bx(113a^2d^2-100abcd+32b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int \frac{45a^3d^3}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\int\frac{1}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$\big\downarrow$ 902

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$\big\downarrow$ 756

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$\big\downarrow$ 221

3.198. $\displaystyle\int\frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + ((b*(8*b*c - 17*a*d)*x)/(5*a*(
b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d^
2)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (45*a^2*d^3*(ArcTan[((b*c - a*d)
^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + Arc
Tanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
a*d)^(1/4))))/(b*c - a*d))/(5*a*(b*c - a*d)))/(9*a*(b*c - a*d))
```

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.198.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$-\frac{3\left(\frac{a^3 d^3 \left(b x^4+a\right)^{\frac{9}{4}}\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b x^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b x^4+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x-\sqrt{2}\left(b x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)+2\,\mathrm{arcta}\cdots}{24}\cdots\right)}{\cdots\left(\frac{ad\cdots}{c}\cdots\right.}$

input
```
int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-3/((a*d-b*c)/c)^(1/4)*(1/24*a^3*d^3*(b*x^4+a)^(9/4)*(ln((-((a*d-b*c)/c)^(
1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/((
(a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^
4+a)^(1/2)))-2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*
d-b*c)/c)^(1/4)/x)+2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4)
)/((a*d-b*c)/c)^(1/4)/x))*2^(1/2)+x*((a*d-b*c)/c)^(1/4)*b*(a^4*d^2-b*(-9/5
*d*x^4+c)*d*a^3+1/3*b^2*(113/45*d^2*x^8-5*c*d*x^4+c^2)*a^2+8/15*x^4*b^3*(-
25/18*d*x^4+c)*c*a+32/135*b^4*c^2*x^8)*c)/(b*x^4+a)^(9/4)/(a*d-b*c)^3/c/a^
3
```

3.198.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.198.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{13}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

3.198.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

3.198.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{13/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

3.199 $\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx$

3.199.1 Optimal result

Integrand size = 21, antiderivative size = 316

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}+\frac{\sqrt{a}b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12d^2(a+bx^4)^{3/4}}+\frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2}+\frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2}$$

output
```
-1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/d^2+1/6*b*x*(b*x^4+a)^(5/4)/d+1/
12*b^(3/2)*(-11*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1
/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d^2/(b*x^4+a)^(3/4)+1/2*(
-a*d+b*c)^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)
/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2+1/2*(-a*d+b*
c)^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2)
,I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2
```

3.199.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.65 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \frac{x\left(5b(a+bx^4)\left(-6bc+13ad+2bdx^4\right)+\frac{b(12b^2c^2-30abcd+23a^2d^2)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4}\right.}{c}}{}$$

input
```
Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

output
```
(x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*
b*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2
*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)
*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a
*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(60*d^2*(a + b*x^4)^(3
/4))
```

3.199.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {933, 25, 1025, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\frac{\sqrt[4]{bx^4+a}\,(b(6bc-11ad)x^4+a(bc-6ad))}{dx^4+c}dx}{6d}+\frac{bx(a+bx^4)^{5/4}}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\int \frac{\sqrt[4]{bx^4+a}\,(b(6bc-11ad)x^4+a(bc-6ad))}{dx^4+c}dx}{6d}$$

$\big\downarrow$ 1025

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{\int -\frac{b\left(12b^2c^2-30abdc+23a^2d^2\right)x^4+a\left(6b^2c^2-13abdc+12a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}+\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}}{6d}$$

$\big\downarrow$ 25

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{\int \frac{b\left(12b^2c^2-30abdc+23a^2d^2\right)x^4+a\left(6b^2c^2-13abdc+12a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}}{6d}$$

$\big\downarrow$ 404

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-ab(6bc-11ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2d}}{6d}$$

$\big\downarrow$ 768

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\big\downarrow$ 858

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\big\downarrow$ 807

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\big\downarrow$ 229

3.199. $\int \frac{\left(a+bx^4\right)^{9/4}}{c+dx^4}dx$

$$\frac{\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}}}{2d}}{6d}}{}$$

$\downarrow$ 923

$$\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\text{Ellipti}}{(a+bx^4)^{3/4}}}{2d}}{6d}$$

$\downarrow$ 925

$$\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{}{\sqrt[4]{}}}{2c}\right.}{2d}}{6d}$$

$\downarrow$ 27

$$\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{}{\sqrt[4]{}}}{2\sqrt{c}}\right.}{2d}}{6d}$$

$\downarrow$ 1542

$$\frac{bx(a+bx^4)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{}}{\sqrt[4]{bx}}\right.\right.}{2\sqrt[4]{b}c}\right.}{2d}}{6d}$$

input
```
Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

3.199. $\int\frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx$

output
```
(b*x*(a + b*x^4)^(5/4))/(6*d) - ((b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/
(2*d) - ((Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a + b*x^4)^(3/4) + 12*(b*c - a*
d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)
+ EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*
x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(2*d))/(6*d)
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.199. $\int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx$

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.199.4 Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{9}{4}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

3.199.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.199.6 Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{9}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(9/4)/(c + d*x**4), x)
```

3.199.7 Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

3.199.8 Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{9/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(9/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(9/4)/(c + d*x^4), x)
```

3.200 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

3.200.1 Optimal result

Integrand size = 21, antiderivative size = 274

$$\begin{aligned}\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx &= \frac{bx\sqrt[4]{a+bx^4}}{2d} \\ &- \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2d\,(a+bx^4)^{3/4}} \\ &- \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd} \\ &- \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd}\end{aligned}$$

output
```
1/2*b*x*(b*x^4+a)^(1/4)/d-1/2*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d/(b*x^4+a)^(3
/4)-1/2*(-a*d+b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/
b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d-1/2*(-a
*d+b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1
/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d
```

3.200.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(-5ac\left(2a^2d+abdx^4+b^2x^4\left(c+dx^4\right)\right)\text{AppellF1}}{\left(c+dx^4\right)\left(-5ac\,\text{AppellF1}\left(\right.\right.}\right.}{10d}$$

input
```
Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*
x^4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] +
b*x^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/
a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^
4)/c)])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/
(10*d*(a + b*x^4)^(3/4))
```

3.200.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {924, 748, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}dx$$

$$\Big\downarrow \text{924}$$

$$\frac{b\int \sqrt[4]{bx^4+a}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{748}$$

$$\frac{b\left(\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{768}$$

$$\frac{b\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2(a+bx^4)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{807}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{229}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{923}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}$$

$$\Big\downarrow \text{925}$$

3.200. $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}dx$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{d}$$

$\downarrow$ 1542

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{d}$$

input

```
Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

output

```
(b*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*E
llipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))))/d -
((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c -
a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(
1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/d
```

3.200. $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 924
```
Int[((a_) + (b_.)*(x_)^4)^(5/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/
d   Int[(a + b*x^4)^(1/4), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^4)^(1
/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.200. $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.200.4 Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

3.200.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.200.6 Sympy [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{5}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

3.200.7 Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

3.200.8 Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{d\,x^4+c}\,dx$$

input

```
int((a + b*x^4)^(5/4)/(c + d*x^4),x)
```

output

```
int((a + b*x^4)^(5/4)/(c + d*x^4), x)
```

3.201 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

3.201.1 Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c}$$

output
```
1/2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2)
,I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c+1/2*EllipticPi(b^(1/4)*x
/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(
b*x^4+a)^(1/2)/b^(1/4)/c
```

3.201.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

$$= \frac{5acx\sqrt[4]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(-4ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,\text{App}\right.\right.}$$

input
```
Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

output
```
(5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*
x^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x
^4)/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

3.201.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}dx$$

$$\Big\downarrow \text{ 923}$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}$$

$$\Big\downarrow \text{ 925}$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{ 1542}$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right.\right.}{2\sqrt[4]{b}c}\right.$$

input
```
Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

output
```
Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]
*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + Ell
ipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^
(1/4)], -1]/(2*b^(1/4)*c))
```

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.201.4 Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

3.201.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.201.6 Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

3.201.7 Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

3.201.8 Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\int \frac{(b\,x^4+a)^{1/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(1/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/4)/(c + d*x^4), x)
```

3.202 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx$

3.202.1 Optimal result

Integrand size = 21, antiderivative size = 259

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

output
```
-b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1
/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1
/2)/a^(1/2))),2^(1/2))/(-a*d+b*c)/(b*x^4+a)^(3/4)/a^(1/2)-1/2*d*EllipticPi
(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+
a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)-1/2*d*EllipticPi(b^(1/4)*x/
(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b
*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)
```

3.202.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^{3/4}(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right.\right.}$$

input
```
Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*
x^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

3.202.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {926, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{926}$$

$$\frac{b\int \frac{1}{(bx^4+a)^{3/4}}dx}{bc-ad}-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{768}$$

$$\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{858}$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 807$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 229$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 923$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 1542$$

3.202. $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

output
```
-((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin
[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c -
 a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(
1/4)*c)))/(b*c - a*d)
```

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 926
```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.202.4 Maple [F]

$$\int \frac{1}{(b\,x^{4}+a)^{\frac{3}{4}}\,(d\,x^{4}+c)}dx$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

3.202.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.202.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

3.202.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

3.202.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)
```

3.203 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

3.203.1 Optimal result

Integrand size = 21, antiderivative size = 304

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(bc-ad)^2(a+bx^4)^{3/4}} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}$$

output
```
1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/3*b^(3/2)*(-5*a*d+2*b*c)*(1+a/b/x^4
)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(
x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/a^(3/2)/(-a*d+b*c)^2/(b*x^4+a)^(3/4)+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x
^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^
4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/
2)/b^(1/4)/c/(-a*d+b*c)^2
```

3.203.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \frac{x\left(-\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(3ad-b(3c+dx^4))\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1\right.\right.}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1\right.\right.}\right.}{15a(-bc+}$$

input
```
Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

output
```
(x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)
/a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4,
3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4,
 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4,
2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

3.203.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {931, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)}-\frac{\int -\frac{2bdx^4+2bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bdx^4+2bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}+\frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{3ad^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{b(2bc-5ad)\int\frac{1}{(bx^4+a)^{3/4}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{3ad^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{3ad^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{3ad^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{3ad^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{923}$$

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{925}$$

3.203. $\int\frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2}{\sqrt{}}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2}{\sqrt{}}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$\downarrow$ 1542

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/}}{}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(2*b*c - 5*a*d)*(1
 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sq
rt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) + (3*a*d^2*Sqrt[a/(a + b*x^4)]*Sqrt[
a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(
Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)
))/(b*c - a*d))/(3*a*(b*c - a*d))
```

3.203. $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

3.203. $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.203.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(d\,x^{4}+c\right)}dx$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

3.203.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.203.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{7}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

3.203.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

3.203.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{7/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)), x)
```

3.204 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

3.204.1 Optimal result

Integrand size = 21, antiderivative size = 357

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{b^{3/2}(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}(bc-ad)^3(a+bx^4)^{3/4}} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}$$

output
```
1/7*b*x/a/(-a*d+b*c)/(b*x^4+a)^(7/4)+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^4+a)^(3/4)-1/21*b^(3/2)*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/
b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),
2^(1/2))/a^(5/2)/(-a*d+b*c)^3/(b*x^4+a)^(3/4)-1/2*d^3*EllipticPi(b^(1/4)*x
/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*
(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^3-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x^4
+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a
)^(1/2)/b^(1/4)/c/(-a*d+b*c)^3
```

3.204.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.71 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{x\left(-\frac{2bd(-6bc+13ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac\left(21a^3d^2+6b^3cx^4(3c\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

output
```
(x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*
x^4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4
 - 13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b
*x^4*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*A
ppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4
, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*A
ppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4
, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(a
+ b*x^4)^(3/4))
```

3.204.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {931, 25, 1024, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)} - \frac{\int -\frac{6bdx^4+6bc-7ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{7a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6bdx^4+6bc-7ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}-\frac{\int -\frac{2bd(6bc-13ad)x^4+12b^2c^2+21a^2d^2-26abcd}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{3a(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2bd(6bc-13ad)x^4+12b^2c^2+21a^2d^2-26abcd}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 404$$

$$\frac{\frac{\frac{b\left(47a^2d^2-38abcd+12b^2c^2\right)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 768$$

$$\frac{\frac{\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 858$$

$$\frac{\frac{-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \; 807$$

3.204. $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}dx$

$$\frac{\frac{-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\big\downarrow$ 229

$$\frac{\frac{-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\big\downarrow$ 923

$$\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}-}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\big\downarrow$ 925

$$\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4}{}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\big\downarrow$ 27

3.204. $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

$$-\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4\cdots}{\cdots}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$$\Big\downarrow \ 1542$$

$$-\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\cdots}{\cdots}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + ((b*(6*b*c - 13*a*d)*x)/(3*a*(
b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*
a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]
/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (21*a^2*d^3*Sqrt[a/(a +
 b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])),
 ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]
/(2*b^(1/4)*c)))/(b*c - a*d))/(3*a*(b*c - a*d)))/(7*a*(b*c - a*d))
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.204. $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.204.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{11}{4}}\left(d\,x^{4}+c\right)}d x$$

input
```
int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

3.204.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.204.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{11}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(11/4)*(c + d*x**4)), x)
```

3.204.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{11}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

3.204.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{11}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{11/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(11/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(11/4)*(c + d*x^4)), x)
```

3.205 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

3.205.1 Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx = \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{b^{7/4}(8bc-11ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc-ad)^{7/4}(8bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} - \frac{b^{7/4}(8bc-11ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc-ad)^{7/4}(8bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

output
```
1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^(3/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(7/4
)/c/d/(d*x^4+c)-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/
4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(1/
4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctanh(b^(1/4
)*x/(b*x^4+a)^(1/4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctanh((-a*d+
b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3
```

3.205.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)dx(a+bx^4)^{3/4}(-2abcd+a^2d^2+b^2c(2c+dx^4))}{c(c+dx^4)} - (1-i)b^{7/4}(8bc-11ad)\arctan\left(\right.\right.}{}$$

input
```
Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

output
```
((1/16 + I/16)*(((2 - 2*I)*d*x*(a + b*x^4)^(3/4)*(-2*a*b*c*d + a^2*d^2 + b
^2*c*(2*c + d*x^4)))/(c*(c + d*x^4)) - (1 - I)*b^(7/4)*(8*b*c - 11*a*d)*Ar
cTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*A
rcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 +
 I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(7/4) - (1 - I
)*b^(7/4)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(
1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^
(1/4))/(2*x)])/c^(7/4)))/d^3
```

3.205.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {930, 1025, 27, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{(bx^4+a)^{3/4}\left(4b(2bc-ad)x^4+a(bc+3ad)\right)}{dx^4+c}dx}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{\int -\frac{4\left(b^2c(8bc-11ad)x^4+a\left(2b^2c^2-2abdc-3a^2d^2\right)\right)}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{4d}+\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{d}-\frac{\int \frac{b^2c(8bc-11ad)x^4+a\left(2b^2c^2-2abdc-3a^2d^2\right)}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 1026$$

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{d}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 770$$

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{d}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx}}{d}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 216$$

3.205. $\int \frac{\left(a+bx^4\right)^{11/4}}{\left(c+dx^4\right)^2}dx$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 219

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 902

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+}}}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 756

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4}}}{2\sqrt{c}}\right.}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 218

3.205. $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4\cdots}}}{2\sqrt{c}}\cdots\right)}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\cdots\right)}{d}}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

input
```
Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(7/4))/(c*d*(c + d*x^4)) + ((b*(2*b*c - a*
d)*x*(a + b*x^4)^(3/4))/d - ((b^2*c*(8*b*c - 11*a*d)*(ArcTan[(b^(1/4)*x)/(
a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*
b^(1/4))))/d - ((b*c - a*d)^2*(8*b*c + 3*a*d)*(ArcTan[((b*c - a*d)^(1/4)*x
)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/
4))))/d)/d)/(4*c*d)
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.205. $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx$

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(232) = 464$.

Time = 5.36 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.81

method	result
pseudoelliptic	$-\frac{-8\left(2b^2c^2-2b\left(-\frac{bx^4}{2}+a\right)dc+a^2d^2\right)xd\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c\left(bx^4+a\right)^{\frac{3}{4}}+\left(16\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^3\left(\ln\left(\frac{-b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\right.\right.\right.}{}$

input
```
int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-8*(2*b^2*c^2-2*b*(-1/2*b*x^4+a)*d*c+a^2*d^2)*x*d*((a*d-b*c)/c)^(1/
4)*c*(b*x^4+a)^(3/4)+(16*((a*d-b*c)/c)^(1/4)*c^3*(ln((-b^(1/4)*x-(b*x^4+a)
^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))
)*b^(11/4)-22*((a*d-b*c)/c)^(1/4)*a*c^2*d*(ln((-b^(1/4)*x-(b*x^4+a)^(1/4))
/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b^(7/
4)+2^(1/2)*(3*a*d+8*b*c)*(a*d-b*c)^2*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(
1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/
4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*a
rctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/
x)+2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^
(1/4)/x)))*(d*x^4+c))/((a*d-b*c)/c)^(1/4)/d^3/c^2/(d*x^4+c)
```

3.205.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 2764, normalized size of antiderivative = 9.87

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*((c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464
*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^
12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*
a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5
*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a
^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*
c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4
+ c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2
- 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7
931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^
9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log((c
^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 -
37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 793
1*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*
b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) - (512*b
^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 ...
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

output
```
Timed out
```

3.205.7 Maxima [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

3.205.8 Giac [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{11/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

3.206 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx$

3.206.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx = -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd(c+dx^4)} + \frac{b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} + \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^4+a)^(3/4)/c/d/(d*x^4+c)+1/2*b^(7/4)*arctan(b^(1/4)
*x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctan((-a*d+b*
c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2+1/2*b^(7/4)*arctanh(b^(1/4
)*x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctanh((-a*d+
b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2
```

3.206.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)d(bc-ad)x(a+bx^4)^{3/4}}{c(c+dx^4)} + (4-4i)b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - \frac{(4b^2c^2-abcd-3a\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

output
```
((1/16 + I/16)*(((-2 + 2*I)*d*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*(c + d*x
^4)) + (4 - 4*I)*b^(7/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c
^2 - a*b*c*d - 3*a^2*d^2)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)
*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4
))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4)) + (4 - 4*I)*b^(7/4)*ArcTanh[(b^(1/4
)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c^2 - a*b*c*d - 3*a^2*d^2)*ArcTanh[(((1
- I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)
*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4))
))/d^2
```

3.206.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {930, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{4b^2cx^4+a(bc+3ad)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 1026

$$\frac{\frac{4b^2c\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 770

$$\frac{\frac{4b^2c\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 756

$$\frac{\frac{4b^2c\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 216

$$\frac{\frac{4b^2c\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 219

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 902

3.206. $\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}dx$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx}\ldots}}{2\sqrt{c}}\ldots\right.}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\ldots\right.}{2c^{3/4}\sqrt[4]{bc-ad}}\ldots\right.}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\ldots\right.}{2c^{3/4}\sqrt[4]{bc-ad}}\ldots\right.}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

3.206. $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*d*(c + d*x^4)) + ((4*b^2*c*(ArcT
an[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b
*x^4)^(1/4)]/(2*b^(1/4))))/d - ((b*c - a*d)*(4*b*c + 3*a*d)*(ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))
 + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(
b*c - a*d)^(1/4))))/d)/(4*c*d)
```

3.206.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.206. $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

3.206.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(186) = 372.

Time = 4.51 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.77

method	result
pseudoelliptic	$-\frac{-(b\,x^4+a)^{\frac{3}{4}}x(ad-bc)dc\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}+\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^2\left(2\arctan\left(\frac{(b\,x^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\ln\left(\frac{-b^{\frac{1}{4}}x-(b\,x^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-(b\,x^4+a)^{\frac{1}{4}}}\right)\right)b^{\frac{7}{4}}+\frac{\sqrt{2}\,(3a^2d^2}{}\right.}{}$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

3.206. $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

output
```
-1/4*(-(b*x^4+a)^(3/4)*x*(a*d-b*c)*d*c*((a*d-b*c)/c)^(1/4)+(((a*d-b*c)/c)^
(1/4)*c^2*(2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^
(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))))*b^(7/4)+1/8*2^(1/2)*(3*a^2*d^2+a*b*c*
d-4*b^2*c^2)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c
)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/
2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((((a*d-b*c)/c)^(1/
4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan((((a*d-b*c)/
c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)))*(d*x^4+c))/((
a*d-b*c)/c)^(1/4)/d^2/c^2/(d*x^4+c)
```

3.206.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 1440, normalized size of antiderivative = 6.26

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + (c*d^2*x^4 + c^2*d)*((256*b^7*c
^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((c^5
*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4
*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^
2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) - (c*d^2*x^
4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609
*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^
7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^
3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^
6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*
b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a
)^(1/4))/x) + (-I*c*d^2*x^4 - I*c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2
 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a
^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((I*c^5*d^6*x*((256*b^7*c^7 -
 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5
*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^
5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c
*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) + (I*c*d^2*x^4 + I*c^2*d)*((25...
```

3.206.6 Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{7}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(7/4)/(c + d*x**4)**2, x)
```

3.206.7 Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

3.206.8 Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx=\int \frac{(b\,x^4+a)^{7/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

3.207 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx$

3.207.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}+\frac{3a\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

output
```
1/4*x*(b*x^4+a)^(3/4)/c/(d*x^4+c)+3/8*a*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/
(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)+3/8*a*arctanh((-a*d+b*c)^(1/4)*x
/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)
```

3.207.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\frac{4c^{3/4}\sqrt[4]{bc-ad}x(a+bx^4)^{3/4}+(3+3i)a(c+dx^4)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a}\cdots}{\sqrt[4]{bc-\cdots}}}{2x}\cdots\right.}{16c^{7/4}\sqrt[4]{bc-ad}\,(\cdots}$$

input
```
Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

output
```
(4*c^(3/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + (3 + 3*I)*a*(c + d*x^4)
*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1
 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (3 + 3*I)*a*(
c + d*x^4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(
1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(16
*c^(7/4)*(b*c - a*d)^(1/4)*(c + d*x^4))
```

3.207.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {903, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \ 903$$

$$\frac{3a\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{4c}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \ 902$$

$$\frac{3a\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4c}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \ 756$$

$$\frac{3a\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{4c}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{3a\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \ 221$$

$$\frac{3a\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

input
```
Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*(ArcTan[((b*c - a*d)^(1/4)*
x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1
/4))))/(4*c)
```

3.207.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.207.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(107) = 214$.

Time = 4.43 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.17

method	result
pseudoelliptic	$\frac{\frac{\left(b\,x^4+a\right)^{\frac{3}{4}} x c \left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}}{4}+\frac{3\sqrt{2}\,a\left(d\,x^4+c\right)\left(2\arctan\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}\right)-2\arctan\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x+\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}\right)-\ln\left(\right.}{32}}{c^2\left(d\,x^4+c\right)\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}}$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/16/((a*d-b*c)/c)^(1/4)*(4/3*(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/4)+1/2*
2^(1/2)*a*(d*x^4+c)*(2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/
4))/((a*d-b*c)/c)^(1/4)/x)-2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+
a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*
2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b
*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))))/c^2/(d
*x^4+c)
```

3.207.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.207.6 Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(a+bx^4)^{\frac{3}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

3.207.7 Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

3.207.8 Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

3.208 $\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$

3.208.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)\,(c+dx^4)}+\frac{(4bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

output
```
-1/4*d*x*(b*x^4+a)^(3/4)/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*arctan(
(-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(5/4)+1/8*(
-3*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/
(-a*d+b*c)^(5/4)
```

3.208.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$
$$= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)c^{3/4}dx(a+bx^4)^{3/4}}{(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{5/4}}+\frac{(4bc-3ad)\text{arctanh}}{}\right.}{c^{7/4}}$$

input
```
Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

output
```
((1/16 + I/16)*(((-2 + 2*I)*c^(3/4)*d*x*(a + b*x^4)^(3/4))/((b*c - a*d)*(c
 + d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(
1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^
(1/4))/(2*x)])/(b*c - a*d)^(5/4) + ((4*b*c - 3*a*d)*ArcTanh[(((1 - I)*(b*c
 - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x
^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(5/4)))/c^(7/4)
```

3.208.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {907, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$
$$\Big\downarrow \; 907$$
$$\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx(a+bx^4)^{3/4}}{4c(c+dx^4)(bc-ad)}$$
$$\Big\downarrow \; 902$$

$$\frac{(4bc-3ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{756}$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(4bc-3ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x*(a + b*x^4)^(3/4))/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d
)*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b
*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4
))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(4*c*(b*c - a*d))
```

3.208.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.208. $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2}\,dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.208.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(134) = 268$.

Time = 4.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.12

method	result
pseudoelliptic	$-\frac{\frac{3\left(ad-\frac{4bc}{3}\right)\sqrt{2}\left(d\,x^4+c\right)\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}\right)}{32}+\frac{3\left(ad-\frac{4bc}{3}\right)\sqrt{2}\left(d\,x^4+c\right)\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x \cdots}{\left(\frac{ad \cdots}{}\right.}\right.}{16}}{c^2(ad-bc)(d\,x^4+c)\left(\frac{ad-\cdots}{c}\right.}$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/16*(-1/2*(a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4
+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c
)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))
)+(a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*arctan(((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b
*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-(a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*arct
an(((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+
4/3*d*(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/4))/((a*d-b*c)/c)^(1/4)/c^2/(a*
d-b*c)/(d*x^4+c)
```

3.208.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.208.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

3.208.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

3.208.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx=\int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx=\int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

3.209 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2} dx$

3.209.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2} dx = \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}(c+dx^4)} - \frac{d(8bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}$$

output
```
1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d+b*c)/
(b*x^4+a)^(1/4)/(d*x^4+c)-1/8*d*(-3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c
^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)-1/8*d*(-3*a*d+8*b*c)*arct
anh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)
```

3.209.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)c^{3/4}x\left(a^2d^2+abd^2x^4+4b^2c\left(c+dx^4\right)\right)}{a(bc-ad)^2\sqrt[4]{a+bx^4}\left(c+dx^4\right)}+\frac{d(-8bc+3ad)\arctan\left(\frac{(1-i)\sqrt[4]{bc-a}}{\sqrt[4]{c}\sqrt[4]{a+b}}\right.}{(bc-ad)^{9}}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

output
```
((1/16 + I/16)*(((2 - 2*I)*c^(3/4)*x*(a^2*d^2 + a*b*d^2*x^4 + 4*b^2*c*(c +
 d*x^4)))/(a*(b*c - a*d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) + (d*(-8*b*c + 3
*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4))
- ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*
d)^(9/4) + (d*(-8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c
^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d
)^(1/4))/(2*x)])/(b*c - a*d)^(9/4)))/c^(7/4)
```

3.209.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {931, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-4bdx^4+4bc-3ad}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 902$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 221$$

3.209. $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) + ((b*(4*b*c + a*
d)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*(8*b*c - 3*a*d)*(ArcTan[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)
) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*
(b*c - a*d)^(1/4))))/(b*c - a*d))/(4*c*(b*c - a*d))
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(173) = 346$.

Time = 4.48 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.73

method	result
pseudoelliptic	$\frac{\frac{\sqrt{2}\, d a \left(d\,x^{4}+c\right)\left(3 a d-8 b c\right)\left(2 \arctan\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}\right)-2 \arctan\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x+\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}\right)-\ln\left(\frac{-\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}\right.}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}\right.}\right.}{8}}{4\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}} c^{2}\left(d\,x^{4}+c\right)(a c}$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(1/4)*(1/8*2^(1/2)*d*a*(d*x^4+c)*(3*a*d-
8*b*c)*(2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c
)/c)^(1/4)/x)-2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a
*d-b*c)/c)^(1/4)/x)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a
*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)
*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))))*(b*x^4+a)^(1/4)+x*((
a*d-b*c)/c)^(1/4)*c*(4*b^2*c^2+4*b^2*c*d*x^4+d^2*a*(b*x^4+a)))/c^2/(d*x^4+
c)/(a*d-b*c)^2/a
```

3.209.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.209.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)
```

3.209.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

3.209.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

3.210 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

3.210.1 Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)\,x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{5/4}(c+dx^4)} + \frac{3d^2(4bc-ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}$$

output
```
1/20*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(5/4)+1/20*b*(-5*a^2*d^2
-56*a*b*c*d+16*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a
*d+b*c)/(b*x^4+a)^(5/4)/(d*x^4+c)+3/8*d^2*(-a*d+4*b*c)*arctan((-a*d+b*c)^(
1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13/4)+3/8*d^2*(-a*d+4*
b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c
)^(13/4)
```

3.210.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{80}+\frac{i}{80}\right)\left(\frac{(2-2i)c^{3/4}x(5a^4d^3+10a^3bd^3x^4-16b^4c^2x^4(c+dx^4)+5a^2b^2d(12c^2+12cdx^4+d^2x^8)+4ab\cdots}{a^2(-bc+ad)^3(a+bx^4)^{5/4}(c+dx^4)}\cdots\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

output
```
((1/80 + I/80)*(((2 - 2*I)*c^(3/4)*x*(5*a^4*d^3 + 10*a^3*b*d^3*x^4 - 16*b^
4*c^2*x^4*(c + d*x^4) + 5*a^2*b^2*d*(12*c^2 + 12*c*d*x^4 + d^2*x^8) + 4*a*
b^3*c*(-5*c^2 + 9*c*d*x^4 + 14*d^2*x^8)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^4
)^(5/4)*(c + d*x^4)) + (15*d^2*(4*b*c - a*d)*ArcTan[(((1 - I)*(b*c - a*d)^
(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4
))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13/4) + (15*d^2*(4*b*c - a*d)*A
rcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1
+ I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13
/4)))/c^(7/4)
```

3.210.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {931, 1024, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-8bdx^4+4bc-3ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bd(4bc+5ad)x^4+16b^2c^2+15a^2d^2-40abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4bd(4bc+5ad)x^4+16b^2c^2+15a^2d^2-40abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int -\frac{15a^2d^2(4bc-ad)}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{bc-ad}+\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{902}$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}+\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{756}$$

3.210. $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{15ad^2(4bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}+\frac{15ad^2(4bc-ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + ((b*(4*b*c + 5*
a*d)*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(16*b^2*c^2 - 56*a*b*c*d
 - 5*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) + (15*a*d^2*(4*b*c - a*
d)*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(
b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(b*c - a*d))/(5*a*(b*c - a*d)))/(4*c*
(b*c - a*d))
```

3.210. $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.210. $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2} dx$

3.210.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$-\frac{3\left(\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2} d^{2} \left(d\,x^{4}+c\right)\left(a d-4 b c\right)\left(\ln\left(\frac{-\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a d-b c}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a d-b c}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)-2\arctan\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}x-}{\left(\frac{a d-b}{c}\right.}\right.\right.\right.}{}$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/32/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(5/4)*((b*x^4+a)^(5/4)*a^2*d^2*(d*x^4+
c)*(a*d-4*b*c)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b
*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan(((((a*d-b*c)/c)^(
1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan(((((a*d-b*c
)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x))*2^(1/2)-8/15
*(5*a^2*b^2*d^3*x^8+56*a*b^3*c*d^2*x^8-16*b^4*c^2*d*x^8+10*a^3*b*d^3*x^4+6
0*a^2*b^2*c*d^2*x^4+36*a*b^3*c^2*d*x^4-16*b^4*c^3*x^4+5*a^4*d^3+60*a^2*b^2
*c^2*d-20*a*b^3*c^3)*x*c*((a*d-b*c)/c)^(1/4))/c^2/(d*x^4+c)/(a*d-b*c)^3/a^
2
```

3.210.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.210.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

3.210.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

3.210.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

3.211 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$

3.211.1 Optimal result

Integrand size = 21, antiderivative size = 353

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}$$
$$-\frac{\sqrt{a}b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd^2\,(a+bx^4)^{3/4}}$$
$$-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2}$$
$$-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2}$$

output
```
1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^(1/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(5/4
)/c/d/(d*x^4+c)-1/4*b^(3/2)*(-a*d+3*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*ar
ccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*E
llipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/d^2/(b*x^
4+a)^(3/4)-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4)
,-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^
(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1
/2)/b^(1/4)/c^2/d^2
```

3.211.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.48 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{2b(-3b^2c^2+3abcd+a^2d^2)\,x^5\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5acx\right.}{}}{}$$

input
```
Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

output
```
(2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1
[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2
 + a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -
((b*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c
*(3*c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((
c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]
+ x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*
c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(
a + b*x^4)^(3/4))
```

3.211.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {930, 1025, 27, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{\sqrt[4]{bx^4+a}\,(2b(3bc-ad)x^4+a(bc+3ad))}{dx^4+c}dx}{4cd}-\frac{x(a+bx^4)^{5/4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{\int -\frac{2\left(2b\left(3b^2c^2-3abdc-a^2d^2\right)x^4+a\left(3b^2c^2-2abdc-3a^2d^2\right)\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}+\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{\int \frac{2b\left(3b^2c^2-3abdc-a^2d^2\right)x^4+a\left(3b^2c^2-2abdc-3a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-ab(3bc-ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{229}$$

3.211. $\int \frac{\left(a+bx^4\right)^{9/4}}{\left(c+dx^4\right)^2}dx$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 923

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\,\text{Elli}}{\left(a+bx^4\right)^{3/}}}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 925

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}}{2c}\right.}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 27

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}}{2\sqrt{c}}\right.}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\downarrow$ 1542

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\right.\right.}{2\sqrt[4]{b}c}\right.}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

3.211. $\int\frac{\left(a+bx^4\right)^{9/4}}{\left(c+dx^4\right)^2}dx$

input
```
Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(5/4))/(c*d*(c + d*x^4)) + ((b*(3*b*c - a*
d)*x*(a + b*x^4)^(1/4))/d - ((Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4)
)^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a + b*x^4)^(3/
4) + 3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(Elli
pticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4
)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/d)/(4*c*d)
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.211.4 Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{9}{4}}}{(d\,x^4+c)^2}dx$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

3.211.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.211.6 Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{9}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(9/4)/(c + d*x**4)**2, x)
```

3.211.7 Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

3.211.8 Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{9/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(9/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(9/4)/(c + d*x^4)^2, x)
```

3.212 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx$

3.212.1 Optimal result

Integrand size = 21, antiderivative size = 298

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd(a+bx^4)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^4+a)^(1/4)/c/d/(d*x^4+c)+1/4*b^(3/2)*(1+a/b/x^4)^(3
/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*
b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*
a^(1/2)/c/d/(b*x^4+a)^(3/4)+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+
a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a
)^(1/2)/b^(1/4)/c^2/d+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)
/b^(1/4)/c^2/d
```

3.212.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \frac{x\left(2b(bc+ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d-b^2cx^4+abd\ldots\right.}{(c+dx^4)\left(-\ldots\right.}\ldots\right.}{\ldots}$$

input
```
Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

output
```
(x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4
)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*
x^4*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/
c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c
+ d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] +
x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*
AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*d*(a +
 b*x^4)^(3/4))
```

3.212.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {930, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{2b(bc+ad)x^4+a(bc+3ad)}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{404}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx - ab\int \frac{1}{(bx^4+a)^{3/4}}dx}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{768}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx - \frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{858}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx + \frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx + \frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{229}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx + \frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{923}$$

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}} + \frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c} + \frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right) + \sqrt{\cdots}}{4cd} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

3.212. $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)+\sqrt{\ }\cdots}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\cdots\right.}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(1/4))/(c*d*(c + d*x^4)) + ((Sqrt[a]*b^(3/
2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(a + b*x^4)^(3/4) + (2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*
(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqr
t[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(4*c*d)
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.212. $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.212.4 Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{(d\,x^4+c)^2}dx$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

3.212.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.212.6 Sympy [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{5}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(5/4)/(c + d*x**4)**2, x)
```

3.212.7 Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

3.212.8 Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx=\int \frac{(b\,x^4+a)^{5/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(5/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(5/4)/(c + d*x^4)^2, x)
```

3.213 $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$

3.213.1 Optimal result

Integrand size = 21, antiderivative size = 308

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4c(bc-ad)(a+bx^4)^{3/4}} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$

output
```
1/4*x*(b*x^4+a)^(1/4)/c/(d*x^4+c)-1/4*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1
/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2
)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/(-a*
d+b*c)/(b*x^4+a)^(3/4)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1
/2)/b^(1/4)/c^2/(-a*d+b*c)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+
a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)
^(1/2)/b^(1/4)/c^2/(-a*d+b*c)
```

3.213.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \frac{x\left(\frac{2bx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c} \cdots\right.}{-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},- \cdots\right.\right.}\right.}{c+dx^4}\cdots}{20\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

output
```
(x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5
/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x
^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)
/c)]))))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

3.213.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {929, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 929$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)}-\frac{\int -\frac{2bx^4+3a}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4c}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2bx^4+3a}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{ab\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}+\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{923}$$

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{925}$$

3.213. $\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}dx$

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}\right.}{}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}\right.}{}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\downarrow$ 1542

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{}{b}\right.}{}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) + (-((Sqrt[a]*b^(3/2)*(1 + a/(b*x^
4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/((b*c - a*d)*
(a + b*x^4)^(3/4))) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]
*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sq
rt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(b*c -
a*d))/(4*c)
```

3.213. $\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

3.213. $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2} \, dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.213.4 Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(d\,x^{4}+c\right)^{2}}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

3.213.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.213.6 Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

3.213.7 Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

3.213.8 Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(1/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/4)/(c + d*x^4)^2, x)
```

3.214 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx$

3.214.1 Optimal result

Integrand size = 21, antiderivative size = 330

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx =& -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)}\\ &-\frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4\sqrt{a}c(bc-ad)^2(a+bx^4)^{3/4}}\\ &-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}\\ &-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}\end{aligned}$$

output
```
-1/4*d*x*(b*x^4+a)^(1/4)/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^(3/2)*(-a*d+4*b*c)*(
1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/
2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2
))),2^(1/2))/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)/a^(1/2)-3/8*d*(-a*d+2*b*c)*Ell
ipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/
(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*
c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I
)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2
```

3.214.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \frac{x\left(\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad}+\frac{c\left(25ac(-4bc+4ad+bdx^4)\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},\right.\right.}{(bc-ad)(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\right.\right.}\right.}{20c^2}$$

input
```
Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

output
```
(x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*
AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^
4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*A
ppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c +
 d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x
^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*A
ppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*(a + b*
x^4)^(3/4))
```

3.214.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {931, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-2bdx^4+4bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{b(4bc-ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 768

$$\frac{\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 858

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 807

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 229

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 923

$$\frac{-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 925

3.214. $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}dx$

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1542

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x*(a + b*x^4)^(1/4))/(c*(b*c - a*d)*(c + d*x^4)) + (-((b^(3/2)*(4*
b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (3*d*(2*b*c - a*d)*Sq
rt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*S
qrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + Ellip
ticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1
/4)], -1]/(2*b^(1/4)*c)))/(b*c - a*d))/(4*c*(b*c - a*d))
```

3.214. $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}\,dx$

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.214. $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx$

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.214.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(d\,x^{4}+c\right)^{2}}d x$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

3.214.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}\,(c+dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.214.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)**2), x)
```

3.214.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

3.214.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{3/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x)
```

3.215 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

3.215.1 Optimal result

Integrand size = 21, antiderivative size = 390

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx &= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} \\
&- \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} \\
&- \frac{b^{3/2}(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}c(bc-ad)^3(a+bx^4)^{3/4}} \\
&+ \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} \\
&+ \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}
\end{aligned}$$

output
```
1/12*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-1/4*d*x/c/(-a*d+b*
c)/(b*x^4+a)^(3/4)/(d*x^4+c)-1/12*b^(3/2)*(3*a^2*d^2-32*a*b*c*d+8*b^2*c^2)
*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(
1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1
/2))),2^(1/2))/a^(3/2)/c/(-a*d+b*c)^3/(b*x^4+a)^(3/4)+1/8*d^2*(-3*a*d+10*b
*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2)
,I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3+1/8*d^2*(
-3*a*d+10*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/
2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3
```

3.215.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.54 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \frac{x\left(2bd(4bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c(25ac(}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

output
```
(x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c
+ d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a),
 -((d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*A
ppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF
1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x
^4)^(3/4))
```

3.215.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {931, 1024, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-6bdx^4+4bc-3ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{4c(bc-ad)} - \frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}-\frac{\int -\frac{2bd(4bc+3ad)x^4+8b^2c^2+9a^2d^2-24abcd}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2bd(4bc+3ad)x^4+8b^2c^2+9a^2d^2-24abcd}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{\frac{b(3a^2d^2-32abcd+8b^2c^2)\int \frac{1}{(bx^4+a)^{3/4}}dx}{bc-ad}+\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3a^2d^2-32abcd+8b^2c^2)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}+\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3a^2d^2-32abcd+8b^2c^2)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3a^2d^2-32abcd+8b^2c^2)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

3.215. $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

$$\Big\downarrow \; 229$$

$$\frac{\frac{\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 923$$

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2 \ldots\right.}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)} \ldots}{4c(bc-ad)} \ldots$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right) \ldots}{\ldots}}{3a(bc-ad)} \ldots}{4c(bc-ad)} \ldots$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right) \ldots}{\ldots}}{3a(bc-ad)} \ldots}{4c(bc-ad)} \ldots$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1542$$

3.215. $\displaystyle\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx$

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}\cdots\right.}{}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) + ((b*(4*b*c + 3*
a*d)*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(8*b^2*c^2 - 32*
a*b*c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(S
qrt[b]*x^2)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) + (3*a*d^2*(1
0*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c
- a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b
^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*
x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(b*c - a*d))/(3*a*(b*c - a*d)))
/(4*c*(b*c - a*d))
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.215. $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.215.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(d\,x^{4}+c\right)^{2}}d x$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

3.215.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{7/4} \left(c+dx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.215.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^4\right)^{7/4} \left(c+dx^4\right)^2}\,dx = \int \frac{1}{\left(a+bx^4\right)^{\frac{7}{4}} \left(c+dx^4\right)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)**2), x)
```

3.215.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx=\int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

3.215.8 Giac [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx=\int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx=\int \frac{1}{(b\,x^4+a)^{7/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x)
```

3.216 $\int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)}\,dx$

3.216.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

output
```
1/4*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+1/4*arctanh(1/2*x*2^(3/4)/
(x^4+1)^(1/4))*2^(1/4)
```

3.216.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)+\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

input
```
Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

output
```
(ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))
])/(2*2^(3/4))
```

3.216.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{2-\frac{x^4}{x^4+1}}d\frac{x}{\sqrt[4]{x^4+1}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}}+\sqrt{2}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

input
```
Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

output
```
ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 +
x^4)^(1/4))]/(2*2^(3/4))
```

3.216.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

3.216.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{4}} \left(2 \arctan\left(\frac{2^{\frac{1}{4}} \left(x^{4}+1\right)^{\frac{1}{4}}}{x}\right)-\ln\left(\frac{2^{\frac{3}{4}} x+2 \left(x^{4}+1\right)^{\frac{1}{4}}}{-2^{\frac{3}{4}} x+2 \left(x^{4}+1\right)^{\frac{1}{4}}}\right)\right)}{8}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{4}-2\right) \ln\left(\frac{2 \sqrt{x^{4}+1}\, \operatorname{RootOf}\left(\_Z^{4}-2\right)^{3} x^{2}+2 \left(x^{4}+1\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^{4}-2\right)^{2} x^{3}+3 \operatorname{RootOf}\left(\_Z^{4}-2\right) x^{4}+4 \left(x^{4}+1\right)^{\frac{3}{4}} x}{x^{4}+2}\right.}{8}$

input
```
int(1/(x^4+1)^(1/4)/(x^4+2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/4)*(2*arctan(1/x*2^(1/4)*(x^4+1)^(1/4))-ln((2^(3/4)*x+2*(x^4+1)^
(1/4))/(-2^(3/4)*x+2*(x^4+1)^(1/4))))
```

3.216.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 5.09

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx$$
$$= \frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2+8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)+\frac{1}{64}i$$
$$\cdot 8^{\frac{3}{4}}\log\left(-\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8i\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(3ix^4+2i)-16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$
$$-\frac{1}{64}i$$
$$\cdot 8^{\frac{3}{4}}\log\left(-\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8i\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(-3ix^4-2i)-16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$
$$-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

output
```
1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x^4 + 1)*
x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) + 1/64*I*8^(3
/4)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*I*8^(1/4)*sqrt(x^4 + 1)*x^2 -
8^(3/4)*(3*I*x^4 + 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64*I*8^(3/4
)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*I*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^
(3/4)*(-3*I*x^4 - 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64*8^(3/4)*l
og((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*
(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

3.216.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}\,dx$$

input
```
integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

output
```
Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

3.216.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

3.216.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+1)^{1/4}\,(x^4+2)}\,dx$$

input
```
int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
```

output
```
int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

3.217 $\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$

3.217.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

output
```
1/2*arctan(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)+1/2*arctanh(a^(1/4)*x/(b*x^4
+a)^(1/4))/a^(5/4)
```

3.217.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right) + \text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

input
```
Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

output
```
(ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(
1/4)])/(2*a^(5/4))
```

3.217.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4(a-b))\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \ 902$$

$$\int \frac{1}{a-\frac{x^4(ab-a(b-a))}{a+bx^4}}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 756$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}+\frac{\int \frac{1}{\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}$$

$$\Big\downarrow \ 216$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}+\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

input
```
Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

output
```
ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*a^(5/4))
```

3.217.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

3.217.4 Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)+\ln\left(\frac{-a^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{4a^{\frac{5}{4}}}$	65

input
```
int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*arctan(1/a^(1/4)/x*(b*x^4+a)^(1/4))+ln((-a^(1/4)*x-(b*x^4+a)^(1/4)
)/(a^(1/4)*x-(b*x^4+a)^(1/4))))/a^(5/4)
```

3.217.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.217.6 Sympy [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = -\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

input
```
integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

output
```
-Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(
a + b*x**4)**(1/4)), x)
```

3.217.7 Maxima [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

3.217.8 Giac [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(a-x^4\,(a-b))}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

3.218 $\int (a+bx^4)^p (c+dx^4)^q \, dx$

3.218.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

output
```
x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^4/a,-d*x^4/c)/((1+b*
x^4/a)^p)/((1+d*x^4/c)^q)
```

3.218.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = \frac{5acx(a+bx^4)^p (c+dx^4)^q \text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ac\,\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(bcp\,\text{AppellF1}\left(\frac{5}{4},1-p,-q,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adq\,\text{AppellF}\right.}$$

input
```
Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
(5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/
a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^
4)/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

3.218.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p (c+dx^4)^q \, dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \left(\frac{bx^4}{a}+1\right)^p (dx^4+c)^q dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} (c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} \int \left(\frac{bx^4}{a}+1\right)^p \left(\frac{dx^4}{c}+1\right)^q dx$$

$$\Big\downarrow \; 936$$

$$x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} (c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

input
```
Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
(x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

3.218.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.218.4 Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)^{q} dx$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

output
```
int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

3.218.5 Fricas [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx=\int \left(bx^4+a\right)^p\left(dx^4+c\right)^q\,dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
```

output
```
Timed out
```

3.218.7 Maxima [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

3.218.8 Giac [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^q \, dx$$

input
```
int((a + b*x^4)^p*(c + d*x^4)^q,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^4)^q, x)
```

3.219 $\int (a+bx^4)^2 (c+dx^4)^q \, dx$

3.219.1 Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^4)^2 (c+dx^4)^q \, dx = -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+16q^2))\,x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\right.}{d^2(5+4q)(9+4q)}$$

output
```
-b*(5*b*c-a*d*(13+4*q))*x*(d*x^4+c)^(1+q)/d^2/(16*q^2+56*q+45)+b*x*(b*x^4+
a)*(d*x^4+c)^(1+q)/d/(9+4*q)+(5*b^2*c^2-2*a*b*c*d*(9+4*q)+a^2*d^2*(16*q^2+
56*q+45))*x*(d*x^4+c)^q*hypergeom([1/4, -q],[5/4],-d*x^4/c)/d^2/(16*q^2+56
*q+45)/((1+d*x^4/c)^q)
```

3.219.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int (a+bx^4)^2 (c+dx^4)^q \, dx = \frac{1}{45}x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\left(45a^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)+bx^4\left(18a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-q,\frac{9}{4},-\frac{dx^4}{c}\right)+5bx^4\,\text{Hypergeometric2F1}\left(\frac{9}{4},-q,\frac{13}{4},-\frac{dx^4}{c}\right)\right)\right)$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

output
```
(x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)] + b
*x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)] + 5*b*x^4*Hyperge
ometric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*(1 + (d*x^4)/c)^q)
```

3.219.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\left(dx^4+c\right)^q \left(b(5bc-ad(4q+13))x^4+a(bc-ad(4q+9))\right)dx}{d(4q+9)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{q+1}}{d(4q+9)}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{q+1}}{d(4q+9)}-\frac{\int \left(dx^4+c\right)^q \left(b(5bc-ad(4q+13))x^4+a(bc-ad(4q+9))\right)dx}{d(4q+9)}$$

$$\Big\downarrow \text{913}$$

$$\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{q+1}}{d(4q+9)}-\frac{\frac{bx\left(c+dx^4\right)^{q+1}(5bc-ad(4q+13))}{d(4q+5)}-\frac{\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right)\int \left(dx^4+c\right)^q dx}{d(4q+5)}}{d(4q+9)}$$

$$\Big\downarrow \text{779}$$

$$\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{q+1}}{d(4q+9)}-\frac{\frac{bx\left(c+dx^4\right)^{q+1}(5bc-ad(4q+13))}{d(4q+5)}-\frac{\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right)\int \left(\frac{dx^4}{c}+1\right)^q dx}{d(4q+5)}}{d(4q+9)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx(a+bx^4)\left(c+dx^4\right)^{q+1}}{d(4q+9)}-\frac{\frac{bx(c+dx^4)^{q+1}(5bc-ad(4q+13))}{d(4q+5)}-\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)}{d(4q+5)}}{d(4q+9)}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

output
```
(b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) - ((b*(5*b*c - a*d*(13
 + 4*q))*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) - ((5*b^2*c^2 - 2*a*b*c*d*(9
 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hypergeometric2F1[
1/4, -q, 5/4, -((d*x^4)/c)])/(d*(5 + 4*q)*(1 + (d*x^4)/c)^q))/(d*(9 + 4*q)
)
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.219. $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx$

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.219.4 Maple [F]

$$\int \left(b\,x^{4}+a\right)^{2} \left(d\,x^{4}+c\right)^{q}dx$$

input
```
int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

output
```
int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

3.219.5 Fricas [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^2 \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)
```

3.219.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.69 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \frac{a^2c^qx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -q \\ \frac{5}{4} \end{matrix} \middle| \frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{abc^qx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -q \\ \frac{9}{4} \end{matrix} \middle| \frac{dx^4e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{b^2c^qx^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{9}{4}, -q \\ \frac{13}{4} \end{matrix} \middle| \frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**q,x)
```

output
```
a**2*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/
(4*gamma(5/4)) + a*b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*
exp_polar(I*pi)/c)/(2*gamma(9/4)) + b**2*c**q*x**9*gamma(9/4)*hyper((9/4,
-q), (13/4,), d*x**4*exp_polar(I*pi)/c)/(4*gamma(13/4))
```

3.219.7 Maxima [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^2 \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

3.219.8 Giac [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^2 \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(b\,x^4+a\right)^2 \left(d\,x^4+c\right)^q \, dx$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^q,x)
```

output
```
int((a + b*x^4)^2*(c + d*x^4)^q, x)
```

3.220 $\int (a+bx^4)(c+dx^4)^q\,dx$

3.220.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^4)(c+dx^4)^q\,dx = \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \frac{(bc-ad(5+4q))x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)}{d(5+4q)}$$

output
```
b*x*(d*x^4+c)^(1+q)/d/(5+4*q)-(b*c-a*d*(5+4*q))*x*(d*x^4+c)^q*hypergeom([1
/4, -q],[5/4],-d*x^4/c)/d/(5+4*q)/((1+d*x^4/c)^q)
```

3.220.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^4)(c+dx^4)^q\,dx = \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\left(b(c+dx^4)\left(1+\frac{dx^4}{c}\right)^q+(-bc+ad(5+4q))\,\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)\right)}{d(5+4q)}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
```

output
```
(x*(c + d*x^4)^q*(b*(c + d*x^4)*(1 + (d*x^4)/c)^q + (-(b*c) + a*d*(5 + 4*q
))*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]))/(d*(5 + 4*q)*(1 + (d*x^
4)/c)^q)
```

3.220.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(a-\frac{bc}{4dq+5d}\right)\int \left(dx^4+c\right)^q dx+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

$$\Big\downarrow \text{779}$$

$$\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right)\int \left(\frac{dx^4}{c}+1\right)^q dx+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

$$\Big\downarrow \text{778}$$

$$x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

output
```
(b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*q))*x*(c
+ d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(1 + (d*x^4)/c)^
q
```

3.220.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.220.4 Maple [F]

$$\int \left(b\,x^{4}+a\right)\left(d\,x^{4}+c\right)^{q}dx$$

input
```
int((b*x^4+a)*(d*x^4+c)^q,x)
```

output
```
int((b*x^4+a)*(d*x^4+c)^q,x)
```

3.220.5 Fricas [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(bx^4+a\right)\left(dx^4+c\right)^q dx$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

3.220.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\frac{ac^q x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-q\\\frac{5}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{bc^q x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},-q\\\frac{9}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

output
```
a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*
gamma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_p
olar(I*pi)/c)/(4*gamma(9/4))
```

3.220.7 Maxima [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

3.220.8 Giac [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(b\,x^4+a\right)\,\left(d\,x^4+c\right)^q\,dx$$

input
```
int((a + b*x^4)*(c + d*x^4)^q,x)
```

output
```
int((a + b*x^4)*(c + d*x^4)^q, x)
```

3.221 $\int \frac{(c+dx^4)^q}{a+bx^4} \, dx$

3.221.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^4)^q}{a+bx^4} \, dx = \frac{x(c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{4},1,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

output
```
x*(d*x^4+c)^q*AppellF1(1/4,1,-q,5/4,-b*x^4/a,-d*x^4/c)/a/((1+d*x^4/c)^q)
```

3.221.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^4)^q}{a+bx^4} \, dx$$
$$= \frac{5acx(c+dx^4)^q \text{AppellF1}\left(\frac{1}{4},-q,1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-q,1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+4x^4\left(adq\,\text{AppellF1}\left(\frac{5}{4},1-q,1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bc\,\text{Ap}\right.\right.}$$

input
```
Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

output
```
(5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a
)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/
a)] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)
] - b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

3.221.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^q}{a+bx^4}dx$$

$$\Big\downarrow \text{937}$$

$$(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\int \frac{\left(\frac{dx^4}{c}+1\right)^q}{bx^4+a}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{4},1,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

input
```
Int[(c + d*x^4)^q/(a + b*x^4),x]
```

output
```
(x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a
*(1 + (d*x^4)/c)^q)
```

3.221.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.221.4 Maple [F]

$$\int \frac{\left(d\,x^{4}+c\right)^{q}}{b\,x^{4}+a}dx$$

input
```
int((d*x^4+c)^q/(b*x^4+a),x)
```

output
```
int((d*x^4+c)^q/(b*x^4+a),x)
```

3.221.5 Fricas [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**q/(b*x**4+a),x)
```

output
```
Timed out
```

3.221.7 Maxima [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

3.221.8 Giac [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(d\,x^4+c)^q}{b\,x^4+a}\,dx$$

input
```
int((c + d*x^4)^q/(a + b*x^4),x)
```

output
```
int((c + d*x^4)^q/(a + b*x^4), x)
```

3.222 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx$

3.222.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

output
```
x*(d*x^4+c)^q*AppellF1(1/4,2,-q,5/4,-b*x^4/a,-d*x^4/c)/a^2/((1+d*x^4/c)^q)
```

3.222.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx$$
$$= \frac{5acx(c+dx^4)^q\operatorname{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^2\left(5ac\operatorname{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(adq\operatorname{AppellF1}\left(\frac{5}{4},2,1-q,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bc\ldots\right.\right.}$$

input
```
Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

output
```
(5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c
)])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/
c)] - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

3.222.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}dx$$

$$\Big\downarrow \text{937}$$

$$(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\int \frac{\left(\frac{dx^4}{c}+1\right)^q}{(bx^4+a)^2}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

input
```
Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

output
```
(x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a
^2*(1 + (d*x^4)/c)^q)
```

3.222.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.222.4 Maple [F]

$$\int \frac{(d\,x^{4}+c)^{q}}{(b\,x^{4}+a)^{2}}dx$$

input
```
int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

output
```
int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

3.222.5 Fricas [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

3.222.8 Giac [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

input
```
integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(d\,x^4+c)^q}{(b\,x^4+a)^2}\,dx$$

input
```
int((c + d*x^4)^q/(a + b*x^4)^2,x)
```

output
```
int((c + d*x^4)^q/(a + b*x^4)^2, x)
```

3.223 $\int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)}\,dx$

3.223.1 Optimal result

Integrand size = 21, antiderivative size = 545

$$\begin{aligned}
\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx &= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\sqrt[5]{bc-ad}x}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}+\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sqrt[5]{bc-ad}x}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&-\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}-\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+2c^{2/5}(a+bx^5)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+2c^{2/5}(a+bx^5)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}
\end{aligned}$$

output
```
-1/5*ln(c^(1/5)-(-a*d+b*c)^(1/5)*x/(b*x^5+a)^(1/5))/c^(4/5)/(-a*d+b*c)^(1/
5)+1/20*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1
/5)+2*c^(2/5)*(b*x^5+a)^(2/5)-c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5
^(1/2))/(b*x^5+a)^(2/5))*(-5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/20*ln((2*
(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)+2*c^(2/5)*
(b*x^5+a)^(2/5)+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1/2))/(b*x^5
+a)^(2/5))*(5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(1/5*(-a*d+b*c)
^(1/5)*x*(50+10*5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^(1/5)+1/5*(25+10*5^(1/2))
^(1/2))*(10-2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(-1/5*(25
-10*5^(1/2))^(1/2)+2*(-a*d+b*c)^(1/5)*x*2^(1/2)/(5+5^(1/2))^(1/2)/c^(1/5)/
(b*x^5+a)^(1/5))*(10+2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)
```

3.223.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{5},1,\frac{6}{5},\frac{(bc-ad)x^5}{c(a+bx^5)}\right)}{c\sqrt[5]{a+bx^5}}$$

input
```
Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

output
```
(x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(
a + b*x^5)^(1/5))
```

3.223.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {902, 752, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

$$\Big\downarrow \text{ 902}$$

$$\int \frac{1}{c-\frac{x^5(bc-ad)}{a+bx^5}}d\frac{x}{\sqrt[5]{a+bx^5}}$$

$$\Big\downarrow \text{752}$$

$$\frac{\int \frac{1}{\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{2\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{2\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{2\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \text{1142}$$

3.223. $\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$

$$\frac{\frac{1}{2}(5+\sqrt{5})\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1-\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{(1-\sqrt{5})\int \frac{\sqrt[5]{bc-ad}\left(\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+(1-\sqrt{5})\sqrt[5]{\cdots}\right.}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1-\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\cdots}}{4\sqrt[5]{bc-ad}\cdots}}{5c^{4/5}}$$

$$\frac{\frac{1}{2}(5-\sqrt{5})\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1+\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{(1+\sqrt{5})\int \frac{\sqrt[5]{bc-ad}\left(\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+(1+\sqrt{5})\sqrt[5]{\cdots}\right.}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1+\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\cdots}}{4\sqrt[5]{bc-ad}\cdots}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}(5+\sqrt{5})\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1-\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{1}{4}(1-\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+(1-\sqrt{5})\sqrt[5]{\cdots}}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1-\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-\cdots}}{\sqrt[5]{bx^5+a}}\cdots}}{5c^{4/5}}$$

$$\frac{\frac{1}{2}(5-\sqrt{5})\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1+\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{1}{4}(1+\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+(1+\sqrt{5})\sqrt[5]{\cdots}}{\frac{2(bc-ad)^{2/5}x^2}{(bx^5+a)^{2/5}}+\frac{(1+\sqrt{5})\sqrt[5]{c}\sqrt[5]{bc-\cdots}}{\sqrt[5]{bx^5+a}}\cdots}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \; 1083$$

3.223. $\int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)}\,dx$

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}-\left(5+\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{-\frac{x^2}{\left(bx^5+a\right)^{2/5}}-2\left(5+\sqrt{5}\right)c^{2/5}(bc-ad)^{2/\cdots}}\cdots}{5c^{4/5}}$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}-\left(5-\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{-\frac{x^2}{\left(bx^5+a\right)^{2/5}}-2\left(5-\sqrt{5}\right)c^{2/5}(bc-ad)^{2/\cdots}}\cdots}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$\downarrow$ 217

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-a\cdots}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\cdots\right)}{\sqrt[5]{bc-ad}}\cdots}{5c^{4/5}}$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-a\cdots}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\cdots\right)}{\sqrt[5]{bc-ad}}\cdots}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$\downarrow$ 1103

3.223. $\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$

$$\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}+\frac{2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}+2c^{2/5}\right)}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}+\frac{\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}+\frac{2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}+2c^{2/5}\right)}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

input
```
Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

output
```
-1/5*Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)]/(c^(4/5)*(b*c
- a*d)^(1/5)) + ((Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*c^(1/5)*(b*c
 - a*d)^(1/5) + (4*(b*c - a*d)^(2/5)*x)/(a + b*x^5)^(1/5))/(Sqrt[2*(5 + Sq
rt[5])]*c^(1/5)*(b*c - a*d)^(1/5))])/(b*c - a*d)^(1/5) + ((1 - Sqrt[5])*Lo
g[2*c^(2/5) + (2*(b*c - a*d)^(2/5)*x^2)/(a + b*x^5)^(2/5) + ((1 - Sqrt[5])
*c^(1/5)*(b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)])/(4*(b*c - a*d)^(1/5)))/(
5*c^(4/5)) + ((Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 + Sqrt[5])*c^(1/5)*(b*c -
a*d)^(1/5) + (4*(b*c - a*d)^(2/5)*x)/(a + b*x^5)^(1/5))/(Sqrt[2*(5 - Sqrt[
5])]*c^(1/5)*(b*c - a*d)^(1/5))])/(b*c - a*d)^(1/5) + ((1 + Sqrt[5])*Log[2
*c^(2/5) + (2*(b*c - a*d)^(2/5)*x^2)/(a + b*x^5)^(2/5) + ((1 + Sqrt[5])*c^
(1/5)*(b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)])/(4*(b*c - a*d)^(1/5)))/(5*c
^(4/5))
```

3.223.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 752
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n
)   Int[1/(r - s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; F
reeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.223.4 Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{\left(\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}+1\right)\ln\left(\frac{2\left(\frac{ad-bc}{c}\right)^{\frac{2}{5}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{5}}\left(b\,x^{5}+a\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)x+2\left(b\,x^{5}+a\right)^{\frac{2}{5}}}{x^{2}}\right)}{2}-\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}-1\right)\ln\left(2(\right.}{}\right.}{}$

input
```
int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x,method=_RETURNVERBOSE)
```

output
```
-1/100*(1/2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)*(5^(1/2)+1)*ln((2*((a*d-b*
c)/c)^(2/5)*x^2-((a*d-b*c)/c)^(1/5)*(b*x^5+a)^(1/5)*(5^(1/2)+1)*x+2*(b*x^5
+a)^(2/5))/x^2)-1/2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)*(5^(1/2)-1)*ln((2*
((a*d-b*c)/c)^(2/5)*x^2+((a*d-b*c)/c)^(1/5)*(b*x^5+a)^(1/5)*(5^(1/2)-1)*x+
2*(b*x^5+a)^(2/5))/x^2)+2^(1/2)*(5-5^(1/2))^(1/2)*(5+5^(1/2))*arctan(1/2*(
((a*d-b*c)/c)^(1/5)*(5^(1/2)-1)*x+4*(b*x^5+a)^(1/5))/((a*d-b*c)/c)^(1/5)/(
5+5^(1/2))^(1/2)*2^(1/2)/x)+(5+5^(1/2))^(1/2)*(-2*(5-5^(1/2))^(1/2)*ln(((
a*d-b*c)/c)^(1/5)*x+(b*x^5+a)^(1/5))/x)+arctan(1/2/((a*d-b*c)/c)^(1/5)*2^(
1/2)*(((a*d-b*c)/c)^(1/5)*(5^(1/2)+1)*x-4*(b*x^5+a)^(1/5))/(5-5^(1/2))^(1/
2)/x)*2^(1/2)*(5^(1/2)-5)))*5^(1/2)/((a*d-b*c)/c)^(1/5)/c
```

3.223.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (trace 0)
```

3.223.6 Sympy [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

input
```
integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

output
```
Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

3.223.7 Maxima [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

3.223.8 Giac [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(b\,x^5+a)^{1/5}\,(d\,x^5+c)}\,dx$$

input
```
int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
```

output
```
int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

3.224 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$

3.224.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x + \frac{c^2(bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
c^2*(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-7/5*d*(c+d/x)^2*(a+
b/x)^(1/2)-1/15*d*(-4*a^2*d^2+30*a*b*c*d+114*b^2*c^2+b*d*(2*a*d+33*b*c)/x)
*(a+b/x)^(1/2)/b^2+(c+d/x)^3*x*(a+b/x)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$
$$=\frac{\sqrt{a+\frac{b}{x}}(4a^2d^3x^2-2abd^2x(d+15cx)-3b^2(2d^3+10cd^2x+30c^2dx^2-5c^3x^3))}{15b^2x^2}$$
$$+\frac{c^2(bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 +
10*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

3.224.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {899, 108, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\int \frac{\left(c+\frac{d}{x}\right)^2\left(bc+6ad+\frac{7bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{1}{2}\int \frac{\left(c+\frac{d}{x}\right)^2\left(bc+6ad+\frac{7bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{170}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{b\left(c+\frac{d}{x}\right)\left(\frac{d(33bc+2ad)}{x}+5c(bc+6ad)\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{5b}-\frac{14}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\right)+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{5}\int \frac{\left(c+\frac{d}{x}\right)\left(\frac{d(33bc+2ad)}{x}+5c(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{14}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\right)+$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-5c^2(6ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\sqrt{a+\right.$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-\frac{10c^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\sqrt{\right.$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(\frac{10c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)}{\sqrt{a}}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\sqrt{a\right.$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

3.224. $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx$

input
```
Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

output
```
Sqrt[a + b/x]*(c + d/x)^3*x + ((-14*d*Sqrt[a + b/x]*(c + d/x)^2)/5 + ((-2*
d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c +
2*a*d))/x))/(3*b^2) + (10*c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/Sqrt[a])/5)/2
```

3.224.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.224.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\left(15b^2c^3x^3+4a^2d^3x^2-30abc\,d^2x^2-90b^2c^2d\,x^2-2a\,d^3xb-30c\,d^2x\,b^2-6b^2d^3\right)\sqrt{\frac{ax+b}{x}}}{15x^2b^2}+\frac{(6ad+bc)c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}}{2\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(180a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b\,c^2d\,x^4+30\sqrt{a}\,\sqrt{a\,x^2+bx}\,b^2c^3x^4+90\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^2c^2d\,x^4+15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right.}{30x^3\sqrt{x(ax+b)}\,\sqrt{a}\,b^2}$

input
```
int((c+d/x)^3*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*b^2*c^3*x^3+4*a^2*d^3*x^2-30*a*b*c*d^2*x^2-90*b^2*c^2*d*x^2-2*a*b
*d^3*x-30*b^2*c*d^2*x-6*b^2*d^3)/x^2/b^2*((a*x+b)/x)^(1/2)+1/2*(6*a*d+b*c)
*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)/(a*x+b)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.14

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx$$

$$= \left[\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2}{30\,ab^2x^2}\right.$$

$$-\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2bcd^2-2}{15\,ab^2x^2}$$

input
```
integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2
*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*s
qrt((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a
)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d
^3 - 2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d
^2 + a^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

3.224.6 Sympy [A] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.22

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1} - 3c^2d\left(\begin{cases} \frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0 \\ -\sqrt{a}\log(x) & \text{otherwise} \end{cases}\right) + 3cd^2\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b=0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) + \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

output
```
4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b*
*(7/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)
*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(1
5*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(b)*c**3*sqrt(
x)*sqrt(a*x/b + 1) - 3*c**2*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/
sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 3*c*d**2
*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b*
c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3 - 3\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)c^2d - \frac{2}{15}d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^2}\right) - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}cd^2}{b}$$

input
```
integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/
b^2 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b
```

3.224.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.224.9 Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right) + \sqrt{a+\frac{b}{x}}\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2} + \frac{2\,a^2\,d^3}{b^2}\right) + c^3\,x\,\sqrt{a+\frac{b}{x}} - \frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{5/2}}{5\,b^2} - \frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,(6\,a\,d+b\,c)\,1\mathrm{i}}{\sqrt{a}}$$

input
```
int((a + b/x)^(1/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a +
 b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d -
 b*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)
^(5/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i
)/a^(1/2)
```

3.225 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$

3.225.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2/3*d^2*(a+b/x)^(3/2)/b+c^2*(a+b/x)^(3/2)*x/a+c*(4*a*d+b*c)*arctanh((a+b/
x)^(1/2)/a^(1/2))/a^(1/2)-c*(4*a*d+b*c)*(a+b/x)^(1/2)/a
```

3.225.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}(-2ad^2x+b(-2d^2-12cdx+3c^2x^2))}{3bx} + \frac{c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
 (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

3.225.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\int \frac{1}{2}\sqrt{a+\frac{b}{x}}\left(\frac{2ad^2}{x}+c(bc+4ad)\right)xd\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\int \sqrt{a+\frac{b}{x}}\left(\frac{2ad^2}{x}+c(bc+4ad)\right)xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(4ad+bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

input
```
Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

output
```
(c^2*(a + b/x)^(3/2)*x)/a - ((4*a*d^2*(a + b/x)^(3/2))/(3*b) + c*(b*c + 4*
a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))/(2*a)
```

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.225.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\left(-3b\,c^{2}x^{2}+2xa\,d^{2}+12bcdx+2b\,d^{2}\right)\sqrt{\frac{ax+b}{x}}}{3xb}+\frac{(4ad+bc)c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(24a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,cd\,x^{3}+6\sqrt{a}\,\sqrt{a\,x^{2}+bx}\,b\,c^{2}x^{3}+12\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abcd\,x^{3}+3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}c^{2}x^{3}-\right.}{6x^{2}\sqrt{x(ax+b)}\,\sqrt{a}\,b}$

input
```
int((c+d/x)^2*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b*c^2*x^2+2*a*d^2*x+12*b*c*d*x+2*b*d^2)/x/b*((a*x+b)/x)^(1/2)+1/2
*(4*a*d+b*c)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.10

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$$
$$= \left[\frac{3\,(b^2c^2+4\,abcd)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{6\,abx},\right.$$
$$\left.-\frac{3\,(b^2c^2+4\,abcd)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{3\,abx}\right]$$

input
```
integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*s
qrt((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d
 + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

3.225.6 Sympy [A] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}$$
$$-2cd\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)$$
$$+d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

output
```
sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*c*d*Piecewise((2*a*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), Tr
ue)) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b),
True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2 - 2\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)cd - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3\,b}$$

input
```
integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

3.225.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.225.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\sqrt{a+\frac{b}{x}}+c^2\,x\,\sqrt{a+\frac{b}{x}} - \frac{2\,d^2\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b}-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+b\,c\right)\,1\mathrm{i}}{\sqrt{a}}$$

input

```
int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

output

```
((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1
/2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2)
)*(4*a*d + b*c)*1i)/a^(1/2)
```

3.226 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$

3.226.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=-\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
c*(a+b/x)^(3/2)*x/a+(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-(2*
a*d+b*c)*(a+b/x)^(1/2)/a
```

3.226.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\sqrt{a+\frac{b}{x}}(-2d+cx)+\frac{(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

output
```
Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/Sqrt[a]
```

3.226.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {899, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$$

$$\Big\downarrow \text{899}$$

$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)x^2d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)}{2a}$$

$$\Big\downarrow \text{73}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(2ad+bc)}{2a}$$

input
```
Int[Sqrt[a + b/x]*(c + d/x),x]
```

output
```
(c*(a + b/x)^(3/2)*x)/a - ((b*c + 2*a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcT
anh[Sqrt[a + b/x]/Sqrt[a]]))/(2*a)
```

3.226. $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$

3.226.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

method	result
risch	$(cx-2d)\sqrt{\frac{ax+b}{x}}+\frac{\left(\frac{bc}{2}+ad\right)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,(4a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,d\,x^{2}+2\sqrt{a}\,\sqrt{a\,x^{2}+bx}\,bc\,x^{2}+2\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd\,x^{2}+\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}c\,x^{2}-4\sqrt{a}\,(a}{2x\sqrt{x(ax+b)}\,b\sqrt{a}}$

input
```
int((c+d/x)*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x-2*d)*((a*x+b)/x)^(1/2)+(1/2*b*c+a*d)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\left[\frac{(bc+2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a},\ -\frac{(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{a}\right]$$

input
```
integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
 + 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

3.226.6 Sympy [A] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-d\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((c+d/x)*(a+b/x)**(1/2),x)
```

output
```
sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - d*Piecewise((2*a*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) +
b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c-\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)d$$

input
```
integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a))) + 2*sqrt(a + b/x))*d
```

3.226.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.226.9 Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=2\sqrt{a}\,d\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,d\sqrt{a+\frac{b}{x}}+c\,x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

input
```
int((a + b/x)^(1/2)*(c + d/x),x)
```

output
```
2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*
x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*
x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.227 $\int \sqrt{a+\frac{b}{x}}\,dx$

3.227.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x*(a+b/x)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

3.227.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \sqrt{a+\frac{b}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\sqrt{a+\frac{b}{x}}-\frac{1}{2}b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$x\sqrt{a+\frac{b}{x}}-\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}$$

input
```
Int[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

3.227.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.227.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

method	result	size
risch	$x\sqrt{\frac{ax+b}{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$	72
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\,\sqrt{a}}$	74

input
```
int((a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*((a*x+b)/x)^(1/2)+1/2*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2
)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int \sqrt{a+\frac{b}{x}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a]
```

3.227.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{a+\frac{b}{x}}x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqr
t(a)))/sqrt(a)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \sqrt{a+\frac{b}{x}}\,dx = -\frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \frac{b\log(|b|)\operatorname{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2+bx}\operatorname{sgn}(x)$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \sqrt{a+\frac{b}{x}}\,dx = x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}} + \frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

input
```
int((a + b/x)^(1/2),x)
```

output
```
x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
 a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.228 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x}{c}+\frac{2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

output
```
(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^2/a^(1/2)+2*arctan(d^(1/2)*(
a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)*(-a*d+b*c)^(1/2)/c^2+x*(a+b/x)^(1/2
)/c
```

3.228.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{c\sqrt{a+\frac{b}{x}}x+2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

output
```
(c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/
x])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt
[a])/c^2
```

3.228.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {899, 110, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(bc-2ad-\frac{bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(bc-2ad-\frac{bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{ 174}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2d(bc-ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4d(bc-ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c}}{2c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{-\frac{4\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{\sqrt{a}c}}{2c}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x),x]
```

output
```
(Sqrt[a + b/x]*x)/c - ((-4*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a
+ b/x])/Sqrt[b*c - a*d]])/c - (2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/(Sqrt[a]*c))/(2*c)
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.228. $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.228.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(86) = 172$.

Time = 0.22 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.23

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c}-\frac{\left(\frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{c\sqrt{a}}+\frac{2(ad-bc)d\ln\left(\frac{\frac{2(ad-bc)d}{c^{2}}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\sqrt{a\left(x+\frac{d}{c}\right)^{2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}\cdots}}{x+\frac{d}{c}}\right.\cdots}{c^{2}\sqrt{\frac{(ad-bc)d}{c^{2}}}\cdots}\cdots\right.}{2c(ax+b)\cdots}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{3}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{\frac{(ad-bc)d}{c^{2}}}\,c-2adx+bcx-bd}{cx+d}\right)d^{2}-2\sqrt{x(ax+b)}\,c^{2}\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^{2}}}-2\sqrt{a}\,\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{\frac{(ad-bc)d}{c^{2}}}\,c-\cdots}{cx+d}\cdots\right.\right.}{2\sqrt{x(ax+b)}\,c^{3}\sqrt{a}\,\sqrt{\frac{(ad-\cdots}{\cdots}}\cdots}$

input
```
int((a+b/x)^(1/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/c*x*((a*x+b)/x)^(1/2)-1/2/c*((2*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2
+b*x)^(1/2))/a^(1/2)+2*(a*d-b*c)*d/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-
b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2
*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)^(1/2)*(x
*(a*x+b))^(1/2)/(a*x+b)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.63

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$
$$= \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right) + 2\,\sqrt{-bcd+ad^2}a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd}}{cx+d}\right.}{2\,ac^2}\right.$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c -
2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2)
, 1/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c
*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b
)/x) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(
-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sq
rt((a*x + b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*
c*d - a*d^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d
)) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

3.228.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x),x)
```

output
```
Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

3.228.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x), x)
```

3.228.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.228.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \frac{x\sqrt{a+\frac{b}{x}}}{c} + \frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2} - \frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)(2\,a\,d-b\,c)}{2\,\sqrt{a}\,c^2} - \frac{\mathrm{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\sqrt{a\,d^2-b\,c\,d}\,2\mathrm{i}}{c^2}$$

input
```
int((a + b/x)^(1/2)/(c + d/x),x)
```

output
```
(x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/
2)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((
a + b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^
(1/2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

3.228. $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

3.229 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$

3.229.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

output
```
(-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^3/a^(1/2)+(-4*a*d+3*b*c)*arc
tan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/c^3/(-a*d+b*c)^(1/2)+2
*d*(a+b/x)^(1/2)/c^2/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)
```

3.229.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(2d+cx)}{d+cx}+\frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcT
an[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

3.229.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {899, 110, 27, 168, 25, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\left(bc-4ad-\frac{3bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\left(bc-4ad-\frac{3bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{-\frac{\int -\frac{(bc-ad)\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{(bc-ad)\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{174}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{(bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{d(3bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2d(3bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{-\frac{2\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}c}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*x)/(c*(c + d/x)) - ((-4*d*Sqrt[a + b/x])/(c*(c + d/x)) + ((
-2*Sqrt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (2*(b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(
Sqrt[a]*c))/c)/(2*c)
```

3.229. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$

3.229.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.229. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.229.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(127) = 254$.

Time = 0.24 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.24

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{(4ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2d^2(ad-bc)\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\cdots}{\cdots}\right)}{\cdots}\right)}{c^3}\right)}{\cdots}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc}{c^2}}}{cx+\cdots}\right.\cdots\right)}{\cdots}$

input
```
int((a+b/x)^(1/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*((4*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a
*x^2+b*x)^(1/2))/a^(1/2)+2*d^2*(a*d-b*c)/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(
a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c
/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(
x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b
*c)*d/c^2)^(1/2))/(x+d/c)))+2/c^2*d*(3*a*d-2*b*c)/((a*d-b*c)*d/c^2)^(1/2)*
ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(
x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.45

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$
$$= \left[-\frac{(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(3\,abcd-4\,a^2d^2+(3\,abc^2-4\,a^2cd}{2\,(ac^4x+ac^3d)}\right.$$
$$-\frac{2\,(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(3\,abcd-4\,a^2d^2+(3\,abc^2-4\,a^2cd)x)\sqrt{-}}{2\,(ac^4x+ac^3d)}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*
c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sq
rt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c
*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (
b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b
*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2
*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x +
a*c^3*d), 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(
d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a
*x + b)/x))/(a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*
a^2*c*d)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*
sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*s
qrt((a*x + b)/x))/(a*c^4*x + a*c^3*d)]
```

3.229.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

3.229.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(127) = 254$.

Time = 0.34 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

$$= \frac{\sqrt{ax^2+bx}\,\mathrm{sgn}(x)}{c^2} - \frac{(3\,bcd\mathrm{sgn}(x) - 4\,ad^2\mathrm{sgn}(x))\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3}$$
$$- \frac{(bc\mathrm{sgn}(x) - 4\,ad\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,\sqrt{a}c^3}$$
$$- \frac{\left(6\,\sqrt{a}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 8\,a^{\frac{3}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - \sqrt{bcd-ad^2}bc\log(|b|) + 4\,\sqrt{bcd-ad^2}ad\log\right.}{2\,\sqrt{bcd-ad^2}\sqrt{a}c^3}$$
$$- \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd\mathrm{sgn}(x) - 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^2\mathrm{sgn}(x) - \sqrt{a}bd^2\mathrm{sgn}(x)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d + bd\right)c^3}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*sgn(x)/c^2 - (3*b*c*d*sgn(x) - 4*a*d^2*sgn(x))*arctan(-(
(sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/(sqrt(
b*c*d - a*d^2)*c^3) - 1/2*(b*c*sgn(x) - 4*a*d*sgn(x))*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^3) - 1/2*(6*sqrt(a)*b*c*d*a
rctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(3/2)*d^2*arctan(sqrt(a)*d/sqrt
(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b*c*log(abs(b)) + 4*sqrt(b*c*d - a*
d^2)*a*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*d)*sgn(x)/(sqrt(b*c*d - a*d
^2)*sqrt(a)*c^3) - ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d*sgn(x) - 2*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*a*d^2*sgn(x) - sqrt(a)*b*d^2*sgn(x))/(((sqrt(a)
*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d
+ b*d)*c^3)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 1195, normalized size of antiderivative = 8.13

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^2,x)
```

output
```
- ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((
a + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^
3*(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a
 + b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(
a^(1/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b
^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*
b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(
d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b
*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*
(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^
2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b
^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d
*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*
c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(
b*c^4 - a*c^3*d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/
c^6 - ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c
^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^
6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(
1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*
d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d...
```

3.230 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

3.230.1 Optimal result

Integrand size = 21, antiderivative size = 213

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

output
```
(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^4/a^(1/2)+1/4*(24*a^2*d^2-40
*a*b*c*d+15*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2
)/c^4/(-a*d+b*c)^(3/2)+3/2*d*(a+b/x)^(1/2)/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11
*b*c)*(a+b/x)^(1/2)/c^3/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

3.230.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$
$$= \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-2ad(6d^2+9cdx+2c^2x^2)+bc(11d^2+17cdx+4c^2x^2))}{(bc-ad)(d+cx)^2}+\frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{4(bc-6ad)\text{arctanh}}{\sqrt{a}}}{4c^4}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-2*a*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + b*c*(11*d^2 +
17*c*d*x + 4*c^2*x^2)))/((b*c - a*d)*(d + c*x)^2) + (Sqrt[d]*(15*b^2*c^2 -
 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/(b*c - a*d)^(3/2) + (4*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt
[a])/(4*c^4)
```

3.230.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {899, 110, 27, 168, 25, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

$$\Big\downarrow \ 899$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \ 110$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(bc-6ad-\frac{5bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(bc-6ad-\frac{5bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \; 168$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{-\frac{\int -\frac{(bc-ad)\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{(bc-ad)\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 168$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{-\frac{\int -\frac{\left(4(bc-6ad)(bc-ad)-\frac{bd(11bc-12ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\int \frac{\left(4(bc-6ad)(bc-ad)-\frac{bd(11bc-12ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 174$$

3.230. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{4(bc-6ad)(bc-ad)\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{d\left(24a^2d^2-40abcd+15b^2c^2\right)\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8(bc-6ad)(bc-ad)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2d\left(24a^2d^2-40abcd+15b^2c^2\right)\int\frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8(bc-6ad)(bc-ad)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{-\frac{2\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)(bc-ad)}{\sqrt{a}c}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

input

```
Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

3.230. $\int\frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}dx$

output
```
(Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - ((-3*d*Sqrt[a + b/x])/(c*(c + d/x)^2)
+ (-((d*(11*b*c - 12*a*d)*Sqrt[a + b/x])/(c*(b*c - a*d)*(c + d/x))) + ((-2
*Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (8*(b*c - 6*a*d)*(b*c - a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(2*c*(b*c - a*d)))/(2*c))/(2*
c)
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.230. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} \, dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.230.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(185) = 370$.

Time = 0.26 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.55

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^3} - \frac{\left(\frac{(6ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2d^2(4ad-3bc)\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}\ldots}{\ldots}\right.}{\ldots}\right.}{c^3}\ldots}{\ldots}$
default	Expression too large to display

input
```
int((a+b/x)^(1/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

3.230. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

output
```
1/c^3*x*((a*x+b)/x)^(1/2)-1/2/c^3*((6*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a
*x^2+b*x)^(1/2))/a^(1/2)+2/c^3*d^2*(4*a*d-3*b*c)*(-1/(a*d-b*c)/d*c^2/(x+d/
c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*
c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)
/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a
*d-b*c)*d/c^2)^(1/2))/(x+d/c)))-2*d^3*(a*d-b*c)/c^4*(-1/2/(a*d-b*c)/d*c^2/
(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2
*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*
c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*
c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d
/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+
d/c)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2
-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/
c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+6/c^2*d*(2*a*d-b*c)/((a*d-b*c)
*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c
^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/
c)))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(185) = 370$.

Time = 0.41 (sec) , antiderivative size = 1749, normalized size of antiderivative = 8.21

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(
a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^
2)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(
b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) -
 b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*
a*b*c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt(
(a*x + b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(
a*b*c^6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*
a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b
^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(
-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b
^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*
d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (1
7*a*b*c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2
*(a*b*c^6*d - a^2*c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*
d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*...
```

3.230.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{x^3\sqrt{a+\frac{b}{x}}}{(cx+d)^3}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

output
```
Integral(x**3*sqrt(a + b/x)/(c*x + d)**3, x)
```

3.230.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(185) = 370.

Time = 0.34 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\frac{\left(15\sqrt{a}b^2c^2d\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 40\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 24\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 2\sqrt{bcd-\right.}{4\left(\sqrt{bcd-\right.}$$
$$-\frac{(15\,b^2c^2d\text{sgn}(x) - 40\,abcd^2\text{sgn}(x) + 24\,a^2d^3\text{sgn}(x))\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\,(bc^5-ac^4d)\sqrt{bcd-ad^2}}$$
$$+\frac{\sqrt{ax^2+bx}\text{sgn}(x)}{c^3}$$
$$-\frac{9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3d\text{sgn}(x) - 32\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d^2\text{sgn}(x) + 24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3 a}{}$$
$$-\frac{(bc\text{sgn}(x) - 6\,ad\text{sgn}(x))\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\sqrt{a}c^4}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
-1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3
/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(
sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b))
 + 14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2
*d^2*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*
a^2*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a
^(3/2)*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c
*d - a*d^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*s
gn(x)/c^3 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d*sgn(x) - 32
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x - sq
rt(a*x^2 + b*x))^3*a^2*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*
sqrt(a)*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*
b*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^4*sgn(x) +
 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sq
rt(a*x^2 + b*x))*a*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a
^2*b*d^4*sgn(x) - 9*sqrt(a)*b^3*c*d^3*sgn(x) + 10*a^(3/2)*b^2*d^4*sgn(x))/
((b*c^5 - a*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - s
qrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn(x))*lo
g(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 1895, normalized size of antiderivative = 8.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

output
```
(log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c
*d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3
*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2
)*(12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*
a*d^3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2
+ 24*a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2
 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^
3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*
c)^3)^(1/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a
^2*d^2 + 15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5
*d^2 - 3*a*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241
*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*
d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c
^11*d^2 - 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^
2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*
b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*
d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*
c^4))*(6*a*d - b*c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2
*d^7 + 241*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^
2*b^4*c^2*d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b...
```

3.230. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

3.231 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx = -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2} + \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x + 3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-9/7*d*(a+b/x)^(3/2)*(c+d/x)^2-1/35*d*(a+b/x)^(3/2)*(2*(-a*d+13*b*c)*(2*a*
d+5*b*c)+3*b*d*(2*a*d+19*b*c)/x)/b^2+(a+b/x)^(3/2)*(c+d/x)^3*x+3*c^2*(2*a*
d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*c^2*(2*a*d+b*c)*(a+b/x)^(1
/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx = \frac{\sqrt{a+\frac{b}{x}}(4a^3d^3x^3 - 2a^2bd^2x^2(d+21cx) + ab^2x(-16d^3 - 84cd^2x - 280c^2dx^2 + 35c^3x^3) - 2b^3(5c}{35b^2x^3} + 3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-1
6*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2
*x + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

3.231.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 108, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx$$

$$\Big\downarrow \; 899$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$

$$\Big\downarrow \; 108$$

$$x\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 - \int \frac{3}{2}\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 \left(bc+2ad+\frac{3bd}{x}\right) x d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\int\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\left(bc+2ad+\frac{3bd}{x}\right)xd\frac{1}{x}$$

$\downarrow$ 170

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\left(\frac{2\int\frac{1}{2}b\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\left(7c(bc+2ad)+\frac{d(19bc+2ad)}{x}\right)xd\frac{1}{x}}{7b}+\frac{6}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\right)$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\left(\frac{1}{7}\int\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\left(7c(bc+2ad)+\frac{d(19bc+2ad)}{x}\right)xd\frac{1}{x}+\frac{6}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\right)$$

$\downarrow$ 164

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\int\sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{15b^2}\right)+\frac{6}{7}d\left(a+\frac{b}{x}\right)^{3}\right.$$

$\downarrow$ 60

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\left(a\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{15b^2}\right.\right.$$

$\downarrow$ 73

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5b\right.}{15b^2}\right.\right.$$

$\downarrow$ 221

3.231. $\int\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx$

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\left(7c^2\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(2ad+bc)+\frac{2d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad\right.}{15b^2}\right.\right.$$

input
```
Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

output
```
(a + b/x)^(3/2)*(c + d/x)^3*x - (3*((6*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 +
((2*d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c +
 2*a*d))/x))/(15*b^2) + 7*c^2*(b*c + 2*a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]]))/7))/2
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.231.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(35a\,b^2c^3x^4+4x^3a^3d^3-42x^3a^2bc\,d^2-280x^3a\,b^2c^2d-70x^3b^3c^3-2x^2a^2b\,d^3-84x^2a\,b^2c\,d^2-70x^2b^3c^2d-16xa\,b^2d^3-42x\,b^3c\,d^2-10b^3d^3}{35x^3b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(420a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b\,c^2d\,x^5+210a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b^2c^3x^5+210\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^2c^2d\,x^5+105\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax}{2\sqrt{a}}\right.}{}$

input
```
int((a+b/x)^(3/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/35*(35*a*b^2*c^3*x^4+4*a^3*d^3*x^3-42*a^2*b*c*d^2*x^3-280*a*b^2*c^2*d*x^
3-70*b^3*c^3*x^3-2*a^2*b*d^3*x^2-84*a*b^2*c*d^2*x^2-70*b^3*c^2*d*x^2-16*a*
b^2*d^3*x-42*b^3*c*d^2*x-10*b^3*d^3)/x^3/b^2*((a*x+b)/x)^(1/2)+3/2*(2*a*d+
b*c)*a^(1/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/
2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = \left[\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c\right.\right.}{}\right.$$

$$-\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c^3+140\,ab^2c^2d+21\,a^2\right.\right.}{35\,b^2x^3}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*s
qrt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 +
 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x
 + b)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arc
tan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35
*b^3*c^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c
^2*d + 42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)
*sqrt((a*x + b)/x))/(b^2*x^3)]
```

3.231.6 Sympy [A] (verification not implemented)

Time = 33.36 (sec) , antiderivative size = 1828, normalized size of antiderivative = 11.15

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

output
```
-16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(
13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d*
*3*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4
0*a**(13/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19
/2)*d**3*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2
)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 12*a**(11/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt
(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a
**(9/2)*b**(21/2)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 6*a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/
2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**...
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = -\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b\right)c^3 - \left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{b}{x}}a\right)c^2d - \frac{2}{35}\left(\frac{5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{b^2} - \frac{7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^2}\right)d^3$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

output
```
-6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*
c^3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
+ 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b
^2 - 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

3.231.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.231.9 Mupad [B] (verification not implemented)

Time = 8.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.99

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx=\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right)$$
$$+\sqrt{a+\frac{b}{x}}\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\left(\frac{6\,a\,d^3-6\,b\right.\right.$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}-\frac{2\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{3\,b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{7/2}}{7\,b^2}+a\,c^3\,x\,\sqrt{a+\frac{b}{x}}-2\,c^2$$

input
```
int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
 b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 -
 (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d
^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 -
6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)
/(3*b^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*
c^2*atan((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2
*d + 3*a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

3.232 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx$

3.232.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx = -c(3bc+4ad)\sqrt{a+\frac{b}{x}}-\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}+\frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}+\sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-1/3*c*(4*a*d+3*b*c)*(a+b/x)^(3/2)/a-2/5*d^2*(a+b/x)^(5/2)/b+c^2*(a+b/x)^(
5/2)*x/a+c*(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-c*(4*a*d+3
*b*c)*(a+b/x)^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx = \frac{\sqrt{a+\frac{b}{x}}\left(-6a^2d^2x^2+abx(-12d^2-80cdx+15c^2x^2)-2b^2(3d^2+10cdx+15c^2x^2)\right)}{15bx^2} +\sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-6*a^2*d^2*x^2 + a*b*x*(-12*d^2 - 80*c*d*x + 15*c^2*x^2) -
 2*b^2*(3*d^2 + 10*c*d*x + 15*c^2*x^2)))/(15*b*x^2) + Sqrt[a]*c*(3*b*c + 4
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

3.232.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {899, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2x^2d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\int \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2ad^2}{x}+c(3bc+4ad)\right)xd\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\int \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2ad^2}{x}+c(3bc+4ad)\right)xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{90}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\left(a\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$\downarrow$ 73

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$\downarrow$ 221

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)(4ad+3bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

input

```
Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

output

```
(c^2*(a + b/x)^(5/2)*x)/a - ((4*a*d^2*(a + b/x)^(5/2))/(5*b) + c*(3*b*c +
4*a*d)*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])))/(2*a)
```

3.232.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.232.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\left(-15ab\,c^2x^3+6a^2d^2x^2+80abcd\,x^2+30b^2c^2x^2+12xab\,d^2+20x\,b^2cd+6b^2d^2\right)\sqrt{\frac{ax+b}{x}}}{15x^2b}+\frac{(4ad+3bc)\sqrt{a}\,c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{}}{2ax+2b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(120a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,cd\,x^4+90a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b\,c^2x^4-120a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}cd\,x^2+60\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bcd\,x^4+45\ln\left(2\right.\right.}{30x^3b\sqrt{x(a}}$

input
```
int((a+b/x)^(3/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-15*a*b*c^2*x^3+6*a^2*d^2*x^2+80*a*b*c*d*x^2+30*b^2*c^2*x^2+12*a*b*
d^2*x+20*b^2*c*d*x+6*b^2*d^2)/x^2/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+3*b*c)*a^
(1/2)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*
x+b))^(1/2)/(a*x+b)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.13

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+\right.}{30\,bx^2}$$

$$-\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^2-}{15\,bx^2}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*
b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(
b*x^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c
*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2)]
```

3.232.6 Sympy [A] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.33

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \sqrt{a}bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1} - 2acd\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) - bc^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + 2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

output
```
4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b*
*(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(
x)/sqrt(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) + a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) -
 2*a*c*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a +
 b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + a*d**2*Piecewise((-sqrt(a)/x,
 Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - b*c**2*Piecewise((2*a*ata
n(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)
*log(x), True)) + 2*b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)*
*(3/2)/(3*b), True))
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b\right)c^2 - \frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{b}{x}}a\right)cd$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((s
qrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
2 - 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))
) + 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

3.232.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.232.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \sqrt{a+\frac{b}{x}}\left(2a\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)-\frac{2(ad-bc)^2}{b}+\frac{2a^2d^2}{b}\right)+\left(\frac{4ad^2-4bcd}{3b}-\frac{4ad^2}{3b}\right)\left(a+\frac{b}{x}\right)^{3/2}-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}+ac^2x\sqrt{a+\frac{b}{x}}-2c\,\mathrm{atan}\left(\frac{2c\sqrt{a+\frac{b}{x}}\,(4ad+3bc)\sqrt{-\frac{a}{4}}}{4da^2c+3bac^2}\right)(4ad+3bc)\sqrt{-\frac{a}{4}}$$

input
```
int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

output
```
(a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c
)^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a
 + b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) -
2*c*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4
*a^2*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

3.233 $\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx$

3.233.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=-\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right)-\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a}+\frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a}+\sqrt{a}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-1/3*(2*a*d+3*b*c)*(a+b/x)^(3/2)/a+c*(a+b/x)^(5/2)*x/a+(2*a*d+3*b*c)*arcta
nh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-(2*a*d+3*b*c)*(a+b/x)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\frac{\sqrt{a+\frac{b}{x}}(ax(-8d+3cx)-2b(d+3cx))}{3x}+\sqrt{a}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

output
```
(Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*
b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

3.233.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {899, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \, dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\int \left(a+\frac{b}{x}\right)^{3/2} xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \text{73}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)(2ad+3bc)}{2a}$$

input
```
Int[(a + b/x)^(3/2)*(c + d/x),x]
```

output
```
(c*(a + b/x)^(5/2)*x)/a - ((3*b*c + 2*a*d)*((2*(a + b/x)^(3/2))/3 + a*(2*S
qrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/(2*a)
```

3.233.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.233. $\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.233.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\left(3ac\,x^{2}-8adx-6bcx-2bd\right)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{\left(2ad+3bc\right)\sqrt{a}\,\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\left(ax+b\right)}}{2ax+2b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(12a^{\frac{5}{2}}\sqrt{a\,x^{2}+bx}\,d\,x^{3}+18a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,bc\,x^{3}-12a^{\frac{3}{2}}\left(a\,x^{2}+bx\right)^{\frac{3}{2}}dx+6\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}bd\,x^{3}+9\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{}}{2\sqrt{}}\right.\right.}{6x^{2}\sqrt{x\left(ax+b\right)}\,\sqrt{a}\,b}$

input
```
int((a+b/x)^(3/2)*(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a*c*x^2-8*a*d*x-6*b*c*x-2*b*d)/x*((a*x+b)/x)^(1/2)+1/2*(2*a*d+3*b*c
)*a^(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(
a*x+b))^(1/2)/(a*x+b)
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.64

$$\int\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,acx^{2}-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{6\,x},\right.$$
$$\left.-\frac{3\,(3\,bc+2\,ad)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,acx^{2}-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

output
```
[1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x
) + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x,
 -1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) -
 (3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

3.233.6 Sympy [A] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx = \sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1} - ad\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) - bc\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x),x)
```

output
```
sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/
b + 1) - a*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt
(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) - b*c*Piecewise((2*a*atan(s
qrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*lo
g(x), True)) + b*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/
(3*b), True))
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax-3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}b\right)c-\frac{1}{3}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}a\right)d$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt
(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a
+ b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(
a + b/x)*a)*d
```

3.233.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.233.9 Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = 2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-2\,a\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x)^(3/2)*(c + d/x),x)
```

output
```
2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a
*d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -
(a*x)/b))/((a*x)/b + 1)^(3/2)
```

3.234 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

3.234.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2} x+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
(a+b/x)^(3/2)*x+3*b*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*b*(a+b/x)^(1/
2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}(-2b+ax)+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2),x]
```

output
```
Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

3.234.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{73}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{221}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)$$

input
```
Int[(a + b/x)^(3/2),x]
```

output
```
(a + b/x)^(3/2)*x - (3*b*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]]))/2
```

3.234.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.234.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

method	result	size
risch	$(ax-2b)\sqrt{\frac{ax+b}{x}}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	78
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(6a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,x^{2}+3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2x\sqrt{x(ax+b)}\,\sqrt{a}}$	100

input
```
int((a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-2*b)*((a*x+b)/x)^(1/2)+3/2*a^(1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right) + (ax-2\,b)\sqrt{\frac{ax+b}{x}}, -3\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b
)*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(a*x - 2*b)*sqrt((a*x + b)/x)]
```

3.234.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 1.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = 3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate((a+b/x)**(3/2),x)
```

output
```
3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a
*x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(
a*x/b + 1))
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}ax - \frac{3}{2}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}b$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

3.234.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.234.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -\frac{2\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x)^(3/2),x)
```

output
```
-(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1
)^(3/2)
```

3.235 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

3.235.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\frac{a\sqrt{a+\frac{b}{x}}\,x}{c}-\frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

output
```
(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^2-2*(-a*d+b*c)^(3/
2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(1/2)+a*x*(a+b/x)^
(1/2)/c
```

3.235.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\frac{ac\sqrt{a+\frac{b}{x}}\,x-\frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}-\sqrt{a}(-3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

output
```
(a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/Sqrt[d] - Sqrt[a]*(-3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]])/c^2
```

3.235.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 109, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int -\frac{\left(a(3bc-2ad)+\frac{b(2bc-ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(a(3bc-2ad)+\frac{b(2bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{ 174}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-ad)^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{a(3bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{4(bc-ad)^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a(3bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2a(3bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{4(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{2c}$$

$$\Big\downarrow \text{221}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{4(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-2ad)}{c}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x),x]
```

output
```
(a*Sqrt[a + b/x]*x)/c - ((4*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c)/(2*c)
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

3.235. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(88) = 176$.

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.33

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c}-\frac{\left(\frac{\sqrt{a}\,(2ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c}-\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)}}{x+\frac{d}{c}}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right)}{2c(ax+b)}$
default	$-\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^3-2a^{\frac{3}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d-4a^{\frac{3}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-$

input
```
int((a+b/x)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

3.235. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

output
```
1/c*x*a*((a*x+b)/x)^(1/2)-1/2/c*(a^(1/2)*(2*a*d-3*b*c)/c*ln((1/2*b+a*x)/a^
(1/2)+(a*x^2+b*x)^(1/2))-(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)/c^2/((a*d-b*c)*d
/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2
)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)
))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.90

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right) - 2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}\,\mathrm{lo}}{2\,c^2}\right.$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2
*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x
+ d)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*s
qrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))
/c^2, 1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*a
rctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a
*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sq
rt((a*x + b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c -
a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a))/c^2]
```

3.235.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x),x)
```

output
```
Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

3.235.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x), x)
```

3.235.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.235.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.25

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c}$$
$$-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}+\frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d}+\frac{12\,a^{7/2}\,b^{4}\cdots}{12\,a^4\,b^4\,d^4-46\,a^3\,b^5\,c\,d^3\cdots}\cdots}{c^2}$$
$$+\frac{2\,\mathrm{atanh}\left(\frac{12\,a^2\,b^4\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{12\,a^4\,b^4\,d^4-40\,a^3\,b^5\,c\,d^3+44\,a^2\,b^6\,c^2\,d^2-16\,a\,b^7\,c^3\,d}+\frac{16\,a\,b^5\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{40\,a^3\,b^5\,d^3-44\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+16\,a\,b^7\,c^2\,d}\right)\,\sqrt{d\,(a\,d\cdots}{c^2\,d}$$

input
```
int((a + b/x)^(3/2)/(c + d/x),x)
```

output
```
(a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/
2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)
/c^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*
c*d^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/
x)^(1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b
^7*c^3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^
7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 +
 (2*atanh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c
^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2
*b^6*c^2*d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^
3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2
*b^6*c*d^2 - (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2)
)/(c^2*d)
```

3.235. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

3.236 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} \, dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} \, dx = -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} -\frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

output
```
(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^3-(-4*a*d+b*c)*arc
tan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^3/d^(1/2)-(
-2*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)
```

3.236.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} \, dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-bc+2ad+acx)}{d+cx}-\frac{(b^2c^2-5abcd+4a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}-\sqrt{a}(-3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right.}{c^3}$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*
b*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]) - Sqrt[a]*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/c^3
```

3.236.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {899, 109, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\left(a(3bc-4ad)+\frac{b(2bc-3ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\left(a(3bc-4ad)+\frac{b(2bc-3ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{\left(a(3bc-4ad)(bc-ad)+\frac{b(bc-2ad)(bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c}$$

$$\Big\downarrow \ 25$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(a(3bc-4ad)(bc-ad)+\frac{b(bc-2ad)(bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \ 174$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{(bc-4ad)(bc-ad)^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{a(3bc-4ad)(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \ 73$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)(bc-ad)^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a(3bc-4ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \ 218$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a(3bc-4ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-4ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \ 221$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)(bc-ad)}{c}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

output
```
(a*Sqrt[a + b/x]*x)/(c*(c + d/x)) - ((2*(b*c - 2*a*d)*Sqrt[a + b/x])/(c*(c
 + d/x)) + ((2*(b*c - 4*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/
x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*Sqrt[a]*(3*b*c - 4*a*d)*(b*c - a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(c*(b*c - a*d)))/(2*c)
```

3.236. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}dx$

3.236.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.236. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(136) = 272$.

Time = 0.23 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.22

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{\sqrt{a}\,(4ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c} - \frac{\left(-6a^2d^2+8abcd-2b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)\cdots}}{x+\frac{d}{c}}\right.\cdots}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\cdots\right.}{\cdots}$
default	$-\frac{\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx\cdots}{cx+d}\right.\cdots\right.}{\cdots}$

input
```
int((a+b/x)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*x*a*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(1/2)*(4*a*d-3*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))-(-6*a^2*d^2+8*a*b*c*d-2*b^2*c^2)/c^2/((a*d-b*
c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d
/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+
d/c))+2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(
x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a
*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d
/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)
*d/c^2)^(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 769, normalized size of antiderivative = 4.93

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \left[-\frac{(3\,bcd-4\,ad^2+(3\,bc^2-4\,acd)x)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(bcd-4\,ad^2+\right.$$

$$-\frac{2\,(3\,bcd-4\,ad^2+(3\,bc^2-4\,acd)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{-\frac{bc-ad}{d}}\log}{2\,(c^4x+c^3d)}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*
sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqr
t((a*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*
d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/
(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a
*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)
*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*
c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d -
 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c
 - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^
2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d -
 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c -
a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4
*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b
*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

3.236.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{(cx+d)^2}\,dx$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

3.236.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(136) = 272$.

Time = 0.34 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\sqrt{ax^2+bx}\,a\operatorname{sgn}(x)}{c^2} + \frac{\left(b^2c^2\operatorname{sgn}(x)-5\,abcd\operatorname{sgn}(x)+4\,a^2d^2\operatorname{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3} - \frac{\left(3\,abc\operatorname{sgn}(x)-4\,a^2d\operatorname{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^3} + \frac{\left(2\sqrt{a}b^2c^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-10\,a^{\frac{3}{2}}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+8\,a^{\frac{5}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+3\sqrt{bcd-ad^2}abc\cdots}{2\sqrt{bcd-ad^2}\sqrt{a}c^3} + \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)b^2c^2\operatorname{sgn}(x)-3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)abcd\operatorname{sgn}(x)+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^2d^2\operatorname{sgn}(x)-\cdots}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d+bd\right)c^3}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a*sgn(x)/c^2 + (b^2*c^2*sgn(x) - 5*a*b*c*d*sgn(x) + 4*a^
2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt
(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3) - 1/2*(3*a*b*c*sgn(x) - 4*a^2*d
*sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)
*c^3) + 1/2*(2*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 10*
a^(3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 8*a^(5/2)*d^2*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 3*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b))
- 4*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*b*c -
2*sqrt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3) + ((
sqrt(a)*x - sqrt(a*x^2 + b*x))*b^2*c^2*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2
+ b*x))*a*b*c*d*sgn(x) + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*d^2*sgn(x)
- sqrt(a)*b^2*c*d*sgn(x) + a^(3/2)*b*d^2*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)*c^3)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-b\,c\right)}{c^3\,d} - \frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{6\,\sqrt{a}\,b^7\,d\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,d-\frac{14\,a^2\,b^6\,d^2}{c}+\frac{8\,a^3\,b^5\,d^3}{c^2}}-\frac{14\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,c\,d-14\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}+\frac{8\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{8\,a^3\,b^5\,d^3-14\,a^2\,b^6\,c\,d^2+6\,a\,b^7\,c^2\,d}\right)\left(4\,a\,d-3\,b\,c\right)}{c^3} - \frac{\frac{2\,\left(a\,b^2\,c-a^2\,b\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2}+\frac{b\,\left(a+\frac{b}{x}\right)^{3/2}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,\left(a+\frac{b}{x}\right)^2-a^2\,d+a\,b\,c}$$

input
```
int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

output
```
(atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^
3 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2
- b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*
d - b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(
a + b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (
14*a^(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3
*b^5*d^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2
*b^6*c*d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d
)*(a + b/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)
*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

3.237 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx = -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a}(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

output
```
3*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^4-3/4*(8*a^2*d^2-8
*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(1/
2)/(-a*d+b*c)^(1/2)-1/2*(-3*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)^2-3/4*(-4*a
*d+b*c)*(a+b/x)^(1/2)/c^3/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

3.237.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(-bc(3d+5cx)+2a\left(6d^2+9cdx+2c^2x^2\right)\right)}{(d+cx)^2} - \frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} - 12\sqrt{a}(-bc+2}{4c^4}$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x
^2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) - 12*Sqrt[a]*(-(
b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

3.237.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 109, 27, 168, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\left(\frac{b(2bc-5ad)}{x}+3a(bc-2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(\frac{b(2bc-5ad)}{x}+3a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int -\frac{3(bc-ad)\left(\frac{b(bc-3ad)}{x}+2a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2c}$$

$\downarrow$ 27

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\int \frac{\left(\frac{b(bc-3ad)}{x}+2a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 168

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{(bc-ad)\left(\frac{b(bc-4ad)}{x}+4a(bc-2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 27

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\int \frac{\left(\frac{b(bc-4ad)}{x}+4a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 174

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4a(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 73

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8a(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 218

3.237. $\displaystyle\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{8a(bc-2ad)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}\sqrt{bc-ad}}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 221

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}\sqrt{bc-ad}}-\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{c}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

output
```
(a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (((b*c - 3*a*d)*Sqrt[a + b/x])/(c*(c
 + d/x)^2) + (3*(((b*c - 4*a*d)*Sqrt[a + b/x])/(c*(c + d/x)) + ((2*(b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[d]*Sqrt[b*c - a*d]) - (8*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c)/(2*c)))/(2*c))/(2*c)
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.237. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.237. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(181) = 362$.

Time = 0.27 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.82

method	result	size
risch	Expression too large to display	1008
default	Expression too large to display	1817

input
```
int((a+b/x)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*x*a*((a*x+b)/x)^(1/2)-1/2/c^3*(3*a^(1/2)*(2*a*d-b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))-(-12*a^2*d^2+12*a*b*c*d-2*b^2*c^2)/c^2/((a*d-
b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)
*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(
x+d/c))+4/c^3*d*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)*(-1/(a*d-b*c)/d*c^2/(x+d/c)*
(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*
c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*
(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-
b*c)*d/c^2)^(1/2))/(x+d/c)))-2*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^4*(-1/2/(
a*d-b*c)/d*c^2/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^
2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d
/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-
b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)
+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/
c^2)^(1/2))/(x+d/c)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*
(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)
^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))))*((a*x+b)/x)^(1
/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(181) = 362$.

Time = 0.34 (sec) , antiderivative size = 1765, normalized size of antiderivative = 8.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*
d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x
)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2
- 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b
*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sq
rt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b
*c^6*d^2 - a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5
+ (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b
*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2
*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d
 + a*d^2)*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d -
23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*
a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6
*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^
3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^...
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

3.237.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(181) = 362$.

Time = 0.38 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\sqrt{ax^2+bx}\,a\mathrm{sgn}(x)}{c^3}$$
$$+ \frac{3\left(b^2c^2\mathrm{sgn}(x) - 8\,abcd\mathrm{sgn}(x) + 8\,a^2d^2\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\sqrt{bcd-ad^2}\,c^4}$$
$$- \frac{3\left(abc\mathrm{sgn}(x) - 2\,a^2d\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^4}$$
$$+ \frac{\left(3\sqrt{a}b^2c^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 24\,a^{\frac{3}{2}}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 24\,a^{\frac{5}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 6\sqrt{bcd-ad^2}ab\right.}{4\sqrt{bcd-ad^2}\sqrt{a}c^4}$$
$$+ \frac{5\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3\mathrm{sgn}(x) - 24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d\mathrm{sgn}(x) + 24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^2cd^2\mathrm{s}}{}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) +
8*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/
sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2
*d*sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(
a)*c^4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2
4*a^(3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arc
tan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b
)) - 12*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*
c - 10*sqrt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4)
 + 1/4*(5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*sgn(x) - 24*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^3*a*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^3*a^2*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2
*d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^2*sgn(x) +
40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^3*sgn(x) + 3*(sqrt(a)*x - s
qrt(a*x^2 + b*x))*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*
b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^3*sgn(x) - 5
*sqrt(a)*b^3*c*d^2*sgn(x) + 10*a^(3/2)*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*
c^4)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 1664, normalized size of antiderivative = 7.96

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(3/2)/(c + d/x)^3,x)
```

output
```
- ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
 - ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (
3*b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*
b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a
^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a
+ b/x)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*
b^6*d^2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a
*d - b*c))/c^4 - (atan(((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5
 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6)
- (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (
3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/
2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a
^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^
(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((
((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 172
8*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)
*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*
b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 -
8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c
*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*...
```

3.237. $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

3.238 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx$

3.238.1 Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx = -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2}$$
$$-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2} + \left(a+\frac{b}{x}\right)^{5/2}\left(c\right.$$

output
```
-1/3*c^2*(6*a*d+5*b*c)*(a+b/x)^(3/2)-11/9*d*(a+b/x)^(5/2)*(c+d/x)^2-1/315*
d*(a+b/x)^(5/2)*(-20*a^2*d^2+270*a*b*c*d+938*b^2*c^2+5*b*d*(10*a*d+89*b*c)
/x)/b^2+(a+b/x)^(5/2)*(c+d/x)^3*x+a^(3/2)*c^2*(6*a*d+5*b*c)*arctanh((a+b/x
)^(1/2)/a^(1/2))-a*c^2*(6*a*d+5*b*c)*(a+b/x)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx = \frac{\sqrt{a+\frac{b}{x}}\left(20a^4d^3x^4-10a^3bd^2x^3(d+27cx)-3a^2b^2x^2(50d^3+270cd^2x+966c^2dx^2-105c^3x^3)-\right.}{315b^2}$$
$$+a^{3/2}c^2(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2
*x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3
+ 135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d
^2*x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c +
 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

3.238.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {899, 108, 27, 170, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$x\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3-\int \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \left(5bc+6ad+\frac{11bd}{x}\right) x d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$x\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3-\frac{1}{2}\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \left(5bc+6ad+\frac{11bd}{x}\right) x d\frac{1}{x}$$

$$\Big\downarrow \text{ 170}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{2}b\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \left(9c(5bc+6ad)+\frac{d(89bc+10ad)}{x}\right) x d\frac{1}{x}}{9b}-\frac{22}{9}d\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2\right)+ x\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\left(9c(5bc+6ad)+\frac{d(89bc+10ad)}{x}\right)xd\frac{1}{x}-\frac{22}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\right)+$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{35b^2}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)\right.}{35b^2}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+13\right.\right.}{3}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2\right.\right.}{}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \text{221}$$

3.238. $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx$

$$\frac{1}{2}\left(\frac{1}{9}\left(-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{35b^2}-9c^2\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\operatorname{arctanh}\left(\right.\right.\right.\right.\right.$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

output
```
(a + b/x)^(5/2)*(c + d/x)^3*x + ((-22*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 + (
(-2*d*(a + b/x)^(5/2)*(2*(469*b^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d
*(89*b*c + 10*a*d))/x))/(35*b^2) - 9*c^2*(5*b*c + 6*a*d)*((2*(a + b/x)^(3/
2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/9
)/2
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.238. $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

3.238.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(315a^2b^2c^3x^5+20a^4d^3x^4-270a^3bc\,d^2x^4-2898a^2b^2c^2d\,x^4-1470a\,b^3c^3x^4-10a^3b\,d^3x^3-810a^2b^2c\,d^2x^3-1386a\,b^3c^2d\,x^3-210b^4c^3x^3-}{315x^4b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(3780a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,b\,c^2d\,x^6+3150a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b^2c^3x^6+1890\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c^2d\,x^6+1575\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{}$

input
```
int((a+b/x)^(5/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/315*(315*a^2*b^2*c^3*x^5+20*a^4*d^3*x^4-270*a^3*b*c*d^2*x^4-2898*a^2*b^2
*c^2*d*x^4-1470*a*b^3*c^3*x^4-10*a^3*b*d^3*x^3-810*a^2*b^2*c*d^2*x^3-1386*
a*b^3*c^2*d*x^3-210*b^4*c^3*x^3-150*a^2*b^2*d^3*x^2-810*a*b^3*c*d^2*x^2-37
8*b^4*c^2*d*x^2-190*a*b^3*d^3*x-270*b^4*c*d^2*x-70*b^4*d^3)/x^4/b^2*((a*x+
b)/x)^(1/2)+1/2*(6*a*d+5*b*c)*a^(3/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.49

$$\int\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\left[\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{a}x^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(315\,a^2b^2c^3x^5-70\,b^4d^3-2\,(}{}\right.$$

$$-\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(315\,a^2b^2c^3x^5-70\,b^4d^3-2\,(735\,ab^3c^3+1449\,a^2b^2}{}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(73
5*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(
105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(
63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 +
19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 +
6*a^2*b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*
a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135
*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^
2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^
2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b
^2*x^4)]
```

3.238.6 Sympy [A] (verification not implemented)

Time = 39.99 (sec) , antiderivative size = 5523, normalized size of antiderivative = 27.89

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

output
```
32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a*
*(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**
(29/2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**
(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b
**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**
(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*
sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(
19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) +
4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(
9/2)*b**21*x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(
315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(
17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/
2)) + 210*a**(21/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**
15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(1
7/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1
890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2
)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3...
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = -\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7\,b}$$
$$+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^3$$
$$-\frac{1}{5}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}a^2\right)c^2d$$
$$-\frac{2}{63}\left(\frac{7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{b^2}-\frac{9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{b^2}\right)d^3$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

output
```
-6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2
)*b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*
a + 30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x
)^(7/2)*a/b^2)*d^3
```

3.238.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.238.9 Mupad [B] (verification not implemented)

Time = 10.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.46

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{7\,b^2}-\frac{4\,a\,d^3}{7\,b^2}\right)$$
$$-\sqrt{a+\frac{b}{x}}\left(a^2\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-2\,a\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\right.\right.\right.$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,(a\,d-b\,c)^3}{3\,b^2}+\frac{2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)}{3}-\frac{a^2\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right.}{3}\right.$$

input
```
int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
 b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(
a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2
*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 +
 (2*a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a
 + b/x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2
)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (
a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*
a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*
b^2) + (2*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*
(a + b/x)^(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d +
5*b*c)*1i
```

3.238. $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx$

3.239 $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx$

3.239.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx=-ac(5bc+4ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}+\frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a}+a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-1/3*c*(4*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*c*(4*a*d+5*b*c)*(a+b/x)^(5/2)/a-2/7
*d^2*(a+b/x)^(7/2)/b+c^2*(a+b/x)^(7/2)*x/a+a^(3/2)*c*(4*a*d+5*b*c)*arctanh
((a+b/x)^(1/2)/a^(1/2))-a*c*(4*a*d+5*b*c)*(a+b/x)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx=\frac{\sqrt{a+\frac{b}{x}}\left(-30a^3d^2x^3-2b^3(15d^2+42cdx+35c^2x^2)+a^2bx^2(-90d^2-644cdx+105c^2x^2)-2ab^2\right.}{105bx^3}+a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-30*a^3*d^2*x^3 - 2*b^3*(15*d^2 + 42*c*d*x + 35*c^2*x^2) +
 a^2*b*x^2*(-90*d^2 - 644*c*d*x + 105*c^2*x^2) - 2*a*b^2*x*(45*d^2 + 154*c
*d*x + 245*c^2*x^2)))/(105*b*x^3) + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]]
```

3.239.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {899, 100, 27, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\int \frac{1}{2} \left(a+\frac{b}{x}\right)^{5/2} \left(\frac{2ad^2}{x}+c(5bc+4ad)\right) x d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\int \left(a+\frac{b}{x}\right)^{5/2} \left(\frac{2ad^2}{x}+c(5bc+4ad)\right) x d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc) \int \left(a+\frac{b}{x}\right)^{5/2} x d\frac{1}{x}+\frac{4ad^2 \left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc) \left(a \int \left(a+\frac{b}{x}\right)^{3/2} x d\frac{1}{x}+\frac{2}{5} \left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2 \left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\int\sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\left(a\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$$\Big\downarrow \; 73$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$$\Big\downarrow \; 221$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)(4ad+5bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

output
```
(c^2*(a + b/x)^(7/2)*x)/a - ((4*a*d^2*(a + b/x)^(7/2))/(7*b) + c*(5*b*c +
4*a*d)*((2*(a + b/x)^(5/2))/5 + a*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b
/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))))/(2*a)
```

3.239. $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx$

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.239. $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.239.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\left(-105a^2b\,c^2x^4+30a^3d^2x^3+644a^2bcd\,x^3+490a\,b^2c^2x^3+90x^2a^2b\,d^2+308x^2a\,b^2cd+70x^2b^3c^2+90a\,b^2d^2x+84b^3cdx+30b^3d^2\right)\sqrt{\frac{ax+}{x}}}{105x^3b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(840a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,cd\,x^5+1050a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b\,c^2x^5-840a^{\frac{5}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}cd\,x^3-60a^{\frac{5}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}d^2x^2-840a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}b\,c\right.}{}$

input
```
int((a+b/x)^(5/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-105*a^2*b*c^2*x^4+30*a^3*d^2*x^3+644*a^2*b*c*d*x^3+490*a*b^2*c^2*
x^3+90*a^2*b*d^2*x^2+308*a*b^2*c*d*x^2+70*b^3*c^2*x^2+90*a*b^2*d^2*x+84*b^
3*c*d*x+30*b^3*d^2)/x^3/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+5*b*c)*a^(3/2)*c*ln
((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2
)/(a*x+b)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.30

$$\int\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx=\left[\frac{105\left(5\,ab^2c^2+4\,a^2bcd\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(105\,a^2bc^2x^4-30\,b^3d^2-2\left(245\right.\right.}{}\right.$$

$$-\frac{105\left(5\,ab^2c^2+4\,a^2bcd\right)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(105\,a^2bc^2x^4-30\,b^3d^2-2\left(245\,ab^2c^2+322\,a^2bcd+\right.\right.}{105\,bx^3}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*
x*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^
2*c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d +
45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b
*x^3), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^
2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a
^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
)]
```

3.239.6 Sympy [A] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 1853, normalized size of antiderivative = 12.19

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

output
```
-16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(
13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d*
*2*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4
0*a**(13/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21
/2)*d**2*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2
)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 8*a**(11/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x
/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9
/2)*b**(23/2)*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a
**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*
x**(7/2)) + 4*a**(9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sq...
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.19

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^2}{7\,b} + \frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x - 15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 24\sqrt{a+\frac{b}{x}}ab\right)c^2 - \frac{2}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 6\left(a+\frac{b}{x}\right)^{\frac{5}{2}} + 10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a + 30\sqrt{a+\frac{b}{x}}a^2\right)cd$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log
((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a
 + 30*sqrt(a + b/x)*a^2)*c*d
```

3.239.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.239.9 Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx = \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)}{3}-\frac{2\,{\left(a\,d-b\,c\right)}^2}{3\,b}+\frac{2\,a^2\,d^2}{3\,b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{5\,b}-\frac{4\,a\,d^2}{5\,b}\right)\left(a+\frac{b}{x}\right)^{5/2}-\sqrt{a+\frac{b}{x}}\left(a^2\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-2\,a\left(2\,a\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2}{b}+\frac{2\,a^2\,d^2}{b}\right)\right)-$$

input
```
int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

output
```
(a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
 b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^
2)/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b -
(4*a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d -
b*c)^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a
 + b/x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*
c)*1i
```

3.239. $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx$

3.240 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx$

3.240.1 Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx=-a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad) \left(a+\frac{b}{x}\right)^{3/2} -\frac{(5bc+2ad) \left(a+\frac{b}{x}\right)^{5/2}}{5a}+\frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a}+a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-1/3*(2*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*(2*a*d+5*b*c)*(a+b/x)^(5/2)/a+c*(a+b/
x)^(7/2)*x/a+a^(3/2)*(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))-a*(2*a*d
+5*b*c)*(a+b/x)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx=\frac{\sqrt{a+\frac{b}{x}}\left(-2b^2(3d+5cx)+a^2x^2(-46d+15cx)-2abx(11d+35cx)\right)}{15x^2}+a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

output
```
(Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*
(11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]]
```

3.240.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {899, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)x^2d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\int \left(a+\frac{b}{x}\right)^{5/2}xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \text{60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \text{73}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\left(a\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)(2ad+5bc)}{2a}$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x),x]
```

output
```
(c*(a + b/x)^(7/2)*x)/a - ((5*b*c + 2*a*d)*((2*(a + b/x)^(5/2))/5 + a*((2*
(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]))))/(2*a)
```

3.240.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.240. $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.240.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
risch	$\frac{\left(15a^{2}c\,x^{3}-46a^{2}d\,x^{2}-70abc\,x^{2}-22axbd-10b^{2}cx-6b^{2}d\right)\sqrt{\frac{ax+b}{x}}}{15x^{2}}+\frac{\left(2ad+5bc\right)a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\left(ax+b\right)}}{2ax+2b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(60a^{\frac{7}{2}}\sqrt{a\,x^{2}+bx}\,d\,x^{4}+150a^{\frac{5}{2}}\sqrt{a\,x^{2}+bx}\,bc\,x^{4}-60a^{\frac{5}{2}}\left(a\,x^{2}+bx\right)^{\frac{3}{2}}d\,x^{2}-120a^{\frac{3}{2}}\left(a\,x^{2}+bx\right)^{\frac{3}{2}}bc\,x^{2}+30\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+\cdots}{2\sqrt{a}}\right.\right.}{30x^{3}b\sqrt{x(ax+\cdots}}$

input
```
int((a+b/x)^(5/2)*(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*a^2*c*x^3-46*a^2*d*x^2-70*a*b*c*x^2-22*a*b*d*x-10*b^2*c*x-6*b^2*d
)/x^2*((a*x+b)/x)^(1/2)+1/2*(2*a*d+5*b*c)*a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(
a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.78

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx = \left[\frac{15\,(5\,abc+2\,a^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a^2\right.}{30\,x^2}$$

$$-\frac{15\,(5\,abc+2\,a^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a^2d)x^2-2\,(5\,b^2c+11\,a}{15\,x^2}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

output
```
[1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 -
 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*
a^2*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 -
 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a
*x + b)/x))/x^2]
```

3.240.6 Sympy [A] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.27

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx = \frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a^2\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1} - a^2d\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) - 2abc\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + 2abd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) + b^2c\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x),x)
```

output
```
4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 1
5*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2
)*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/
2)) - 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) -
 4*a**6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(
5/2)) - 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b*
*4*x**(5/2)) + a**2*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - a**2*d*Piecewise((
2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-
sqrt(a)*log(x), True)) - 2*a*b*c*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a
))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 2*a*b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) +
b**2*c*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

3.240.7 Maxima [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

output
```
Timed out
```

3.240.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.240.9 Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{5/2}}{5}-2\,a^2\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int((a + b/x)^(5/2)*(c + d/x),x)
```

output
```
- (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
 b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)
^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

3.241 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

3.241.1 Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-5/3*b*(a+b/x)^(3/2)+(a+b/x)^(5/2)*x+5*a^(3/2)*b*arctanh((a+b/x)^(1/2)/a^(
1/2))-5*a*b*(a+b/x)^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \frac{\sqrt{a+\frac{b}{x}}(-2b^2-14abx+3a^2x^2)}{3x} + 5a^{3/2}b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]
```

3.241.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2),x]
```

output
```
(a + b/x)^(5/2)*x - (5*b*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*S
qrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/2
```

3.241.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.241. $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

3.241.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{(3a^2x^2-14abx-2b^2)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	94
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(30a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,x^3-24a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}x+15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3-4b\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^2\sqrt{x(ax+b)}\,\sqrt{a}}$	120

input
```
int((a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^2*x^2-14*a*b*x-2*b^2)/x*((a*x+b)/x)^(1/2)+5/2*a^(3/2)*b*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+
b)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.96

$$\int\left(a+\frac{b}{x}\right)^{5/2}dx=\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{6\,x},\right.$$
$$\left.-\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3
*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x
*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqr
t((a*x + b)/x))/x]
```

3.241.6 Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}} - \frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2} + 5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{2\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

input
```
integrate((a+b/x)**(5/2),x)
```

output
```
a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3
/2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)
*b**2*sqrt(1 + b/(a*x))/(3*x)
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \sqrt{a+\frac{b}{x}}a^2x - \frac{5}{2}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 4\sqrt{a+\frac{b}{x}}ab$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

3.241.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.241.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -\frac{2\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

input
```
int((a + b/x)^(5/2),x)
```

output
```
-(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
 + 1)^(5/2))
```

3.242 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

3.242.1 Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=-\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}+\frac{a^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

output
```
a*(a+b/x)^(3/2)*x/c+2*(-a*d+b*c)^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+
b*c)^(1/2))/c^2/d^(3/2)+a^(3/2)*(-2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/
2))/c^2-b*(a*d+2*b*c)*(a+b/x)^(1/2)/c/d
```

3.242.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}(-2b^2c+a^2dx)}{d}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}-a^{3/2}(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

output
```
((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(S
qrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 2*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

3.242.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 109, 27, 171, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-2ad)+\frac{b(2bc+ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-2ad)+\frac{b(2bc+ad)}{x}\right)x}{c+\frac{d}{x}}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{2\int \frac{\left(a^2d(5bc-2ad)-\frac{b\left(2b^2c^2-6abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\int \frac{\left(a^2d(5bc-2ad)-\frac{b\left(2b^2c^2-6abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \; 174$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{a^2d(5bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2(bc-ad)^3\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \; 73$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{2a^2d(5bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4(bc-ad)^3\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \; 218$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{2a^2d(5bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \; 221$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{-\frac{2a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-2ad)}{c}-\frac{4(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

input
```
Int[(a + b/x)^(5/2)/(c + d/x),x]
```

output
```
(a*(a + b/x)^(3/2)*x)/c - ((2*b*(2*b*c + a*d)*Sqrt[a + b/x])/d + ((-4*(b*c
 - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d])
 - (2*a^(3/2)*d*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/d)/(2*c
)
```

3.242.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.242. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

3.242.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.08

method	result
risch	$\frac{(a^2dx-2b^2c)\sqrt{\frac{ax+b}{x}}}{dc} - \frac{\left(\frac{a^{\frac{3}{2}} d(2ad-5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c} - \frac{\left(-2a^3d^3+6a^2bc\,d^2-6a\,b^2c^2d+2b^3c^3\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}{}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2cd(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^4x^2-2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d^2x^2-6a^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}}{cx+}\right.\right.}{}$

input
```
int((a+b/x)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
(a^2*d*x-2*b^2*c)/d/c*((a*x+b)/x)^(1/2)-1/2/c/d*(a^(3/2)*d*(2*a*d-5*b*c)/c
*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))-(-2*a^3*d^3+6*a^2*b*c*d^2-6*a*b
^2*c^2*d+2*b^3*c^3)/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a
*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+
d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
/(a*x+b)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 659, normalized size of antiderivative = 4.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \left[-\frac{(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-}{a}}}{2\,c^2d}\right.$$

$$-\frac{(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc}{cx+d}\right.}{c^2d}$$

$$-\frac{4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}\right.}{2\,c^2d}$$

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-}{c^2d}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((
2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x
 + d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*
d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2
*c^2)*sqrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c -
a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x +
 b)/x))/(c^2*d)]
```

3.242.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x),x)
```

output
```
Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

3.242.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x), x)
```

3.242.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.242.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 1427, normalized size of antiderivative = 10.65

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \frac{a^2\,b\,d\,\sqrt{a+\frac{b}{x}}}{c\,\left(d\,\left(a+\frac{b}{x}\right)-a\,d\right)} - \frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{d}$$

$$+\frac{\mathrm{atan}\left(\frac{a^3\,b^5\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^5\,d^8-5\,a^4\,b\,c\,d^7+10\,a^3\,b^2\,c^2\,d^6-10\,a^2\,b^3\,c^3\,d^5+5\,a\,b^4\,c^4\,d^4-b^5\,c^5\,d^3}\,160\mathrm{i}}{448\,a^3\,b^8\,c^3\,d-340\,a^6\,b^5\,d^4-128\,a^2\,b^9\,c^4+740\,a^5\,b^6\,c\,d^3+\frac{16\,a\,b^{10}\,c^5}{d}-796\,a^4\,b^7\,c^2\,d^2+\frac{60\,a^7\,b^4\,d^5}{c}} - \frac{a^2\,b^6\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^5\,d^8-5\,a^4\,b\,c\,d^7+\cdots}}{16\,a\,b^{10}\,c^4+740\,a^5\,b^6\,d^4-128\,a^2\,b^9\,c^3\cdots}\right.}{\cdots}$$

$$+\frac{\mathrm{atan}\left(\frac{b^9\,c^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}\,40\mathrm{i}}{40\,a^2\,b^9\,c^3-790\,a^5\,b^6\,d^3-256\,a^3\,b^8\,c^2\,d+696\,a^4\,b^7\,c\,d^2+\frac{370\,a^6\,b^5\,d^4}{c}-\frac{60\,a^7\,b^4\,d^5}{c^2}} + \frac{a\,b^8\,c^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}\,256\mathrm{i}}{256\,a^3\,b^8\,c^2+790\,a^5\,b^6\,d^2-\frac{40\,a^2\,b^9\,c^3}{d}-\frac{370\,a^6\,b^5\,d^3}{c}+6\cdots}\right.}{\cdots}$$

input
```
int((a + b/x)^(5/2)/(c + d/x),x)
```

output
```
(atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 -
10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*
a^3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (1
6*a*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a
+ b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5
 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5
*b^6*d^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (
340*a^6*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5
*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2
*d^6 - 5*a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a
^4*b^7*c^3*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d
^2 - (128*a^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^
3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*
d^7)^(1/2)*16i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2
 + 448*a^3*b^8*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a
*b^10*c^4*d))*(d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(
1/2))/d + (atan((b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3
- 790*a^5*b^6*d^3 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d
^4)/c - (60*a^7*b^4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b...
```

3.243 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

output
```
a*(a+b/x)^(3/2)*x/c/(c+d/x)-(-a*d+b*c)^(3/2)*(4*a*d+b*c)*arctan(d^(1/2)*(a
+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)+a^(3/2)*(-4*a*d+5*b*c)*arctanh((
a+b/x)^(1/2)/a^(1/2))/c^3+(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d
/x)
```

3.243.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(b^2c^2-2abcd+a^2d(2d+cx))}{d(d+cx)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - a^{3/2}(-5bc+4ad)\text{arctan}}{c^3}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x
)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[
b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/c^3
```

3.243.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 109, 27, 166, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-4ad)+\frac{b(2bc-ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-4ad)+\frac{b(2bc-ad)}{x}\right)x}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{166}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}-\frac{\int -\frac{\left(d(5bc-4ad)a^2+\frac{b\left(b^2c^2+2abdc-2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{cd}}{2c}$$

$$\Big\downarrow \ 25$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(d(5bc-4ad)a^2+\frac{b\left(b^2c^2+2abdc-2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \ 174$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{a^2d(5bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}+\frac{(bc-ad)^2(4ad+bc)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \ 73$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a^2d(5bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^2(4ad+bc)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \ 218$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a^2d(5bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \ 221$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-4ad)}{c}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

input
```
Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

output
```
(a*(a + b/x)^(3/2)*x)/(c*(c + d/x)) - ((2*(3*a*b - (b^2*c)/d - (2*a^2*d)/c
)*Sqrt[a + b/x])/(c + d/x) + ((2*(b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(S
qrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*a^(3/2)*d*(5*b*c
- 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(c*d))/(2*c)
```

3.243. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.243. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.243.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(146) = 292$.

Time = 0.28 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.11

method	result
risch	$\frac{a^2 x\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{a^{\frac{3}{2}}(4ad-5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c} + \frac{6a\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2}}{x+\frac{d}{c}}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{}$
default	Expression too large to display

input
```
int((a+b/x)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*a^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(3/2)*(4*a*d-5*b*c)/c*ln((1/2*b+a
*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+6*a/c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b
*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*
d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x
+d/c))+1/c^3*(2*a^3*d^3-6*a^2*b*c*d^2+6*a*b^2*c^2*d-2*b^3*c^3)*(-1/(a*d-b*
c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)
-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c
^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c
)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b
))^(1/2)/(a*x+b)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1001, normalized size of antiderivative = 6.03

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \left[-\frac{(5\,abcd^2-4\,a^2d^3+(5\,abc^2d-4\,a^2cd^2)x)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(b^2c^2d\right.$$

$$-\frac{2\,(5\,abcd^2-4\,a^2d^3+(5\,abc^2d-4\,a^2cd^2)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(b^2c^2d+3\,abcd^2-4\,a^2d^3+(b^2c^3+}{2}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 -
 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)
*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x
)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)
*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3
+ (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) + (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*
x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2),
1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x +
 b)/x)/(b*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^
2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*
d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d
*x + c^3*d^2), ((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^
2*d - 4*a^2*c*d^2)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4
*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2...
```

3.243.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{(cx+d)^2}\,dx$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

3.243.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(146) = 292$.

Time = 0.34 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\sqrt{ax^2+bx}\,a^2\operatorname{sgn}(x)}{c^2}$$
$$-\frac{\left(5\,a^2bc\operatorname{sgn}(x)-4\,a^3d\operatorname{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^3}$$
$$+\frac{\left(b^3c^3\operatorname{sgn}(x)+2\,ab^2c^2d\operatorname{sgn}(x)-7\,a^2bcd^2\operatorname{sgn}(x)+4\,a^3d^3\operatorname{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3d}$$
$$+\frac{\left(2\sqrt{a}b^3c^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+4\,a^{\frac{3}{2}}b^2c^2d\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-14\,a^{\frac{5}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+8\,a^{\frac{7}{2}}d^3\operatorname{arctar}\right.}{}$$
$$-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c^3\operatorname{sgn}(x)-4\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{3}{2}}b^2c^2d\operatorname{sgn}(x)+5\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{5}{2}}bcd^2}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{\right.\right.}}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^2 - 1/2*(5*a^2*b*c*sgn(x) - 4*a^3*d*sgn(x))
*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^3) +
(b^3*c^3*sgn(x) + 2*a*b^2*c^2*d*sgn(x) - 7*a^2*b*c*d^2*sgn(x) + 4*a^3*d^3*
sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d
 - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3*d) + 1/2*(2*sqrt(a)*b^3*c^3*arctan(sqr
t(a)*d/sqrt(b*c*d - a*d^2)) + 4*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*
c*d - a*d^2)) - 14*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) +
 8*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 5*sqrt(b*c*d - a*d^
2)*a^2*b*c*d*log(abs(b)) - 4*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - 2*s
qrt(b*c*d - a*d^2)*a*b^2*c^2 + 4*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 2*sqrt(b*
c*d - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3*d) - ((sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^3*sgn(x) - 4*(sqrt(a)*x - sqrt(a*x
^2 + b*x))*a^(3/2)*b^2*c^2*d*sgn(x) + 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^
(5/2)*b*c*d^2*sgn(x) - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3*sgn(x
) - a*b^3*c^2*d*sgn(x) + 2*a^2*b^2*c*d^2*sgn(x) - a^3*b*d^3*sgn(x))/(((sqr
t(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a
)*d + b*d)*sqrt(a)*c^3*d)
```

3.243.9 Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.95

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{\sqrt{a+\frac{b}{x}}\,(2\,a^3\,b\,d^2-3\,a^2\,b^2\,c\,d+a\,b^3\,c^2)}{c^2\,d}-\frac{b\left(a+\frac{b}{x}\right)^{3/2}(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{c^2\,d}}{\left(a+\frac{b}{x}\right)(2\,a\,d-b\,c)-d\left(a+\frac{b}{x}\right)^2-a^2\,d+a\,b\,c}$$
$$-\frac{\mathrm{atanh}\left(\frac{10\,b^9\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{10\,a^2\,b^9+\frac{32\,a^3\,b^8\,d}{c}-\frac{132\,a^4\,b^7\,d^2}{c^2}+\frac{130\,a^5\,b^6\,d^3}{c^3}-\frac{40\,a^6\,b^5\,d^4}{c^4}}+\frac{32\,a\,b^8\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{32\,a^3\,b^8+\frac{10\,a^2\,b^9\,c}{d}-\frac{132\,a^4\,b^7\,d}{c}+\frac{130\,a^5\,b^6\,d^2}{c^2}-\frac{40\,a^6\,b^5\,d^3}{c^3}}-\frac{}{32\,a^3\,b^8\,c-1}\right.}{}$$
$$+\frac{\mathrm{atanh}\left(\frac{30\,a^3\,b^6\sqrt{a+\frac{b}{x}}\sqrt{a^3\,d^6-3\,a^2\,b\,c\,d^5+3\,a\,b^2\,c^2\,d^4-b^3\,c^3\,d^3}}{14\,a^2\,b^9\,c^3+110\,a^5\,b^6\,d^3-4\,a^3\,b^8\,c^2\,d-82\,a^4\,b^7\,c\,d^2+\frac{2\,a\,b^{10}\,c^4}{d}-\frac{40\,a^6\,b^5\,d^4}{c}}+\frac{18\,a^2\,b^7\sqrt{a+\frac{b}{x}}\sqrt{a^3\,d^6-3\,a^2\,b\,c\,d^5+3\,a\,b^2\,c^2\,d^4-}}{2\,a\,b^{10}\,c^3-82\,a^4\,b^7\,d^3+14\,a^2\,b^9\,c^2\,d-4\,a^3\,b^8\,c\,d^2+\frac{110\,a^5\,b^6}{c}}\right.}{}$$

input
```
int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

3.243. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

output
```
(((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*
(a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*
a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/
2)*(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (
130*a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(
a^3)^(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*
b^6*d^2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3
)^(1/2))/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6
*d^2)/c - (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(
1/2))/(32*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5
*d^3)/c - 132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/
(32*a^3*b^8*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 +
 (10*a^2*b^9*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^
6*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5
)^(1/2))/(14*a^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*
c*d^2 + (2*a*b^10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/
2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b
^10*c^3 - 82*a^4*b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b
^6*d^4)/c - (40*a^6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 -
 b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 +...
```

3.243. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

3.244 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx = & \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} \\ & + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}(b^2c^2+8abcd-24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}} \\ & + \frac{a^{3/2}(5bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}\end{aligned}$$

output
```
a*(a+b/x)^(3/2)*x/c/(c+d/x)^2+a^(3/2)*(-6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)
/a^(1/2))/c^4-1/4*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(
1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^4/d^(3/2)+1/2*(-3*a*d+b*c)*(-a*d
+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2+7*a*b*c*d+b^2*c^2)*(a
+b/x)^(1/2)/c^3/d/(c+d/x)
```

3.244.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(b^2c^2(-d+cx)-abcd(7d+11cx)+2a^2d\left(6d^2+9cdx+2c^2x^2\right)\right)}{d(d+cx)^2}-\frac{\left(b^3c^3+7ab^2c^2d-32a^2bcd^2+24a^3d^3\right)\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{d^{3/2}\sqrt{bc-ad}}}{4c^4}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d
*(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - ((b^3*c^3 + 7*a*b^2*c^2
*d - 32*a^2*b*c*d^2 + 24*a^3*d^3)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(d^(3/2)*Sqrt[b*c - a*d]) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(4*c^4)
```

3.244.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 109, 27, 166, 25, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-6ad)+\frac{b(2bc-3ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-6ad)+\frac{b(2bc-3ad)}{x}\right)x}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$\big\downarrow$ 166

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}-\frac{\int -\frac{\left(2d(5bc-6ad)a^2+\frac{b\left(b^2c^2+6abdc-9a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2cd}}{2c}$$

$\big\downarrow$ 25

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{\left(2d(5bc-6ad)a^2+\frac{b\left(b^2c^2+6abdc-9a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\big\downarrow$ 168

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{\left(4d(5bc-6ad)(bc-ad)a^2+\frac{b(bc-ad)\left(b^2c^2+7abdc-12a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\big\downarrow$ 27

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\int \frac{\left(4d(5bc-6ad)(bc-ad)a^2+\frac{b(bc-ad)\left(b^2c^2+7abdc-12a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\big\downarrow$ 174

3.244. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{(bc-ad)^2\left(-24a^2d^2+8abcd+b^2c^2\right)\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4a^2d(5bc-6ad)(bc-ad)\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-b\cdots\right.}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 73

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{2(bc-ad)^2\left(-24a^2d^2+8abcd+b^2c^2\right)\int\frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8a^2d(5bc-6ad)(bc-ad)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}\cdots\right.}{\left(c+\frac{d}{x}\right)\cdots}}{2c}$$

$\Big\downarrow$ 218

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8a^2d(5bc-6ad)(bc-ad)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^{3/2}\left(-24a^2d^2+8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}\cdots\right.}{\left(c+\cdots\right)}}{2c}$$

$\Big\downarrow$ 221

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}+\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}+\frac{\frac{2(bc-ad)^{3/2}\left(-24a^2d^2+8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{8a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\cdots}{c}}{2c(bc-ad)}}{2cd}}{2c}$$

input
```
Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

3.244. $\int\frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

output
```
(a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (((4*a*b - (b^2*c)/d - (3*a^2*d)/c
)*Sqrt[a + b/x])/(c + d/x)^2 + (((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a
 + b/x])/(c*(c + d/x)) + ((2*(b*c - a*d)^(3/2)*(b^2*c^2 + 8*a*b*c*d - 24*a
^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (8*
a^(3/2)*d*(5*b*c - 6*a*d)*(b*c - a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(
2*c*(b*c - a*d)))/(2*c*d))/(2*c)
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

3.244. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.244.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(209) = 418$.

Time = 0.34 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.37

method	result	size
risch	Expression too large to display	1035
default	Expression too large to display	1638

input
```
int((a+b/x)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/c^3*x*((a*x+b)/x)^(1/2)-1/2/c^3*(a^(3/2)*(6*a*d-5*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))+6/c^2*a*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)/((a*d-b
*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*
d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x
+d/c))+(8*a^3*d^3-18*a^2*b*c*d^2+12*a*b^2*c^2*d-2*b^3*c^3)/c^3*(-1/(a*d-b*
c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)
-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c
^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c
)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))-2*d*(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c)^2*(a*(x+d/c)^2-(2*
a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-
1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c
^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d
-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(
2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2*a/(a*d-b*c)/d*c
^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*(
(a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)
^(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 1445, normalized size of antiderivative = 6.10

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*
(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b
*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x
) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11
*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d
^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c
^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^
2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/
d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b
*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2
- 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) +
 (4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c
^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*
c^5*d^2*x + c^4*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*
a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2
*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 -
24*a^2*c*d^3)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*s...
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

3.244.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(209) = 418$.

Time = 0.35 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))
*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^4) +
1/4*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^
3*d^3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt
(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqr
t(b*c*d - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2
)) + 24*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d
- a*d^2)*a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)
) - sqrt(b*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*
sqrt(b*c*d - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) -
1/4*((sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b
*x))^3*a^(7/2)*c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^
3*d*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) +
48*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt
(a)*b^4*c^3*d*sgn(x) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*
d^2*sgn(x) + 52*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x...
```

3.244.9 Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 1476, normalized size of antiderivative = 6.23

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

output

```
(atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*
d)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a
+ b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b
^7*d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)
^(1/2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) +
 (45*a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/
(4*(8*a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*
b^7*c*d)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4
*c^3 - 24*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a
 + b/x)^(5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(
1/2)*(12*a^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*
d))/((a + b/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*
a*b*c*d) - d^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (
5*a^2*b^9*c^6 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 -
 59*a^4*b^7*c^4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9
*d) - ((((a + b/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5
- 15*a^2*b^6*c^4*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b
^7*c^5*d))/(8*c^6*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^
3*b^3*c^8*d^4)/(16*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x
)^(1/2)*(d^3*(a*d - b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8...
```

3.244. $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

3.245 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

3.245.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2x}{a} - \frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-c^2*(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-1/3*d*(-4*a^2*d^2
+18*a*b*c*d+6*b^2*c^2+b*d*(2*a*d+3*b*c)/x)*(a+b/x)^(1/2)/a/b^2+c*(c+d/x)^2
*x*(a+b/x)^(1/2)/a
```

3.245.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}(4a^2d^3x+3b^2c^3x^2-2abd^2(d+9cx))}{3ab^2x} + \frac{c^2(-bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a
*b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

3.245.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^3x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(bc-6ad)-\frac{d(3bc+2ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(bc-6ad)-\frac{d(3bc+2ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \text{164}$$

$$\frac{c^2(bc-6ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2c^2(bc-6ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2d\sqrt{a+\frac{b}{x}}\left(2(-2a^2d^2+9abcd+3b^2c^2)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)}{\sqrt{a}}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

input
```
Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*(c + d/x)^2*x)/a + ((-2*d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9
*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*a*d))/x))/(3*b^2) - (2*c^2*(b*c -
6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(2*a)
```

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.245.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(ax+b)\left(3c^3b^2x^2+4x\,a^2d^3-18xabc\,d^2-2ab\,d^3\right)}{3b^2x^2a\sqrt{\frac{ax+b}{x}}}+\frac{(6ad-bc)c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(6a^{\frac{7}{2}}\sqrt{x(ax+b)}\,d^3x^3-18a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bc\,d^2x^3+18a^{\frac{3}{2}}\sqrt{x(ax+b)}\,b^2c^2d\,x^3-6\sqrt{a}\,\sqrt{x(ax+b)}\,b^3c^3x^3+6a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,d^3x^3\right.}{}$

input
```
int((c+d/x)^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x+b)*(3*b^2*c^3*x^2+4*a^2*d^3*x-18*a*b*c*d^2*x-2*a*b*d^3)/b^2/x^2/a
/((a*x+b)/x)^(1/2)+1/2*(6*a*d-b*c)*c^2/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x
^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.85

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$$

$$= \left[-\frac{3\,(b^3c^3-6\,ab^2c^2d)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(3\,ab^2c^3x^2-2\,a^2bd^3-2\,(9\,a^2bcd^2-2\,a^3d^3)x}{6\,a^2b^2x}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 -
2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^
2*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 -
2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2
*x)]
```

3.245.6 Sympy [A] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.10

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}$$
$$-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}$$
$$-\frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-3c^2d\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$+3cd^2\left(\begin{cases}-\frac{1}{\sqrt{ax}} & \text{for } b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

output
```
4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) +
3*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*
a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d
**3*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3
/2)) - 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b*
*4*x**(3/2)) - 3*c**2*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
, Ne(b, 0)), (-log(x)/sqrt(a), True)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x)
, Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b
 + 1)/a - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.32

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}\,c^3\left(\frac{2\,\sqrt{a+\frac{b}{x}}b}{(a+\frac{b}{x})a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2}{3}\,d^3\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{a+\frac{b}{x}}a}{b^2}\right) - \frac{3\,c^2d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{6\,\sqrt{a+\frac{b}{x}}cd^2}{b}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2
- 3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

3.245.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.245.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b^2}+\frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1\mathrm{i}}{a^{3/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a +
b/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/
2)*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

3.246 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}+\frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-c*(-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-2*d^2*(a+b/x)^(1/2)
/b+c^2*x*(a+b/x)^(1/2)/a
```

3.246.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,(-2ad^2+bc^2x)}{ab}+\frac{c(-bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[S
qrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

3.246.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{\int -\frac{\left(c(bc-4ad)-\frac{2ad^2}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(c(bc-4ad)-\frac{2ad^2}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{90}$$

$$\frac{c(bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2c(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2c\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

input
```
Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

output
```
(c^2*Sqrt[a + b/x]*x)/a + ((-4*a*d^2*Sqrt[a + b/x])/b - (2*c*(b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(2*a)
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.246. $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.246.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

method	result
risch	$-\frac{(ax+b)(-bx\,c^2+2a\,d^2)}{bax\sqrt{\frac{ax+b}{x}}}+\frac{(4ad-bc)c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,d^2x^2-4a^{\frac{3}{2}}\sqrt{x(ax+b)}\,bcd\,x^2+2\sqrt{a}\,\sqrt{x(ax+b)}\,b^2c^2x^2+2a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,d^2x^2+4a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,bcd\,x^2+\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$

input
```
int((c+d/x)^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+b)*(-b*c^2*x+2*a*d^2)/b/a/x/((a*x+b)/x)^(1/2)+1/2*(4*a*d-b*c)*c/a^(3
/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+
b))^(1/2)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int\frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$=\left[-\frac{(b^2c^2-4\,abcd)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(abc^2x-2\,a^2d^2)\sqrt{\frac{ax+b}{x}}}{2\,a^2b},\frac{(b^2c^2-4\,abcd)\sqrt{-a}\,\mathrm{arc}}{}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b
)/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^
2 - 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x
- 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

3.246.6 Sympy [A] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = -2cd\left(\begin{cases} \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0 \\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise} \end{cases}\right) + d^2\left(\begin{cases} -\frac{1}{\sqrt{a}x} & \text{for } b=0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

output
```
-2*c*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-lo
g(x)/sqrt(a), True)) + d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt
(a + b/x)/b, True)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - b*c**2*asin
h(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.246.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}\,c^2\left(\frac{2\,\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\,\sqrt{a+\frac{b}{x}}d^2}{b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

3.246.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.246.9 Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{c^2\,x\,\sqrt{a+\frac{b}{x}}}{a} - \frac{2\,d^2\,\sqrt{a+\frac{b}{x}}}{b} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-b\,c)}{a^{3/2}}$$

input
```
int((c + d/x)^2/(a + b/x)^(1/2),x)
```

output
```
(c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)
^(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

3.247 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

3.247.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+c*x*(a+b/x)^(1/2)/a
```

3.247.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} + \frac{(-bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*x)/a + ((-(b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2)
```

3.247.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {899, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{3/2}}$$

input
```
Int[(c + d/x)/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(
3/2)
```

3.247.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.247.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.65

method	result
risch	$\frac{c(ax+b)}{a\sqrt{\frac{ax+b}{x}}}+\frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{3}{2}}\sqrt{x(ax+b)}\,d-2\sqrt{a}\,\sqrt{x(ax+b)}\,bc-2a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,d-\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd-\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\right.}{2\sqrt{x(ax+b)}\,b\,a^{\frac{3}{2}}}$

input
```
int((c+d/x)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2*(2*a*d-b*c)/a^(3/2)*ln((1/2*b+a*x)/a^(
1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.247. $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx$

3.247.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{acx\sqrt{\frac{ax+b}{x}} + (bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\ldots}{\ldots}\right.}{a^2}\right.$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

3.247.6 Sympy [A] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -d\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + \frac{\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)/(a+b/x)**(1/2),x)
```

output
```
-d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(x)
/sqrt(a), True)) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - b*c*asinh(sqrt(a)
*sqrt(x)/sqrt(b))/a**(3/2)
```

3.247.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}\,c\left(\frac{2\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))
/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{(bc\log(|b|)-2\,ad\log(|b|))\text{sgn}(x)}{2\,a^{\frac{3}{2}}}+\frac{\sqrt{ax^2+bx}c}{a\text{sgn}(x)}+\frac{(bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\text{sgn}(x)}$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*c*log(abs(b)) - 2*a*d*log(abs(b)))*sgn(x)/a^(3/2) + sqrt(a*x^2 + b
*x)*c/(a*sgn(x)) + 1/2*(b*c - 2*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a) + b))/(a^(3/2)*sgn(x))
```

3.247.9 Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\,c\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

input
```
int((c + d/x)/(a + b/x)^(1/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x
)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3
/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

3.248 $\int \frac{1}{\sqrt{a+\frac{b}{x}}} \, dx$

3.248.1 Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{\sqrt{a+\frac{b}{x}}x}{a} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+x*(a+b/x)^(1/2)/a
```

3.248.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{\sqrt{a+\frac{b}{x}}x}{a} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

3.248.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[1/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

3.248.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.248.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2\sqrt{x(ax+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\, a^{\frac{3}{2}}}$	71
risch	$\frac{ax+b}{a\sqrt{\frac{ax+b}{x}}}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}\, x\sqrt{\frac{ax+b}{x}}}$	75

input
```
int(1/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(3/2)
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a))/a^2]
```

3.248.6 Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3
/2)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}b}{(a+\frac{b}{x})a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/
(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=-\frac{b\log(|b|)\,\mathrm{sgn}(x)}{2\,a^{\frac{3}{2}}}+\frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\,(x)}+\frac{\sqrt{ax^2+bx}}{a\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) + b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

3.248.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\,x\left(\frac{3\sqrt{b}\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\sqrt{\frac{a\,x}{b}+1}}{3\sqrt{a+\frac{b}{x}}}$$

input
```
int(1/(a + b/x)^(1/2),x)
```

output
```
(2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)
*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(
1/2))
```

3.248. $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

3.249 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

3.249.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\frac{\sqrt{a+\frac{b}{x}}x}{ac}-\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

output
```
-(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^2-2*d^(3/2)*arctan(d
^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(1/2)+x*(a+b/x)^(1/2
)/a/c
```

3.249.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x}{a}-\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^2}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

output
```
((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/a^(3/2))/c^2
```

3.249.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {899, 114, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(bc+2ad+\frac{bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bc+2ad+\frac{bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{(2ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2ad^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(2ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4ad^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2(2ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc)}{\sqrt{a}c}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c) + ((-4*a*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b/
x]/Sqrt[a]])/(Sqrt[a]*c))/(2*a*c)
```

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.249. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.249.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(90) = 180$.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.11

method	result
default	$-\frac{\left(2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,acd+\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,bc^2-2\sqrt{x(ax+b)}\,c^2\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^2}}+2a^{\frac{3}{2}}\ln\right.}{2a^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^3\sqrt{x(ax+b)}}$
risch	$\frac{ax+b}{ca\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2a\,d^2\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(a}{}}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2cax\sqrt{\frac{ax+b}{x}}}$

input
```
int(1/(c+d/x)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*a*c*d+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((
a*d-b*c)*d/c^2)^(1/2)*b*c^2-2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)+2*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*d^2)*x*((a*x+b)/x)^(1/2)/a^(3/2)/((a*d-b*c)*d/c^2)
^(1/2)/c^3/(x*(a*x+b))^(1/2)
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$
$$=\left[\frac{2\,a^2d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,acx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\right.}{2\,a^2c^2}\right.$$
$$-\frac{4\,a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-2\,acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}\right.}{2\,a^2c^2}$$
$$\left.-\frac{2\,a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2c^2}\right]$$

input
```
integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((
a*x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqr
t(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) +
a*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x
 + b)/x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt(
(a*x + b)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b
)/x) + b))/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*
sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c
^2)]
```

3.249.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

input
```
integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

3.249.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

3.249.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.249.9 Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 1183, normalized size of antiderivative = 10.95

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \frac{x\sqrt{a+\frac{b}{x}}}{a\,c} - \frac{\operatorname{atanh}\left(\frac{12\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\left(\frac{12\,b^4\,d^4}{a}+\frac{10\,b^5\,c\,d^3}{a^2}+\frac{2\,b^6\,c^2\,d^2}{a^3}\right)}+\frac{10\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\left(\frac{10\,b^5\,d^3}{a}+\frac{12\,b^4\,d^4}{c}+\frac{2\,b^6\,c\,d^2}{a^2}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\left(\frac{2\,b^6\,d^2}{a}+\frac{10\,b^5\,d^3}{c}+\frac{12\,a\,b^4\,d^4}{c^2}\right)}\right)\left(2\,a\,d+\right.}{c^2\,\sqrt{a^3}} - \frac{\operatorname{atan}\left(\frac{\left(\frac{\left(\frac{2\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\left(b\,c^3-a\,c^2\,d\right)}\right)\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\sqrt{a+\frac{b}{x}}\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4\right.}{a^2\,c^2}\right.}{b\,c^3-a\,c^2\,d}}{\left(\frac{\left(\frac{2\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\left(b\,c^3-a\,c^2\,d\right)}\right)\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\sqrt{a+\frac{b}{x}}\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right.}{a^2\,c^2}\right.}{b\,c^3-a\,c^2\,d}}\right)$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)),x)
```

output
```
(x*(a + b/x)^(1/2))/(a*c) - (atan(((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4
*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1
/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3
)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d
^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2
*d) - (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*
b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/
(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) +
(2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2
))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2
 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d
^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*
(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2
*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(
b*c^3 - a*c^2*d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b
^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3
*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*
(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3
)^(1/2))/(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2...
```

3.249. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

3.250 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

3.250.1 Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}$$

output
```
-d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3
/(-a*d+b*c)^(3/2)-(4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^3+d
*(-2*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/a/c/(
c+d/x)
```

3.250.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-bc(d+cx)+ad(2d+cx))}{a(-bc+ad)(d+cx)}+\frac{d^{3/2}(-5bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^3}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c*(d + c*x)) + a*d*(2*d + c*x)))/(a*(-(b*c) + a*d
)*(d + c*x)) + (d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]])/a^(3/2))/c^3
```

3.250.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {899, 114, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(bc+4ad+\frac{3bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(bc+4ad+\frac{3bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{bd(bc-2ad)}{x}+(bc-ad)(bc+4ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(\frac{bd(bc-2ad)}{x}+(bc-ad)(bc+4ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{(bc-ad)(4ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{ad^2(5bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^2(5bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)(4ad+bc)}{\sqrt{ac}}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

3.250. $\displaystyle\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}dx$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c*(c + d/x)) + ((2*d*(b*c - 2*a*d)*Sqrt[a + b/x])/(c*
(b*c - a*d)*(c + d/x)) + ((-2*a*d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(b*c +
 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(c*(b*c - a*d)))/(2*a
*c)
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.250. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(152) = 304$.

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.73

method	result
risch	$\frac{ax+b}{a c^2 \sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(4ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)}{c\sqrt{a}} + \frac{2ad^3\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad}{}}{}\right.}{}\right.}{c^3}\right.}{}$
default	Expression too large to display

input
```
int(1/(c+d/x)^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a*((4*a*d+b*c)/c*ln((1/2*b+a*x)/
a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+2*a*d^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)
*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)
*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c
*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d
-b*c)*d/c^2)^(1/2))/(x+d/c)))+6*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a
*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2
-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^(1/
2)*(x*(a*x+b))^(1/2)
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1163, normalized size of antiderivative = 6.76

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^
2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^
2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*
d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (
b*c - 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2
*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5
- a^3*c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a
^3*c*d^2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))
*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 +
 (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*
sqrt((a*x + b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2
*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^
3*c^4*d)*x), 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*
b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c
 - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*
d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b
*c^5 - a^3*c^4*d)*x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^
3*c*d^2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d...
```

3.250.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\,(cx+d)^2}\,dx$$

input
```
integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b/x)*(c*x + d)**2), x)
```

3.250.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(152) = 304.

Time = 0.32 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$
$$= \frac{\left(10\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-8\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-\sqrt{bcd-ad^2}b^2c^2\log\left(|b|\right)-3\sqrt{bcd-ad^2}abcd\,\mathrm{l}\right.}{2\left(\sqrt{bcd-ad^2}a^{\frac{3}{2}}bc^4-\sqrt{bcd-ad^2}a^{\frac{5}{2}}c^3d\right)}$$
$$+\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\sqrt{bcd-ad^2}}$$
$$+\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd^2-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^3-\sqrt{a}bd^3}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d+bd\right)}$$
$$+\frac{\sqrt{ax^2+bx}}{ac^2\mathrm{sgn}\left(x\right)}+\frac{\left(bc+4\,ad\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
1/2*(10*a^(3/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(5/2)*
d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b^2*c^2*lo
g(abs(b)) - 3*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) + 4*sqrt(b*c*d - a*d
^2)*a^2*d^2*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a^2*d^2)*sgn(x)/(sqrt(b*c*
d - a*d^2)*a^(3/2)*b*c^4 - sqrt(b*c*d - a*d^2)*a^(5/2)*c^3*d) + (5*b*c*d^2
 - 4*a*d^3)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b
*c*d - a*d^2))/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*sqrt(b*c*d - a*d^2)) + ((s
qrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d^2 - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*
a*d^3 - sqrt(a)*b*d^3)/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*((sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)) +
 sqrt(a*x^2 + b*x)/(a*c^2*sgn(x)) + 1/2*(b*c + 4*a*d)*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(3/2)*c^3*sgn(x))
```

3.250.9 Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 3813, normalized size of antiderivative = 22.17

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)
```

output
```
(((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b
*c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a +
b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4
*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^
5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d +
 b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5
*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d
)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/
2)) + (((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4
 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
a^3*b*c^5*d) - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4
 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a +
b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*
b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*
d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1
i)/(2*c^3*(a^3)^(1/2)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*
d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((
2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*...
```

3.250. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

3.251 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3} \, dx$

3.251.1 Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3} \, dx = \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} - \frac{(bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}$$

output
```
-1/4*d^(3/2)*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/
2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(5/2)-(6*a*d+b*c)*arctanh((a+b/x)^(1/2
)/a^(1/2))/a^(3/2)/c^4+1/2*d*(-3*a*d+2*b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)
/(c+d/x)^2+1/4*d*(-4*a*d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^(1/2)/a/c^3/(-a*d+b*c
)^2/(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d/x)^2
```

3.251.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$
$$=\frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(4b^2c^2(d+cx)^2+2a^2d^2\left(6d^2+9cdx+2c^2x^2\right)-abcd\left(19d^2+29cdx+8c^2x^2\right)\right)}{a(bc-ad)^2(d+cx)^2}-\frac{d^{3/2}\left(35b^2c^2-56abcd+24a^2d^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}-4(\ldots}{4c^4}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x +
2*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/(a*(b*c - a*d)^2*(d
 + c*x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[
d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(4*c^4)
```

3.251.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 114, 27, 168, 25, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$
$$\Big\downarrow \; 899$$
$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,d\frac{1}{x}$$
$$\Big\downarrow \; 114$$
$$\frac{\int \frac{\left(bc+6ad+\frac{5bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(bc+6ad+\frac{5bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{3bd(2bc-3ad)}{x}+2(bc-ad)(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(\frac{3bd(2bc-3ad)}{x}+2(bc-ad)(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(4(bc+6ad)(bc-ad)^2+\frac{bd(bc-4ad)(4bc-3ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{\left(4(bc+6ad)(bc-ad)^2+\frac{bd(bc-4ad)(4bc-3ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{\frac{\frac{4(bc-ad)^2(6ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{ad^2\left(24a^2d^2-56abcd+35b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

3.251. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}dx$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{\frac{8(bc-ad)^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^2\left(24a^2d^2-56abcd+35b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{8(bc-ad)^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{-\frac{2ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)(bc-ad)^2}{\sqrt{a}c}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) + ((d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(
c*(b*c - a*d)*(c + d/x)^2) + ((d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/
x])/(c*(b*c - a*d)*(c + d/x)) + ((-2*a*d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d +
24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c -
 a*d]) - (8*(b*c - a*d)^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(S
qrt[a]*c))/(2*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*a*c)
```

3.251. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.251. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.251.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(222) = 444$.

Time = 0.33 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.81

method	result
risch	$\frac{ax+b}{a\,c^3\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(6ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{8a\,d^3\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right)}{c^3}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int(1/(c+d/x)^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^3*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^3/a*((6*a*d+b*c)/c*ln((1/2*b+a*x)/
a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+8*a*d^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)
*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)
*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c
*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d
-b*c)*d/c^2)^(1/2))/(x+d/c)))-2*a*d^4/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c)^2*
(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*
c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/
c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^
(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2
)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2
*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*
c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+
(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+12*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln(
(2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d
/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x
)^(1/2)*(x*(a*x+b))^(1/2)
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(222) = 444$.

Time = 0.60 (sec) , antiderivative size = 2307, normalized size of antiderivative = 9.23

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3
*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*
d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x
 - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d
^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x
^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(
b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) -
 b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3
*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 +
(4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/
(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*
c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^
3)*x), 1/8*(8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5
+ (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^
3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*ar
ctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4
+ 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2
+ 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c
 - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3...
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(222) = 444.

Time = 0.38 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$
$$= \frac{\left(35\,a^{\frac{3}{2}}b^2c^2d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 56\,a^{\frac{5}{2}}bcd^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 24\,a^{\frac{7}{2}}d^4\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 2\sqrt{bcd-a}\right.}{4\left(\sqrt{b}\right.}$$
$$+ \frac{(35\,b^2c^2d^2 - 56\,abcd^3 + 24\,a^2d^4)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\left(b^2c^6\mathrm{sgn}\,(x) - 2\,abc^5d\mathrm{sgn}\,(x) + a^2c^4d^2\mathrm{sgn}\,(x)\right)\sqrt{bcd-ad^2}}$$
$$+ \frac{13\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3d^2 - 40\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d^3 + 24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^2cd^4 + 7\,(\cdots}{}$$
$$+ \frac{\sqrt{ax^2+bx}}{ac^3\mathrm{sgn}\,(x)} + \frac{(bc+6\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^4\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(35*a^(3/2)*b^2*c^2*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 56*a^(
5/2)*b*c*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(7/2)*d^4*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^3*c^3*log(abs(b)
) - 8*sqrt(b*c*d - a*d^2)*a*b^2*c^2*d*log(abs(b)) + 22*sqrt(b*c*d - a*d^2)
*a^2*b*c*d^2*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^3*log(abs(b)) + 13
*sqrt(b*c*d - a*d^2)*a^2*b*c*d^2 - 10*sqrt(b*c*d - a*d^2)*a^3*d^3)*sgn(x)/
(sqrt(b*c*d - a*d^2)*a^(3/2)*b^2*c^6 - 2*sqrt(b*c*d - a*d^2)*a^(5/2)*b*c^5
*d + sqrt(b*c*d - a*d^2)*a^(7/2)*c^4*d^2) + 1/4*(35*b^2*c^2*d^2 - 56*a*b*c
*d^3 + 24*a^2*d^4)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)
/sqrt(b*c*d - a*d^2))/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*
sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(13*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b
^2*c^3*d^2 - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^3 + 24*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^4 + 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2
*sqrt(a)*b^2*c^2*d^3 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^
4 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^5 + 11*(sqrt(a)*x - sqr
t(a*x^2 + b*x))*b^3*c^2*d^3 - 60*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d
^4 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^5 - 13*sqrt(a)*b^3*c*d^4 +
 10*a^(3/2)*b^2*d^5)/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*s
gn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 +
b*x))*sqrt(a)*d + b*d)^2) + sqrt(a*x^2 + b*x)/(a*c^3*sgn(x)) + 1/2*(b*c...
```

3.251.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 2890, normalized size of antiderivative = 11.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
```

output
```
(log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a
*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b
^2*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*
b^3*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(1
2*a^2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a +
 b/x)^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d)
)/(4*a*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 5
6*a^2*b^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((
a + b/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d) - d^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d
 - b*c)^5)^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*
b*c*d))/(8*(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 -
 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^
6*b^2*d^9 + 16*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129
*a^2*b^6*c^4*d^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b
^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*
d^2)) - (((4*a*b^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 7
4*a^4*b^5*c^10*d^5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^1
3 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*...
```

3.251. $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

3.252 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.252.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3*c^2*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+((-2*a*d+b*c)*(
2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)-a*b*d^2*(2*a*d+b*c)/x)/a^2/b^2/(a+b/x)^(1/2
)+c*(c+d/x)^2*x/a/(a+b/x)^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\left(3b^3c^3x-4a^3d^3x-2a^2bd^2(d-3cx)+ab^2c^2x(-6d+cx)\right)}{a^2b^2(b+ax)} + \frac{3c^2(-bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*(3*b^3*c^3*x - 4*a^3*d^3*x - 2*a^2*b*d^2*(d - 3*c*x) + a*b^
2*c^2*x*(-6*d + c*x)))/(a^2*b^2*(b + a*x)) + (3*c^2*(-(b*c) + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

3.252.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {899, 109, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^3 x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(3c(bc-2ad)-\frac{d(bc+2ad)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(3c(bc-2ad)-\frac{d(bc+2ad)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 163}$$

$$\frac{\frac{3c^2(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{6c^2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}-\frac{6c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

output
```
(c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) + ((2*((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*
b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d))/x))/(a*b^2*Sqrt[a + b/x]) - (
6*c^2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(2*a)
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

3.252. $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.252.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{(ax+b)\left(-b^{2}x\,c^{3}+2a^{2}d^{3}\right)}{b^{2}a^{2}x\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b^{2}c^{3}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{\sqrt{a}}+6\sqrt{a}\,b\,c^{2}d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)+\frac{2\left(-2a^{3}d^{3}+6a^{2}bc\,d^{2}-6a\,b^{2}c^{2}d+2\right.}{ab\left(x+\right.}\right.}{2a^{2}bx\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b^{4}c^{3}x^{4}+6a^{\frac{9}{2}}\sqrt{x(ax+b)}\,bc\,d^{2}x^{4}-12a^{\frac{7}{2}}\sqrt{x(ax+b)}\,b^{2}c^{2}d\,x^{4}-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right.}{}$

input
```
int((c+d/x)^3/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+b)*(-b^2*c^3*x+2*a^2*d^3)/b^2/a^2/x/((a*x+b)/x)^(1/2)+1/2/a^2/b*(-3*
b^2*c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+6*a^(1/2)*b*c^2*
d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+2*(-2*a^3*d^3+6*a^2*b*c*d^2-6*
a*b^2*c^2*d+2*b^3*c^3)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+
b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.252. $\int \frac{\left(c+\frac{d}{x}\right)^{3}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.252.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.55

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{3\,(b^4c^3-2\,ab^3c^2d+(ab^3c^3-2\,a^2b^2c^2d)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2b^2}{2\,(a^4b^2x+a^3b^3)}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(
a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2
*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x
)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d +
(a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) + (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6
*a^3*b*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

3.252.6 Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

output
```
Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.52

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}\,c^3\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - 3\,c^2d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right) - 2\,d^3\left(\frac{\sqrt{a+\frac{b}{x}}}{b^2}+\frac{a}{\sqrt{a+\frac{b}{x}}b^2}\right)+\frac{6\,cd^2}{\sqrt{a+\frac{b}{x}}b}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^
3) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2))
 - 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/
2) + 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^
2)) + 6*c*d^2/(sqrt(a + b/x)*b)
```

3.252.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.252.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}-a\,b^2\,\sqrt{a+\frac{b}{x}}}$$

$$-\frac{2\,d^3\,\sqrt{a+\frac{b}{x}}}{b^2}+\frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(2\,a\,d-b\,c\right)}{a^{5/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(3/2),x)
```

output
```
((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2
*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)
^(3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*ata
nh((a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

3.253 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.253.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-c*(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(2*a^2*d^2+b*c*(-
4*a*d+3*b*c))/a^2/b/(a+b/x)^(1/2)+c^2*x/a/(a+b/x)^(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(3b^2c^2+2a^2d^2+abc(-4d+cx))}{a^2b(b+ax)}+\frac{c(-3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x)))/(a^2*b*(b +
 a*x)) + (c*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

3.253.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2 x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{c^2 x}{a\sqrt{a+\frac{b}{x}}}-\frac{\int -\frac{\left(c(3bc-4ad)-\frac{2ad^2}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(c(3bc-4ad)-\frac{2ad^2}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{c^2 x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{c(3bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}}{2a}+\frac{c^2 x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2c(3bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)}{a^{3/2}}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

output
```
(c^2*x)/(a*Sqrt[a + b/x]) + ((2*((2*a*d^2)/b + (c*(3*b*c - 4*a*d))/a))/Sqr
t[a + b/x] - (2*c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))
/(2*a)
```

3.253.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.253.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(84) = 168$.

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.94

method	result
risch	$\frac{c^2(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b\,c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{2\left(2a^2d^2-4abcd+2b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}\right)\sqrt{x(a}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-4\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^4cd-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b^2c^2x^2+16\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b^2cdx-8(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}bcd-8\ln\left(\frac{2\sqrt{x}}{}\right.\right.}{}$

input
```
int((c+d/x)^2/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(-3*b*c^2*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))+2*(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+
b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.89

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^3c^2 - 4\,ab^2cd + (3\,ab^2c^2 - 4\,a^2bcd)x)\sqrt{a}\log\left(2\,ax + 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2\,(a^2bc^2}{2\,(a^4bx + a^3b^2)}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*l
og(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^
2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)
, ((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arct
an(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b
*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

3.253.6 Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

output
```
Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}\,c^2\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - 2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)+\frac{2\,d^2}{\sqrt{a+\frac{b}{x}}b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^
3) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2))
 - 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

3.253.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(84) = 168.

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^2\mathrm{sgn}\left(x\right)} + \frac{\left(3\,bc^2-4\,acd\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} - \frac{\left(3\,b^2c^2\log\left(|b|\right)-4\,abcd\log\left(|b|\right)+4\,b^2c^2-8\,abcd+4\,a^2d^2\right)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}b} + \frac{2\left(\sqrt{a}b^2c^2-2\,a^{\frac{3}{2}}bcd+a^{\frac{5}{2}}d^2\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^2/(a^2*sgn(x)) + 1/2*(3*b*c^2 - 4*a*c*d)*log(abs(2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) - 1/2*(3*b^2*c
^2*log(abs(b)) - 4*a*b*c*d*log(abs(b)) + 4*b^2*c^2 - 8*a*b*c*d + 4*a^2*d^2
)*sgn(x)/(a^(5/2)*b) + 2*(sqrt(a)*b^2*c^2 - 2*a^(3/2)*b*c*d + a^(5/2)*d^2)
/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x))
```

3.253.9 Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-3\,b\,c)}{a^{5/2}}-\frac{\frac{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2\right)}{a^2}}{b\,\left(a+\frac{b}{x}\right)^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

input
```
int((c + d/x)^2/(a + b/x)^(3/2),x)
```

output
```
(c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d)
)/a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

3.254 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.254.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(-2*a*d+3*b*c)/a^2/
(a+b/x)^(1/2)+c*x/a/(a+b/x)^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(3bc-2ad+acx)}{a^2(b+ax)}+\frac{(-3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*b*c + 2*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

3.254.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {899, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{(3bc-2ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(3bc-2ad)\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)(3bc-2ad)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)/(a + b/x)^(3/2),x]
```

3.254. $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}dx$

output
```
(c*x)/(a*Sqrt[a + b/x]) + ((3*b*c - 2*a*d)*(2/(a*Sqrt[a + b/x]) - (2*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/(2*a)
```

3.254.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.254. $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.254.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.09

method	result
risch	$\frac{c(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{3bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}-\frac{4(ad-bc)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4a^{\frac{7}{2}}\sqrt{x(ax+b)}\,d\,x^2-6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bc\,x^2-4a^{\frac{5}{2}}(x(ax+b))^{\frac{3}{2}}d+8a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bdx+4a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}bc-12a^{\frac{3}{2}}\sqrt{x(ax+\right.}}{}$

input
```
int((c+d/x)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1/2
)+(a*x^2+b*x)^(1/2))-3*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/
2)-4*(a*d-b*c)/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2
)*(x*(a*x+b))^(1/2)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.76

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^2c-2\,abd+(3\,abc-2\,a^2d)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2cx^2+(3\,a\right.}{2\,(a^4x+a^3b)}$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sq
rt((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c
 - 2*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

3.254.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(65) = 130$.

Time = 14.55 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right) + d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^2b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

input
```
integrate((c+d/x)/(a+b/x)**(3/2),x)
```

output
```
c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x
/b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqr
t(1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x
 + a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2
)*b) - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1
 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

3.254.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}c\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3)
 + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(
sqrt(a + b/x)*a))
```

3.254.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(66) = 132$.

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.96

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,bc\log\left(|b|\right)-2\,ad\log\left(|b|\right)+4\,bc-4\,ad)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{\sqrt{ax^2+bx}c}{a^2\mathrm{sgn}\left(x\right)} + \frac{(3\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} + \frac{2\left(\sqrt{a}b^2c-a^{\frac{3}{2}}bd\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*c*log(abs(b)) - 2*a*d*log(abs(b)) + 4*b*c - 4*a*d)*sgn(x)/a^(5/2
) + sqrt(a*x^2 + b*x)*c/(a^2*sgn(x)) + 1/2*(3*b*c - 2*a*d)*log(abs(2*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + 2*(sqrt(a)*b^2
*c - a^(3/2)*b*d)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x
))
```

3.254.9 Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}} + \frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int((c + d/x)/(a + b/x)^(3/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
 (2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a +
b/x)^(3/2))
```

3.254. $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

3.255.1 Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx = \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3*b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b
/x)^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx = \frac{\sqrt{a+\frac{b}{x}}x(3b+ax)}{a^2(b+ax)} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(a + b/x)^(-3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(5/2)
```

3.255.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3b\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(a + b/x)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b/x]) + (3*b*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(3/2)))/(2*a)
```

3.255.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.255. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} \, dx$

3.255.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

method	result
risch	$\frac{ax+b}{a^{2}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{5}{2}}}+\frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{a^{3}\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,x^{2}-4a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}+12a^{\frac{3}{2}}\sqrt{x(ax+b)}\,bx-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}bx^{2}-6\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+\right.}{2\sqrt{a}}\right.}{2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,(ax+b)^{2}}$

input
```
int(1/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/2*b/a^(5/2)*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))+2*b/a^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{3\,(abx+b^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(a^2x^2+3\,abx)\sqrt{\frac{ax+b}{x}}}{2\,(a^4x+a^3b)}, \frac{3\,(abx+b^2)\sqrt{}}{}\right.$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
b) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x +
 b^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*
sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

3.255.6 Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)**(3/2),x)
```

output
```
x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b
+ 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*
log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

3.255.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=&-\frac{(3\,b\log(|b|)+4\,b)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}}\\&+\frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}+\frac{\sqrt{ax^{2}+bx}}{a^{2}\mathrm{sgn}(x)}\\&+\frac{2\,b^{2}}{\left(\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right)a^{\frac{5}{2}}\mathrm{sgn}(x)}\end{aligned}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*log(abs(b)) + 4*b)*sgn(x)/a^(5/2) + 3/2*b*log(abs(2*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a^
2*sgn(x)) + 2*b^2/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^(5/2)*s
gn(x))
```

3.255.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(a + b/x)^(3/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x
)^(3/2))
```

3.256 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

3.256.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

output
```
2*d^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(3
/2)-(2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^2+b*(-a*d+3*b*c
)/a^2/c/(-a*d+b*c)/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-3b^2c+a^2dx+ab(d-cx))}{a^2(-bc+ad)(b+ax)}+\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{c^2}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/(a^2*(-(b*c) + a
*d)*(b + a*x)) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]
])/(b*c - a*d)^(3/2) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^
(5/2))/c^2
```

3.256.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 114, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(3bc+2ad+\frac{3bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3bc+2ad+\frac{3bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\frac{2\int \frac{\left(\frac{bd(3bc-ad)}{x}+(bc-ad)(3bc+2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(\frac{bd(3bc-ad)}{x}+(bc-ad)(3bc+2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow 174$$

$$\frac{\frac{\frac{2a^2d^3\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{(bc-ad)(2ad+3bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow 73$$

$$\frac{\frac{\frac{4a^2d^3\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow 218$$

$$\frac{\frac{\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)(2ad+3bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

output
```
x/(a*c*Sqrt[a + b/x]) + ((2*b*(3*b*c - a*d))/(a*(b*c - a*d)*Sqrt[a + b/x])
 + ((4*a^2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqr
t[b*c - a*d]) - (2*(b*c - a*d)*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(2*a*c)
```

3.256. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}dx$

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.256. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(127) = 254$.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.02

method	result
risch	$\frac{ax+b}{a^2c\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(2ad+3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{4c\,b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{(ad-bc)a\left(x+\frac{b}{a}\right)} + \frac{2a^2d^3\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{\ldots}}{x+\frac{d}{c}}\right)}{c^2(ad-bc)\sqrt{\frac{(ad-b\ldots}{c^2}}}\right.}{2a^2cx\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\left(2a^{\frac{9}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^3x^2-2a^{\frac{7}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c^2dx^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2ad\ldots}{cx+d}\right.\right.}{}$

input
```
int(1/(a+b/x)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^2/c*((2*a*d+3*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+4*c*b^2/(a*d-b*c)/a/(x+b/a)*(a*(x+b/a
)^2-b*(x+b/a))^(1/2)+2/c^2*a^2*d^3/(a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln((2
*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c
)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^
(1/2)*(x*(a*x+b))^(1/2)
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(127) = 254$.

Time = 0.35 (sec) , antiderivative size = 1075, normalized size of antiderivative = 7.31

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \left[\frac{(3\,b^3c^2 - ab^2cd - 2\,a^2bd^2 + (3\,ab^2c^2 - a^2bcd - 2\,a^3d^2)x)\sqrt{a}\log\left(2\,ax - 2\,\sqrt{a}x\right.}{}\right.$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^
4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b
*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2
*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a
^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2
*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arc
tan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d
+ (b*c - 2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2
- a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3
 - a^5*c^2*d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arcta
n(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3
*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)
*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2
 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2
*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2
)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x
+ b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c
^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/...
```

3.256.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)}\,dx$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

output
```
Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

3.256.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

3.256.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.256.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 3000, normalized size of antiderivative = 20.41

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)),x)
```

output
```
(atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 -
66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5
*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10
) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2
- 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a
^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c
^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10
*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/
(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5
*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c
^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2
*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a
*d - b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b
^7*c^10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*
d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56
*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a
^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d -
b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^
5 - 96*a^7*b^7*c^6*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10
*b^4*c^3*d^9 + 8*a^11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b...
```

3.257 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

3.257.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}-\frac{(3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}$$

output
```
d^(5/2)*(-4*a*d+7*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/
(-a*d+b*c)^(5/2)-(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^3+
b*(2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(1/2)+d*(-2
*a*d+b*c)/a/c^2/(-a*d+b*c)/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)/(a+b/x)^(1/
2)
```

3.257.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(3b^3c^2(d+cx)+a^3d^2x(2d+cx)+a^2bd(2d^2-cdx-2c^2x^2)+ab^2c(-2d^2-cdx+c^2x^2))}{a^2(bc-ad)^2(b+ax)(d+cx)}+\frac{d^{5/2}(7bc-4c}{}}{c^3}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(3*b^3*c^2*(d + c*x) + a^3*d^2*x*(2*d + c*x) + a^2*b*d
*(2*d^2 - c*d*x - 2*c^2*x^2) + a*b^2*c*(-2*d^2 - c*d*x + c^2*x^2)))/(a^2*(
b*c - a*d)^2*(b + a*x)*(d + c*x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[
d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - ((3*b*c + 4*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/c^3
```

3.257.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 114, 27, 168, 25, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \; 899$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \; 114$$

$$\frac{\int \frac{\left(3bc+4ad+\frac{5bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(3bc+4ad+\frac{5bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 168

$$\frac{\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{3bd(bc-2ad)}{x}+(bc-ad)(3bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\left(\frac{3bd(bc-2ad)}{x}+(bc-ad)(3bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 169

$$\frac{\frac{\frac{2\int \frac{\left((3bc+4ad)(bc-ad)^2+\frac{bd\left(3b^2c^2-2abdc+2a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\left((3bc+4ad)(bc-ad)^2+\frac{bd\left(3b^2c^2-2abdc+2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 174

3.257. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

$$\frac{\frac{\frac{\frac{a^2d^3(7bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{(bc-ad)^2(4ad+3bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{2a^2d^3(7bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^2(4ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(4ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a^2d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{2a^2d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^2(4ad+3bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

input
```
Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

3.257. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

output
```
x/(a*c*Sqrt[a + b/x]*(c + d/x)) + ((2*d*(b*c - 2*a*d))/(c*(b*c - a*d)*Sqrt
[a + b/x]*(c + d/x)) + ((2*b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a*(b*c
- a*d)*Sqrt[a + b/x]) + ((2*a^2*d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)^2*(3*b
*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/
(c*(b*c - a*d)))/(2*a*c)
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.257. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.257.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(202) = 404$.

Time = 0.38 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.38

method	result
risch	$\frac{ax+b}{a^2c^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^3}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^2}+\frac{2b^3\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3(ad-bc)^2\left(x+\frac{b}{a}\right)}+\frac{d^3\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+}}{c^3(ad-bc)^2\left(x+\frac{d}{c}\right)}\right.}{}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-2/a^(3/2)/c^3*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b+2/a^3*b^3/(a*d-b*c)^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+1/c^3*
d^3/(a*d-b*c)^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2
)^(1/2)-2*a/c^4*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/
c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*
c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+7/2/c^3*d^3/(a*d-b*c)^2/((a*
d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*
c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))
/(x+d/c))*b)/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(202) = 404$.

Time = 0.62 (sec) , antiderivative size = 2321, normalized size of antiderivative = 10.36

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*
a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*
c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a
)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^
4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4
*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-
d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(
(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c
^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d
^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d
^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^
3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3
*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^
2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c
 - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^
3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a
^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3
*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2
*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d...
```

3.257.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)^2}\,dx$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2/((a + b/x)**(3/2)*(c*x + d)**2), x)
```

3.257.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

3.257.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.257.9 Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 4274, normalized size of antiderivative = 19.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
```

output
```
((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b
*c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3
*b^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/
x)^(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^
11*c^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i
- a^10*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*
1694i - a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x
)^(1/2)*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*
(a + b/x)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*
c^4*d^10*(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i +
a^11*b^13*c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^
7*c^7*d^7 - 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d
^10 + 140*a^4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 169
4*a^2*b^10*c^10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^
3*b^14*c^14 + 189*a^4*b^13*c^13*d))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2))
 - (atan((((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*
a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^
9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^
12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768
*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 6...
```

3.257. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

3.258 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

3.258.1 Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{3d^{5/2}\left(21b^2c^2-24abcd+8a^2d^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} - \frac{3(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}$$

output
```
3/4*d^(5/2)*(8*a^2*d^2-24*a*b*c*d+21*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)
/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(7/2)-3*(2*a*d+b*c)*arctanh((a+b/x)^(1/2
)/a^(1/2))/a^(5/2)/c^4+3/4*b*(-a*d+2*b*c)*(4*a^2*d^2-a*b*c*d+2*b^2*c^2)/a^
2/c^3/(-a*d+b*c)^3/(a+b/x)^(1/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(c+
d/x)^2/(a+b/x)^(1/2)+1/4*d*(12*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b
*c)^2/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)^2/(a+b/x)^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(-12b^4c^3(d+cx)^2-4ab^3c^2(-3d+cx)(d+cx)^2+2a^4d^3x\left(6d^2+9cdx+2c^2x^2\right)+a^3bd^2\left(12d^3-9cd^2x-37\right.\right.}{a^2(-bc+ad)^3(b+ax)(d+cx)^2}}{}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-12*b^4*c^3*(d + c*x)^2 - 4*a*b^3*c^2*(-3*d + c*x)*(d
 + c*x)^2 + 2*a^4*d^3*x*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + a^3*b*d^2*(12*d^3
- 9*c*d^2*x - 37*c^2*d*x^2 - 12*c^3*x^3) + a^2*b^2*c*d*(-27*d^3 - 29*c*d^2
*x + 12*c^2*d*x^2 + 12*c^3*x^3)))/(a^2*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x
)^2) + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (12*(b*c + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/(4*c^4)
```

3.258.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {899, 114, 27, 168, 25, 168, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(\frac{7bd}{x}+3(bc+2ad)\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(\frac{7bd}{x}+3(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{5bd(2bc-3ad)}{x}+6(bc-ad)(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(\frac{5bd(2bc-3ad)}{x}+6(bc-ad)(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{3\left(4(bc+2ad)(bc-ad)^2+\frac{bd\left(4b^2c^2-21abdc+12a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{\left(4(bc+2ad)(bc-ad)^2+\frac{bd\left(4b^2c^2-21abdc+12a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{169}$$

3.258. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{\frac{\frac{3\left(\frac{2\int \frac{\left(4(bc+2ad)(bc-ad)^3+\frac{bd(2bc-ad)\left(2b^2c^2-abdc+4a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3a}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\left(\frac{\int \frac{\left(4(bc+2ad)(bc-ad)^3+\frac{bd(2bc-ad)\left(2b^2c^2-abdc+4a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3a}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 174

$$\frac{\frac{\frac{3\left(\frac{\frac{a^2d^3\left(8a^2d^2-24abcd+21b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4(bc-ad)^3(2ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 73

3.258. $\displaystyle\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{\frac{3\left(\frac{\frac{2a^2d^3\left(8a^2d^2-24abcd+21b^2c^2\right)\int\frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8(bc-ad)^3(2ad+bc)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+\cdots\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc\cdots}}{2c(bc-ad)} \cdots$$

$$2ac$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\frac{8(2ad+bc)(bc-ad)^3\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a^2d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+\cdots\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc\cdots}}{2c(bc-ad)} \cdots$$

$$2ac$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(\frac{\frac{2a^2d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^3(2ad+bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21ab\cdots\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\cdots\right.}}{2c(bc-ad)} \cdots$$

$$2ac$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

input

```
Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

3.258. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

output
```
x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + ((d*(2*b*c - 3*a*d))/(c*(b*c - a*d)*Sq
rt[a + b/x]*(c + d/x)^2) + ((d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(c*(
b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + (3*((2*b*(2*b*c - a*d)*(2*b^2*c^2 -
a*b*c*d + 4*a^2*d^2))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((2*a^2*d^(5/2)*(21*
b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(c*Sqrt[b*c - a*d]) - (8*(b*c - a*d)^3*(b*c + 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d))))/(2*c*(b*c - a*d)))/(2*c
*(b*c - a*d)))/(2*a*c)
```

3.258.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.258. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.258.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. 2(288) = 576.

Time = 0.35 (sec) , antiderivative size = 983, normalized size of antiderivative = 3.07

method	result
risch	$\frac{ax+b}{a^2c^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^4}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^3}-\frac{2b^4\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3(ad-bc)^3\left(x+\frac{b}{a}\right)}-\frac{d^4\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+}}{2c^5(ad-bc)^2\left(x+\frac{d}{c}\right)^2}\right.}{}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(3/2)/c^4*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b-2/a^3*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^
5*d^4/(a*d-b*c)^2/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d
/c^2)^(1/2)+5/2*a/c^4*d^4/(a*d-b*c)^3/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(
x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-17/4/c^3*d^3/(a*d-b*c)^3/(x+d/c)*(a*(x+d/c)^
2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2/c^5*d^5/(a*d-b*c)
^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*(
(a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)
^(1/2))/(x+d/c))+19/2*a/c^4*d^4/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*
(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)
^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))*b-63/8/c^3*d^3/(
a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+
d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c
)*d/c^2)^(1/2))/(x+d/c))*b^2+1/2*a/c^5*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(
1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)
*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((
a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(288) = 576$.

Time = 1.45 (sec) , antiderivative size = 4093, normalized size of antiderivative = 12.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^
5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b
*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2
 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a
*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5
*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^
3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*
a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*
d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*
b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*
x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^
5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37
*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^
2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*
c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((
a*x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^
6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d
^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d
^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*...
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

3.258.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(288) = 576$.

Time = 0.40 (sec) , antiderivative size = 1089, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
2*sqrt(a)*b^5/((a^3*b^3*c^3*sgn(x) - 3*a^4*b^2*c^2*d*sgn(x) + 3*a^5*b*c*d^
2*sgn(x) - a^6*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))
- 1/4*(63*a^(5/2)*b^2*c^2*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 72*a
^(7/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(9/2)*d^5*arct
an(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*b^4*c^4*log(abs(
b)) - 6*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 18*sqrt(b*c*d - a*d^
2)*a^2*b^2*c^2*d^2*log(abs(b)) + 30*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3*log(ab
s(b)) - 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) + 8*sqrt(b*c*d - a*d^2)
*b^4*c^4 + 17*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3 - 10*sqrt(b*c*d - a*d^2)*a^4
*d^4)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(5/2)*b^3*c^7 - 3*sqrt(b*c*d - a*d^2)*
a^(7/2)*b^2*c^6*d + 3*sqrt(b*c*d - a*d^2)*a^(9/2)*b*c^5*d^2 - sqrt(b*c*d -
 a*d^2)*a^(11/2)*c^4*d^3) - 3/4*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5
)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b^3*c^7*sgn(x) - 3*a*b^2*c^6*d*sgn(x) + 3*a^2*b*c^5*d^2*sgn(x) - a^
3*c^4*d^3*sgn(x))*sqrt(b*c*d - a*d^2)) - 1/4*(17*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^3*b^2*c^3*d^3 - 48*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^4 +
24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^5 + 11*(sqrt(a)*x - sqrt(a*x^
2 + b*x))^2*sqrt(a)*b^2*c^2*d^4 - 72*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(
3/2)*b*c*d^5 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^6 + 15*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^4 - 76*(sqrt(a)*x - sqrt(a*x^2 + b...
```

3.258.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 8936, normalized size of antiderivative = 27.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)
```

output
```
((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*
b^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(
a*d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^
3 - 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)
^2*(24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72
*a^3*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(
3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d
^2*(a + b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
+ (atan(((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25
*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*
b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 -
65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a
^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^1
5*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 588
16512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*
c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^
2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^1
8*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 944
9472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13
*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 +...
```

3.259 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.259.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-2a^2bd(3d+5cx)+ab^2c(-3d+20cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
c*(c+d/x)^2*x/a/(a+b/x)^(3/2)+1/3*(-a*d+b*c)*(15*b^3*c^2-4*a^3*d^2*x-2*a^2
*b*d*(5*c*x+3*d)+a*b^2*c*(20*c*x-3*d))/a^3/b^2/(a+b/x)^(3/2)/x-c^2*(-6*a*d
+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,x(15b^4c^3+4a^4d^3x+3a^2b^2c^2x(-8d+cx)+6a^3bd^2(d+cx)+2ab^3c^2(-9d+10c}{3a^3b^2(b+ax)^2}$$
$$+\frac{c^2(-5bc+6ad)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^4*c^3 + 4*a^4*d^3*x + 3*a^2*b^2*c^2*x*(-8*d + c*x)
+ 6*a^3*b*d^2*(d + c*x) + 2*a*b^3*c^2*(-9*d + 10*c*x)))/(3*a^3*b^2*(b + a*
x)^2) + (c^2*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

3.259.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 109, 27, 162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^3 x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(5bc-6ad)+\frac{d(bc-2ad)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(5bc-6ad)+\frac{d(bc-2ad)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 162

$$\frac{\frac{c^2(5bc-6ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a^2}+\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{2c^2(5bc-6ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a^2b}+\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-6ad)}{a^{5/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

output
```
(c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((2*(2*a*(b*c - a*d)*(10*b^2*c^2 -
 5*a*b*c*d - 2*a^2*d^2) + (3*b*(b*c - a*d)*(5*b^2*c^2 - a*b*c*d - 2*a^2*d^
2))/x))/(3*a^2*b^2*(a + b/x)^(3/2)) - (2*c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/a^(5/2))/(2*a)
```

3.259. $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.259. $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.259.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(131) = 262$.

Time = 0.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.11

method	result
risch	$\frac{c^3(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5c^3 b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+6\sqrt{a}\,c^2 d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{12c\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}+\frac{\left(2a^3 d\right.}{}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	Expression too large to display

input
```
int((c+d/x)^3/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(-5*c^3*b*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))/a^(1/2)+6*a^(1/2)*c^2*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+
b*x)^(1/2))+12*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x
+b/a))^(1/2)+(2*a^3*d^3-6*a^2*b*c*d^2+6*a*b^2*c^2*d-2*b^3*c^3)/a^2*(2/3/b/
(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(
x+b/a))^(1/2)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.38

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\left(5\,b^5c^3-6\,ab^4c^2d+(5\,a^2b^3c^3-6\,a^3b^2c^2d)x^2+2\,(5\,ab^4c^3-6\,a^2b^3c^2d)x\right)\sqrt{a}\log\left(2\,a\right.}{}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^
2 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*s
qrt((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*
b^2*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^
2*d + 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*
b^4), 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d
)*x^2 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt
((a*x + b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2
*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2
*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

3.259.6 Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

output
```
Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{6}c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - c^2d\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2}{3}d^3\left(\frac{3}{\sqrt{a+\frac{b}{x}}b^2}-\frac{a}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}\right)+\frac{2cd^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2
)*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt
(a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2
/3*d^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b
/x)^(3/2)*b)
```

3.259.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(132) = 264$.

Time = 0.32 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.00

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{ax^2+bx}c^3}{a^3\mathrm{sgn}\left(x\right)} + \frac{\left(5\,bc^3-6\,ac^2d\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$
$$-\frac{\left(15\,b^3c^3\log\left(|b|\right)-18\,ab^2c^2d\log\left(|b|\right)+28\,b^3c^3-48\,ab^2c^2d+12\,a^2bcd^2+8\,a^3d^3\right)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b^2}$$
$$+\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^3-18\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bc^2d+9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3cd^2+15\left(\sqrt{a}\right.\right.}{}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^3/(a^3*sgn(x)) + 1/2*(5*b*c^3 - 6*a*c^2*d)*log(abs(2*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^
3*c^3*log(abs(b)) - 18*a*b^2*c^2*d*log(abs(b)) + 28*b^3*c^3 - 48*a*b^2*c^2
*d + 12*a^2*b*c*d^2 + 8*a^3*d^3)*sgn(x)/(a^(7/2)*b^2) + 2/3*(9*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^2*a*b^2*c^3 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^
2*b*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*c*d^2 + 15*(sqrt(a)*x
- sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^3 - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x))*
a^(3/2)*b^2*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*c*d^2 + 3*
(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3 + 7*b^4*c^3 - 12*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 + 2*a^3*b*d^3)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)
+ b)^3*a^(7/2)*sgn(x))
```

3.259.9 Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.36

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{3\,a}+\frac{\left(a+\frac{b}{x}\right)^2\,(2\,a^3\,d^3-6\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{a^3}-\frac{2\,\left(a+\frac{b}{x}\right)\,(4\,a^3\,d^3-3\,a^2\,b\,c\,d^2-6\,a\,b^2 ...}{3\,a^2}}{b^2\,\left(a+\frac{b}{x}\right)^{5/2}-a\,b^2\,\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(6\,a\,d-5\,b\,c)}{a^{7/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(5/2),x)
```

output
```
((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x
)^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3
 + 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/
2) - a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d -
 5*b*c))/a^(7/2)
```

3.260 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

3.260.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
1/3*(2*a^2*d^2+b*c*(-4*a*d+5*b*c))/a^2/b/(a+b/x)^(3/2)+c^2*x/a/(a+b/x)^(3/
2)-c*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+c*(-4*a*d+5*b*c
)/a^3/(a+b/x)^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^3c^2+2a^3d^2x+a^2bcx(-16d+3cx)+4ab^2c(-3d+5cx))}{3a^3b(b+ax)^2} + \frac{c(-5bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^3*c^2 + 2*a^3*d^2*x + a^2*b*c*x*(-16*d + 3*c*x) + 4
*a*b^2*c*(-3*d + 5*c*x)))/(3*a^3*b*(b + a*x)^2) + (c*(-5*b*c + 4*a*d)*ArcT
anh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

3.260.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2 x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{\int -\frac{\left(c(5bc-4ad)-\frac{2ad^2}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(c(5bc-4ad)-\frac{2ad^2}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{c(5bc-4ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{c(5bc-4ad)\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 73$$

$$\frac{\frac{c(5bc-4ad)\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{c\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)(5bc-4ad)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

output
```
(c^2*x)/(a*(a + b/x)^(3/2)) + ((2*((2*a*d^2)/b + (c*(5*b*c - 4*a*d))/a))/(
3*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a*d)*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[
Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/a)/(2*a)
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.260. $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.260. $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(108) = 216$.

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.35

method	result
risch	$\frac{c^2(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5bc^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{2\left(2a^2d^2-8abcd+6b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}-\frac{2\left(a^2d^2\right.}{}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(24a^{\frac{9}{2}}\sqrt{x(ax+b)}\,cd\,x^3-30a^{\frac{7}{2}}\sqrt{x(ax+b)}\,b\,c^2x^3-24a^{\frac{7}{2}}\left(x(ax+b)\right)^{\frac{3}{2}}cdx+72\sqrt{x(ax+b)}\,a^{\frac{7}{2}}bcd\,x^2-4\left(x(ax+b)\right)^{\frac{3}{2}}a^{\frac{7}{2}}d^2+2\right.}{}$

input
```
int((c+d/x)^2/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(-5*b*c^2*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))+2*(2*a^2*d^2-8*a*b*c*d+6*b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+
b/a))^(1/2)-2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a
)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)))/x/(
(a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.34

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\left(5\,b^4c^2-4\,ab^3cd+(5\,a^2b^2c^2-4\,a^3bcd)x^2+2\,(5\,ab^3c^2-4\,a^2b^2cd)x\right)\sqrt{a}\log\left(2\,ax+\right.}{6\,(a^6}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*
(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4
*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x
^2 + 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*
c^2 - 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8
*a^3*b*c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x +
 b)/x))/(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

3.260.6 Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

output
```
Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.56

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{6}c^2\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - \frac{2}{3}cd\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2d^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2
)*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sq
rt(a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) +
 2/3*d^2/((a + b/x)^(3/2)*b)
```

3.260.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(108) = 216$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.98

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^3\mathrm{sgn}\,(x)} + \frac{(5\,bc^2-4\,acd)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\,(x)} - \frac{(15\,b^2c^2\log\left(|b|\right)-12\,abcd\log\left(|b|\right)+28\,b^2c^2-32\,abcd+4\,a^2d^2)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^2-12\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bcd+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3d^2+15\left(\sqrt{a}x\right.\right.}{3\left(\left(\sqrt{a}x-\sqrt{ax^2\right.\right.}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^2/(a^3*sgn(x)) + 1/2*(5*b*c^2 - 4*a*c*d)*log(abs(2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^2*
c^2*log(abs(b)) - 12*a*b*c*d*log(abs(b)) + 28*b^2*c^2 - 32*a*b*c*d + 4*a^2
*d^2)*sgn(x)/(a^(7/2)*b) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*
c^2 - 12*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d + 3*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*a^3*d^2 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*
c^2 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*c*d + 3*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*a^(5/2)*b*d^2 + 7*b^4*c^2 - 8*a*b^3*c*d + a^2*b^2*d^2)/
(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

3.260.9 Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\frac{2\left(a+\frac{b}{x}\right)\left(a^2\,d^2+4\,a\,b\,c\,d-5\,b^2\,c^2\right)}{3\,a^2}-\frac{2\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a}+\frac{b\left(a+\frac{b}{x}\right)^2\left(5\,b\,c^2-4\,a\,c\,d\right)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-a\,b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\left(4\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

input
```
int((c + d/x)^2/(a + b/x)^(5/2),x)
```

output
```
((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(
a + b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*
(4*a*d - 5*b*c))/a^(7/2)
```

3.261 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.261.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
1/3*(-2*a*d+5*b*c)/a^2/(a+b/x)^(3/2)+c*x/a/(a+b/x)^(3/2)-(-2*a*d+5*b*c)*ar
ctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+(-2*a*d+5*b*c)/a^3/(a+b/x)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(15b^2c+a^2x(-8d+3cx)+ab(-6d+20cx))}{3a^3(b+ax)^2}+\frac{(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2*c + a^2*x*(-8*d + 3*c*x) + a*b*(-6*d + 20*c*x)))/
(3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^
(7/2)
```

3.261.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {899, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{(5bc-2ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(5bc-2ad)\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(5bc-2ad)\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(5bc-2ad)\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)(5bc-2ad)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)/(a + b/x)^(5/2),x]
```

output
```
(c*x)/(a*(a + b/x)^(3/2)) + ((5*b*c - 2*a*d)*(2/(3*a*(a + b/x)^(3/2)) + (2
/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

3.261.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.261. $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.261.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(89) = 178$.

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.47

method	result
risch	$\frac{c(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{5bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+\frac{2(ad-bc)b^2\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\right.}}{3b^2\left(x+\frac{b}{a}\right)}\right.}{a^2}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(12a^{\frac{9}{2}}\sqrt{x(ax+b)}\,d\,x^3-30a^{\frac{7}{2}}\sqrt{x(ax+b)}\,bc\,x^3-12a^{\frac{7}{2}}(x(ax+b))^{\frac{3}{2}}dx+36a^{\frac{7}{2}}\sqrt{x(ax+b)}\,bd\,x^2+24a^{\frac{5}{2}}(x(ax+b))^{\frac{3}{2}}bcx-90a\right.}{}$

input
```
int((c+d/x)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1
/2)+(a*x^2+b*x)^(1/2))-5*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(
1/2)+2*(a*d-b*c)*b^2/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/
3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))-4*(2*a*d-3*b*c)/a/(x+b/a)*(
a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.21

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^3c-2\,ab^2d+(5\,a^2bc-2\,a^3d)x^2+2\,(5\,ab^2c-2\,a^2bd)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\right.}{6\,(a^6x^2+2\,a^5bx}\right.$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
 2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*
(3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*
sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*
b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3
*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5
*b*x + a^4*b^2)]
```

3.261.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(90) = 180.

Time = 33.93 (sec) , antiderivative size = 1479, normalized size of antiderivative = 14.36

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d/x)/(a+b/x)**(5/2),x)
```

output
```
c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*
x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(
33/2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2
)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(s
qrt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a*
*(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**
3) + 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x*
*2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt
(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 4
5*a**14*b**3*x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)
) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a
**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/
2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sq
rt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x + 6*a**(33/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a*...
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.65

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - \frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*
a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
 + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

3.261.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(89) = 178$.

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.51

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,bc\log\left(|b|\right) - 6\,ad\log\left(|b|\right) + 28\,bc - 16\,ad)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{\sqrt{ax^2+bx}c}{a^3\mathrm{sgn}\left(x\right)} + \frac{(5\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c - 6\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bd + 15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c - 9\left(\sqrt{a}x - \right.\right.}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b*c*log(abs(b)) - 6*a*d*log(abs(b)) + 28*b*c - 16*a*d)*sgn(x)/a^(
7/2) + sqrt(a*x^2 + b*x)*c/(a^3*sgn(x)) + 1/2*(5*b*c - 2*a*d)*log(abs(2*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + 2/3*(9*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c - 6*(sqrt(a)*x - sqrt(a*x^2 + b*x))^
2*a^2*b*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c - 9*(sqrt(a)*
x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*d + 7*b^4*c - 4*a*b^3*d)/(((sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

3.261.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2\,d}{3\,a}+\frac{2\,d\left(a+\frac{b}{x}\right)}{a^2}}{\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

input
```
int((c + d/x)/(a + b/x)^(5/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/
x))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2]
, 9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

3.261. $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.262 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

3.262.1 Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
5/3*b/a^2/(a+b/x)^(3/2)+x/a/(a+b/x)^(3/2)-5*b*arctanh((a+b/x)^(1/2)/a^(1/2
))/a^(7/2)+5*b/a^3/(a+b/x)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(15b^2+20abx+3a^2x^2)}{3a^3(b+ax)^2}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(a + b/x)^(-5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2) - (5
*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

3.262.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(a + b/x)^(-5/2),x]
```

output
```
x/(a*(a + b/x)^(3/2)) + (5*b*(2/(3*a*(a + b/x)^(3/2)) + (2/(a*Sqrt[a + b/x
]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

3.262.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.262.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result
risch	$\frac{ax+b}{a^{3}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{7}{2}}}-\frac{2b^{2}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{5}\left(x+\frac{b}{a}\right)^{2}}+\frac{14b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{4}\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}x^{3}-24(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}x+90\sqrt{x(ax+b)}\,a^{\frac{5}{2}}bx^{2}-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}bx^{3}-20b\,a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}}{6}$

input
```
int(1/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-5/2/a^(7/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))-2/3/a^5*b^2/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+14/3/a
^4*b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b)
)^(1/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.85

$$\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\left[\frac{15\left(a^{2}bx^{2}+2\,ab^{2}x+b^{3}\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^{3}x^{3}+20\,a^{2}bx^{2}+15\right.}{6\left(a^{6}x^{2}+2\,a^{5}bx+a^{4}b^{2}\right)}\right.$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x
+ b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x +
 b^3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*
b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

3.262.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(66) = 132$.

Time = 2.71 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{6a^{17}x^4\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{46a^{16}bx^3\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{15a^{16}bx^3\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{30a^{16}bx^3\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{70a^{15}b^2x^2\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{45a^{15}b^2x^2\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{30a^{14}b^3x\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{45a^{14}b^3x\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{90a^{14}b^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{15a^{13}b^4\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{30a^{13}b^4\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}$$

input
```
integrate(1/(a+b/x)**(5/2),x)
```

output
```
6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))
/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b
*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt
(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3)
+ 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1
+ b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x
**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a
**14*b**3*x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) +
 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(
33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*
b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(
1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b
**2*x + 6*a**(33/2)*b**3)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{15\left(a+\frac{b}{x}\right)^2 b - 10\left(a+\frac{b}{x}\right)ab - 2a^2b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3 - \left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4\right)} + \frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
 (a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a)))/a^(7/2)
```

3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,b\log(|b|)+28\,b)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}(x)} + \frac{\sqrt{ax^2+bx}}{a^3\mathrm{sgn}(x)} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3+7\,b^4\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b*log(abs(b)) + 28*b)*sgn(x)/a^(7/2) + 5/2*b*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(
a^3*sgn(x)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2 + 15*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

3.262.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,x\left(\frac{a\,x}{b}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

input
```
int(1/(a + b/x)^(5/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x
)^(5/2))
```

3.263 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

3.263.1 Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}} - \frac{(5bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}$$

output
```
1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(3/2)+x/a/c/(a+b/x)^(3/2)-2*
d^(7/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(5/2
)-(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^2+b*(a^2*d^2-8*a*
b*c*d+5*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(15b^4c^2+3a^4d^2x^2+6a^3bdx(d-cx)+4ab^3c(-6d+5cx)+a^2b^2\left(3d^2-32cdx+3c^2x^2\right)\right)}{a^3(bc-ad)^2(b+ax)^2} - \frac{6d^{7/2}\arctan(\ldots}{(bc-ad\ldots}}{3c^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

output
```
((c*Sqrt[a + b/x]*x*(15*b^4*c^2 + 3*a^4*d^2*x^2 + 6*a^3*b*d*x*(d - c*x) +
4*a*b^3*c*(-6*d + 5*c*x) + a^2*b^2*(3*d^2 - 32*c*d*x + 3*c^2*x^2)))/(a^3*(
b*c - a*d)^2*(b + a*x)^2) - (6*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(5/2) - (3*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(7/2))/(3*c^2)
```

3.263.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.24, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {899, 114, 27, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(5bc+2ad+\frac{5bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5bc+2ad+\frac{5bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\frac{2\int \frac{3\left(\frac{bd(5bc-3ad)}{x}+(bc-ad)(5bc+2ad)\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(\frac{bd(5bc-3ad)}{x}+(bc-ad)(5bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{\frac{2\int \frac{\left((5bc+2ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-8abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left((5bc+2ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-8abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{\frac{(bc-ad)^2(2ad+5bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2a^3d^4\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(2ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4a^3d^4\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 218$$

3.263. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(2ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(a^2d^2-8abcd+5b^2c^2)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{-\frac{4a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^2(2ad+5bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(a^2d^2-8abcd+5b^2c^2)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)) + ((2*b*(5*b*c - 3*a*d))/(3*a*(b*c - a*d)*(a + b/x
)^(3/2)) + ((2*b*(5*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a*(b*c - a*d)*Sqrt[a
+ b/x]) + ((-4*a^3*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)^2*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(2*a*c)
```

3.263.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.263. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.263. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.00

method	result
risch	$\frac{ax+b}{a^3c\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(2ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} - \frac{2c\,b^4\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)a^2} + \frac{2a^3d^4\ln\left(\frac{2(ad-bc)d}{c^2}-\frac{(2}{}\right.}{}\right.}{2a^3cx}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^3/c*((2*a*d+5*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)-2*c*b^4/(a*d-b*c)/a^2*(2/3/b/(x+b/a)^
2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^
(1/2))+2/c^2*a^3*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d
/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b
*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+4*c*b^2*(4*a*d-3*b*c)/(a*d-
b*c)^2/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*
x+b))^(1/2)
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(177) = 354$.

Time = 1.52 (sec) , antiderivative size = 1990, normalized size of antiderivative = 9.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^
2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^
3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*
b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 -
 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3
*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*s
qrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*
b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^
3*d + a^7*b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2
 + 2*a^3*b^2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*
d^3)*x^2 + 2*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)
*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5
*b*d^3*x + a^4*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(
a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*
b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^
2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^
2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^...
```

3.263.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

output
```
Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

3.263.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

3.263.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.263.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 5387, normalized size of antiderivative = 26.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

output
```
- ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2
*d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*
(a^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (ata
n(((((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858
*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 -
5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 7
50*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*
a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14
*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b
^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^1
8*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^2
1*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1
/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^
17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 228
0*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*
a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*
b*c)*1i)/(2*c^2*(a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 4
60*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6
+ 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*...
```

3.263. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

3.264 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

3.264.1 Optimal result

Integrand size = 21, antiderivative size = 287

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx &= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} \\ &+ \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} \\ &+ \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} \\ &- \frac{d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}} - \frac{(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}\end{aligned}$$

output
```
1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+d
*(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(
c+d/x)-d^(7/2)*(-4*a*d+9*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2
))/c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/
2)/c^3+b*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+
b/x)^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(-15b^5c^3(d+cx)+3a^5d^3x^2(2d+cx)+ab^4c^2\left(33d^2+13cdx-20c^2x^2\right)-3a^4bd^2x\left(-4d^2+cdx+3c^2x^2\right)+\right.}{a^3(-bc+ad)^3(b+ax)^2(d+cx)}}{}$$

input
```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-15*b^5*c^3*(d + c*x) + 3*a^5*d^3*x^2*(2*d + c*x) + a
*b^4*c^2*(33*d^2 + 13*c*d*x - 20*c^2*x^2) - 3*a^4*b*d^2*x*(-4*d^2 + c*d*x
+ 3*c^2*x^2) + a^2*b^3*c*(-9*d^3 + 35*c*d^2*x + 41*c^2*d*x^2 - 3*c^3*x^3)
+ 3*a^3*b^2*d*(2*d^3 - 5*c*d^2*x - 3*c^2*d*x^2 + 3*c^3*x^3)))/(a^3*(-(b*c)
 + a*d)^3*(b + a*x)^2*(d + c*x)) + (3*d^(7/2)*(-9*b*c + 4*a*d)*ArcTan[(Sqr
t[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (3*(5*b*c + 4*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(3*c^3)
```

3.264.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {899, 114, 27, 168, 25, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(5bc+4ad+\frac{7bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(5bc+4ad+\frac{7bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{5bd(bc-2ad)}{x}+(bc-ad)(5bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(\frac{5bd(bc-2ad)}{x}+(bc-ad)(5bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 169$$

$$\frac{\frac{\frac{2\int \frac{3\left((5bc+4ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-6abdc+6a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{\left((5bc+4ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-6abdc+6a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 169$$

3.264. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}dx$

$$\frac{\frac{\frac{\frac{2\int \frac{\left((5bc+4ad)(bc-ad)^3+\frac{bd(bc-2ad)\left(5b^2c^2-abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc\cdots}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\left((5bc+4ad)(bc-ad)^3+\frac{bd(bc-2ad)\left(5b^2c^2-abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-\cdots}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 174

$$\frac{\frac{\frac{\frac{\frac{(bc-ad)^3(4ad+5bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{a^3d^4(9bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2a\cdots}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\cdots\right.}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 73

$$\frac{\frac{\frac{\frac{\frac{2(bc-ad)^3(4ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^4(9bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2\cdots}{c\left(a+\frac{b}{x}\right)\cdots}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\downarrow$ 218

3.264. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

$$\frac{\frac{\frac{\frac{\frac{2(bc-ad)^3(4ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{\cdots}{c\left(a+\frac{b}{x}\right)\cdots}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{\frac{-\frac{2a^3d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^3(4ad+5bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}}{c(bc-ad)}+\frac{\cdots}{c\left(a\cdots\right.}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)*(c + d/x)) + ((2*d*(b*c - 2*a*d))/(c*(b*c - a*d)*(a
 + b/x)^(3/2)*(c + d/x)) + ((2*b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a
*(b*c - a*d)*(a + b/x)^(3/2)) + ((2*b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d +
 a^2*d^2))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((-2*a^3*d^(7/2)*(9*b*c - 4*a*d
)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (
2*(b*c - a*d)^3*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c
))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(c*(b*c - a*d)))/(2*a*c)
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.264. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.264.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(261) = 522$.

Time = 0.37 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.32

method	result
risch	$\frac{ax+b}{a^3c^2\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(4ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2c^2b^5\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)^2a^2}+\frac{2a^3d^5\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a^3*((4*a*d+5*b*c)/c*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+2*c^2*b^5/(a*d-b*c)^2/a^2*(2/3/b/
(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(
x+b/a))^(1/2))+2/c^3*a^3*d^5/(a*d-b*c)^2*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x
+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*
d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/
c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*
d/c^2)^(1/2))/(x+d/c))-4*c^2*b^3*(5*a*d-3*b*c)/(a*d-b*c)^3/a/(x+b/a)*(a*(
x+b/a)^2-b*(x+b/a))^(1/2)+2/c^2*a^3*d^4*(3*a*d-5*b*c)/(a*d-b*c)^3/((a*d-b*
c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d
/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+
d/c)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(261) = 522$.

Time = 1.66 (sec) , antiderivative size = 3887, normalized size of antiderivative = 13.54

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*
d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^
2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d
- 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 +
(5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^
4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7
*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*
b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(
-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*
a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2
 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d
^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d
 - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*
b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*s
qrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 -
 a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*
c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*
b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^
5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d...
```

3.264.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)^2}\,dx$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2/((a + b/x)**(5/2)*(c*x + d)**2), x)
```

3.264.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

3.264.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(261) = 522$.

Time = 0.40 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.22

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\left(54\,a^{\frac{7}{2}}bcd^4\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-24\,a^{\frac{9}{2}}d^5\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-15\,\sqrt{bcd-ad^2}b^4c^{\cdots}}{\cdots}$$

$$+\frac{(9\,bcd^4-4\,ad^5)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{(b^3c^6\mathrm{sgn}\,(x)-3\,ab^2c^5d\mathrm{sgn}\,(x)+3\,a^2bc^4d^2\mathrm{sgn}\,(x)-a^3c^3d^3\mathrm{sgn}\,(x))\sqrt{bcd-ad^2}}$$

$$+\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd^4-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^5-\sqrt{a}bd^5}{(b^3c^6\mathrm{sgn}\,(x)-3\,ab^2c^5d\mathrm{sgn}\,(x)+3\,a^2bc^4d^2\mathrm{sgn}\,(x)-a^3c^3d^3\mathrm{sgn}\,(x))\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{a\cdots}\right.\right.}$$

$$+\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^5c-15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2b^4d+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^6c-27\left(\sqrt{a\cdots}\right.\right.}{3\left(a^{\frac{7}{2}}b^3c^3\mathrm{sgn}\,(x)-3\,a^{\frac{9}{2}}b^2c^2d\mathrm{sgn}\,(x)+3\,a^{\frac{11}{2}}bcd^2\mathrm{sgn}\,(x)-a^{\frac{13}{2}}d^3\mathrm{sgn}\,(x)\right)\left(\left(\sqrt{a}x-\sqrt{\cdots}\right.\right.}$$

$$+\frac{\sqrt{ax^2+bx}}{a^3c^2\mathrm{sgn}\,(x)}+\frac{(5\,bc+4\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}c^3\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
1/6*(54*a^(7/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(9/2)
*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 15*sqrt(b*c*d - a*d^2)*b^4*c^
4*log(abs(b)) + 33*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 9*sqrt(b*
c*d - a*d^2)*a^2*b^2*c^2*d^2*log(abs(b)) - 21*sqrt(b*c*d - a*d^2)*a^3*b*c*
d^3*log(abs(b)) + 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) - 28*sqrt(b*c
*d - a*d^2)*b^4*c^4 + 52*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d + 6*sqrt(b*c*d -
a*d^2)*a^4*d^4)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^3*c^6 - 3*sqrt(b*c*d
 - a*d^2)*a^(9/2)*b^2*c^5*d + 3*sqrt(b*c*d - a*d^2)*a^(11/2)*b*c^4*d^2 - s
qrt(b*c*d - a*d^2)*a^(13/2)*c^3*d^3) + (9*b*c*d^4 - 4*a*d^5)*arctan(-((sqr
t(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^3*c^6*
sgn(x) - 3*a*b^2*c^5*d*sgn(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x
))*sqrt(b*c*d - a*d^2)) + ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d^4 - 2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*a*d^5 - sqrt(a)*b*d^5)/((b^3*c^6*sgn(x) - 3*a
*b^2*c^5*d*sgn(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x))*((sqrt(a)
*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d
+ b*d)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*c - 15*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^2*a^2*b^4*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqr
t(a)*b^6*c - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^5*d + 7*b^7*c -
13*a*b^6*d)/((a^(7/2)*b^3*c^3*sgn(x) - 3*a^(9/2)*b^2*c^2*d*sgn(x) + 3*a^(1
1/2)*b*c*d^2*sgn(x) - a^(13/2)*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b...
```

3.264.9 Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 5789, normalized size of antiderivative = 20.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

output

```
((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d
- a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2
- 58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/
x)^3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*
c)^3))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5
/2)*(2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^1
7*c^17*d^2*(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*377
76i + a^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a +
b/x)^(1/2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12
*c^12*d^7*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*1845
63i + a^24*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a +
b/x)^(1/2)*107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c
^7*d^12*(a + b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i
+ a^29*b^5*c^5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(
1/2)*4515i + a^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a
+ b/x)^(1/2)*1700i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107
740*a*b^9*c^9*d^10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 528
36*a^4*b^6*c^6*d^13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420
*a^7*b^3*c^3*d^16) + 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a
^2*b^15*c^15*d^4 - 126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 +...
```

3.264. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

3.265 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

3.265.1 Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{d^{7/2}(99b^2c^2-88abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}} - \frac{(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}$$

output
```
1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*
d+b*c)^3/(a+b/x)^(3/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)
/(c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b
/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+d/x)^2-1/4*d^(7/2)*(24*a^2*d^2-88
*a*b*c*d+99*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-
a*d+b*c)^(9/2)-(6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^4+1/
4*b*(12*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2-56*a*b^3*c^3*d+20*b^4*c^
4)/a^3/c^3/(-a*d+b*c)^4/(a+b/x)^(1/2)
```

3.265.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(60b^6c^4(d+cx)^2+8ab^5c^3(d+cx)^2(-21d+10cx)+6a^6d^4x^2\left(6d^2+9cdx+2c^2x^2\right)+4a^2b^4c^2(d+cx)^2\left(18\right.\right.}{}}{}$$

input
```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(60*b^6*c^4*(d + c*x)^2 + 8*a*b^5*c^3*(d + c*x)^2*(-21
*d + 10*c*x) + 6*a^6*d^4*x^2*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + 4*a^2*b^4*c^2
*(d + c*x)^2*(18*d^2 - 56*c*d*x + 3*c^2*x^2) + 3*a^5*b*d^3*x*(24*d^3 + c*d
^2*x - 45*c^2*d*x^2 - 16*c^3*x^3) + 6*a^4*b^2*d^2*(6*d^4 - 26*c*d^3*x - 39
*c^2*d^2*x^2 + 8*c^3*d*x^3 + 12*c^4*x^4) - 3*a^3*b^3*c*d*(35*d^4 + 5*c*d^3
*x - 64*c^2*d^2*x^2 - 16*c^3*d*x^3 + 16*c^4*x^4)))/(a^3*(b*c - a*d)^4*(b +
 a*x)^2*(d + c*x)^2) - (3*d^(7/2)*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*A
rcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(9/2) - (12*(5
*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(12*c^4)
```

3.265.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {899, 114, 27, 168, 25, 168, 27, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(5bc+6ad+\frac{9bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5bc+6ad+\frac{9bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{7bd(2bc-3ad)}{x}+2(bc-ad)(5bc+6ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(\frac{7bd(2bc-3ad)}{x}+2(bc-ad)(5bc+6ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(4(5bc+6ad)(bc-ad)^2+\frac{5bd\left(4b^2c^2-23abdc+12a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^2+\frac{5bd\left(4b^2c^2-23abdc+12a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{169}$$

3.265. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{\frac{\frac{2\int \frac{3\left(4(5bc+6ad)(bc-ad)^3+\frac{bd\left(20b^3c^3-36ab^2dc^2+87a^2bd^2c-36a^3d^3\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^3+\frac{bd\left(20b^3c^3-36ab^2dc^2+87a^2bd^2c-36a^3d^3\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{\ }{c(}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 169

$$\frac{\frac{\frac{\frac{\frac{2\int \frac{\left(4(5bc+6ad)(bc-ad)^4+\frac{bd\left(20b^4c^4-56ab^3dc^3+24a^2b^2d^2c^2-35a^3bd^3c+12a^4d^4\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(-36}{}}{2c(bc-ad)}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\downarrow$ 27

3.265. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^4+\frac{bd\left(20b^4c^4-56ab^3dc^3+24a^2b^2d^2c^2-35a^3bd^3c+12a^4d^4\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(-36a^3\cdots\right.}{\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{\frac{\frac{\frac{4(bc-ad)^4(6ad+5bc)\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{a^3d^4\left(24a^2d^2-88abcd+99b^2c^2\right)\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(-36a^3\cdots\right.}{\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{\frac{\frac{8(bc-ad)^4(6ad+5bc)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^4\left(24a^2d^2-88abcd+99b^2c^2\right)\int\frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\cdots}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\frac{\frac{8(bc-ad)^4(6ad+5bc)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\cdots}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

3.265. $\displaystyle\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

$$\Big\downarrow 221$$

$$\frac{\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{-\frac{\frac{2a^3d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}-\frac{8\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(bc-}}{2c(bc-ad)}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) + ((d*(2*b*c - 3*a*d))/(c*(b*c - a*d)*
(a + b/x)^(3/2)*(c + d/x)^2) + ((d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/
(c*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)) + ((2*b*(20*b^3*c^3 - 36*a*b^2*c
^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(3*a*(b*c - a*d)*(a + b/x)^(3/2)) + (
(2*b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*b^2*c^2*d^2 - 35*a^3*b*c*d^3 +
12*a^4*d^4))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((-2*a^3*d^(7/2)*(99*b^2*c^2
- 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (8*(b*c - a*d)^4*(5*b*c + 6*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(2*c*(b*c -
a*d)))/(2*c*(b*c - a*d)))/(2*a*c)
```

3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.265. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.265.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. 2(373) = 746.

Time = 0.46 (sec) , antiderivative size = 1138, normalized size of antiderivative = 2.78

method	result	size
risch	Expression too large to display	1138
default	Expression too large to display	7300

input
```
int(1/(a+b/x)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(5/2)/c^4*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-5/2/a^(7/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b+2/3/a^5*b^5/(a*d-b*c)^3/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/
3/a^4*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^5*d^5/(a
*d-b*c)^3/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1
/2)+5/2*a/c^4*d^5/(a*d-b*c)^4/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(
a*d-b*c)*d/c^2)^(1/2)-21/4/c^3*d^4/(a*d-b*c)^4/(x+d/c)*(a*(x+d/c)^2-(2*a*d
-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2/c^5*d^6/(a*d-b*c)^4/((a*d
-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c
)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/
(x+d/c))+23/2*a/c^4*d^5/(a*d-b*c)^4/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c
)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*
d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))*b-99/8/c^3*d^4/(a*d-b*c)
^4/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*(
(a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)
^(1/2))/(x+d/c))*b^2+1/2*a/c^5*d^5/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln(
(2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d
/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))-12/a^3*b^4/(a
*d-b*c)^4/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)*d+6/a^4*c*b^5/(a*d-b*c)^4/
(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^...
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(373) = 746$.

Time = 4.57 (sec) , antiderivative size = 6171, normalized size of antiderivative = 15.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

3.265.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(373) = 746$.

Time = 0.49 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
1/12*(297*a^(7/2)*b^2*c^2*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 264*
a^(9/2)*b*c*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 72*a^(11/2)*d^6*ar
ctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 30*sqrt(b*c*d - a*d^2)*b^5*c^5*log(a
bs(b)) + 84*sqrt(b*c*d - a*d^2)*a*b^4*c^4*d*log(abs(b)) - 36*sqrt(b*c*d -
a*d^2)*a^2*b^3*c^3*d^2*log(abs(b)) - 96*sqrt(b*c*d - a*d^2)*a^3*b^2*c^2*d^
3*log(abs(b)) + 114*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4*log(abs(b)) - 36*sqrt(
b*c*d - a*d^2)*a^5*d^5*log(abs(b)) - 56*sqrt(b*c*d - a*d^2)*b^5*c^5 + 128*
sqrt(b*c*d - a*d^2)*a*b^4*c^4*d + 63*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4 - 30*
sqrt(b*c*d - a*d^2)*a^5*d^5)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^4*c^8 -
 4*sqrt(b*c*d - a*d^2)*a^(9/2)*b^3*c^7*d + 6*sqrt(b*c*d - a*d^2)*a^(11/2)*
b^2*c^6*d^2 - 4*sqrt(b*c*d - a*d^2)*a^(13/2)*b*c^5*d^3 + sqrt(b*c*d - a*d^
2)*a^(15/2)*c^4*d^4) + 1/4*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*ar
ctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))
/((b^4*c^8*sgn(x) - 4*a*b^3*c^7*d*sgn(x) + 6*a^2*b^2*c^6*d^2*sgn(x) - 4*a^
3*b*c^5*d^3*sgn(x) + a^4*c^4*d^4*sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(21*(s
qrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d^4 - 56*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^3*a*b*c^2*d^5 + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^6 + 15
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d^5 - 88*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2
*a^(5/2)*d^7 + 19*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^5 - 92*(sqr...
```

3.265.9 Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 4284, normalized size of antiderivative = 10.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)
```

output
```
((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*
d^2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^
3*(a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4
*d^2 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3
*(a^2*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5
*d + 496*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a
^4*b*c*d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*
(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) +
d^2*(a + b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
 + (atan((a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b
/x)^(1/2)*277440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^2
0*c^20*d^4*(a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8
371440i + a^21*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7
*(a + b/x)^(1/2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i +
 a^24*b^15*c^15*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b
/x)^(1/2)*66665280i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27
*b^12*c^12*d^12*(a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^
(1/2)*85652532i + a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9
*c^9*d^15*(a + b/x)^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)...
```

3.265. $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

3.266 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

3.266.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx=\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x+\frac{(bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

output
```
(a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(
1/2)-2*arctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*b^(1/2)*d^(1/2
)+x*(a+b/x)^(1/2)*(c+d/x)^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$
$$=\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left((bc+ad)\sqrt{b+ax}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)+\sqrt{a}\sqrt{c}\left((b+ax)\sqrt{d+cx}-2\sqrt{b}\sqrt{d}\sqrt{b+ax}\operatorname{arc}\right.\right.}{\sqrt{a}\sqrt{c}(b+ax)\sqrt{d+cx}}$$

input
```
Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x*((b*c + a*d)*Sqrt[b + a*x]*ArcTanh[(Sqrt[a]
*Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])] + Sqrt[a]*Sqrt[c]*((b + a*x)*Sqrt
[d + c*x] - 2*Sqrt[b]*Sqrt[d]*Sqrt[b + a*x]*ArcTanh[(Sqrt[b]*Sqrt[d + c*x]
)/(Sqrt[d]*Sqrt[b + a*x])])))/(Sqrt[a]*Sqrt[c]*(b + a*x)*Sqrt[d + c*x])
```

3.266.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {899, 108, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}-\int \frac{\left(bc+ad+\frac{2bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}-\frac{1}{2}\int \frac{\left(bc+ad+\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(-2bd\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-(ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(-4bd\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-(ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-2(ad+bc)\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-4bd\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-4\sqrt{b}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

input
```
Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

output
```
Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((2*(b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 4*Sqrt[b]*Sqrt[d]*ArcT
anh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/2
```

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}} \left(\sqrt{bd}\, \ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}+ad+bc}{2\sqrt{ac}}\right) ad+\sqrt{bd}\, \ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}+ad+bc}{2\sqrt{ac}}\right) bc-2bd \ln\left(\frac{adx+bcx+\cdots}{\cdots}\right.\right.}{2\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}\, \sqrt{bd}}$

input
```
int((c+d/x)^(1/2)*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(
(a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+(b*d)^(1/2)*l
n(1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))
*b*c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((a*x+b)*(c*x+d))^(1/2)+2*b*d)/x)
*(a*c)^(1/2)+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((a*x+b)*(
c*x+d))^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 890, normalized size of antiderivative = 7.24

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$
$$= \left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 2\sqrt{bd}ac\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,bdx+(bc+ad)x^2)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd+abd^2)}{x^2}\right.}{} \right.$$

input
```
integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(
8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)
*x^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2
)*x)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d
 - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x +
 b)/x)*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a
*d)*x^2)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2
*d^2 + (b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2
- b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2
*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*
d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*
sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x
^2) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^
2)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 +
 (b^2*c*d + a*b*d^2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sq
rt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

3.266.6 Sympy [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx=\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.266.7 Maxima [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx=\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.266.8 Giac [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx=\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 4674, normalized size of antiderivative = 38.00

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

output
```
atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9
/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d
^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1
/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*
b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*
d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d +
 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*
c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^
(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a
^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^
5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7
))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2
) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2
)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/
2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7
*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^
9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5
 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^
(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^
9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 ...
```

3.267 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

3.267.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c}+\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}$$

output
```
(-a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(3/2)/a^
(1/2)+x*(a+b/x)^(1/2)*(c+d/x)^(1/2)/c
```

3.267.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{d+cx}\left(\frac{\sqrt{b+ax}\sqrt{d+cx}}{c}+\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)}{\sqrt{a}c^{3/2}}\right)}{\sqrt{c+\frac{d}{x}}\sqrt{b+ax}}$$

input
```
Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[d + c*x]*((Sqrt[b + a*x]*Sqrt[d + c*x])/c + ((b*c - a*
d)*ArcTanh[(Sqrt[a]*Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])])/(Sqrt[a]*c^(3
/2))))/(Sqrt[c + d/x]*Sqrt[b + a*x])
```

3.267.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {899, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}-\frac{(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}-\frac{(bc-ad)\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}}{c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

input
```
Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + 
 b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

3.267.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.91

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x\sqrt{\frac{cx+d}{x}}\left(-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ad+\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\right)}{2\sqrt{(ax+b)(cx+d)}\,c\sqrt{ac}}$

input
```
int((a+b/x)^(1/2)/(c+d/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*
x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+ln(1/2*(2*a*c*x+2*((a*x+
b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*((a*x+b)*(c*x+d)
)^(1/2)*(a*c)^(1/2))/((a*x+b)*(c*x+d))^(1/2)/c/(a*c)^(1/2)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - \sqrt{ac}(bc-ad)\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 + 4\,(2\,acx^2 + (bc+ad)x)\sqrt{ac}\right.}{4\,ac^2}\right.$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*
log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c +
 a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*
c*d)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(
-a*c)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x
)/(2*a*c*x + b*c + a*d)))/(a*c^2)]
```

3.267.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.267.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.267.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 478, normalized size of antiderivative = 5.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4\,c\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{b^2}{4\,c\,d} + \frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^3}{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^3} + \frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{a+\frac{b}{x}}-\sqrt{a}\,\sqrt{c+\frac{d}{x}}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

output
```
(d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - ((((a
+ b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c +
d/x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^
2*d^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/
2))^2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*
((a + b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)
^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(
1/2))^2)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*
(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b
/x)^(1/2) - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1
/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))
*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

3.268 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

3.268.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}$$

output
```
(-3*a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(5/2)/
a^(1/2)+(a+b/x)^(3/2)*x/a/c/(c+d/x)^(1/2)-(-3*a*d+b*c)*(a+b/x)^(1/2)/a/c^2
/(c+d/x)^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left(\sqrt{a}\sqrt{c}\sqrt{b+ax}(3d+cx)+(bc-3ad)\sqrt{d+cx}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{b+ax}}{\sqrt{a}\sqrt{d+cx}}\right)\right)}{\sqrt{a}c^{5/2}\sqrt{b+ax}(d+cx)}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x*(Sqrt[a]*Sqrt[c]*Sqrt[b + a*x]*(3*d + c*x)
+ (b*c - 3*a*d)*Sqrt[d + c*x]*ArcTanh[(Sqrt[c]*Sqrt[b + a*x])/(Sqrt[a]*Sqr
t[d + c*x])]))/(Sqrt[a]*c^(5/2)*Sqrt[b + a*x]*(d + c*x))
```

3.268.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {899, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\int \frac{\sqrt{a+\frac{b}{x}}x}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{x}}{2ac}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{a\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}}{c}+\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}\right)}{2ac}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{2a\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}}{c}+\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}\right)}{2ac}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{c^{3/2}}\right)}{2ac}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

output
```
((a + b/x)^(3/2)*x)/(a*c*Sqrt[c + d/x]) - ((b*c - 3*a*d)*((2*Sqrt[a + b/x]
)/(c*Sqrt[c + d/x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*
Sqrt[c + d/x])])/c^(3/2)))/(2*a*c)
```

3.268.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.268. $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(102) = 204$.

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\sqrt{\frac{cx+d}{x}}\left(-3\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)acdx+\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)b\,c^2x+2cx\sqrt{(ax+b)(cx+\cdots}}{2\sqrt{ac}\,(cx+d)\sqrt{(ax+\cdots}}$

input
```
int((a+b/x)^(1/2)/(c+d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(
c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*c*d*x+ln(1/2*(2*a*c*x+2*
((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c^2*x+2*c*x*((
a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1
/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((a*x+b)*(c*
x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^
(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \left[-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{ac}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx^2\cdots\right.}{4\,(ac^4x\cdots}\right.$$

$$\left.-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{-ac}\arctan\left(\frac{2\sqrt{-ac}x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)-2\,(ac^2x^2+3\,acdx)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(ac^4x+ac^3d)}\right]$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^
2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c
)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^
2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d
), -1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(
-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*
c^2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3
*d)]
```

3.268.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

output
```
Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

3.268.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

3.268.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)
```

output
```
int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)
```

3.269 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \,dx$

3.269.1 Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \,dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}$$

output
```
-b*(a+b/x)^(p+1)*(c+d/x)^q*AppellF1(p+1,2,-q,2+p,(a+b/x)/a,-d*(a+b/x)/(-a*
d+b*c))/a^2/(p+1)/((b*(c+d/x)/(-a*d+b*c))^q)
```

3.269.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 206 vs. $2(96) = 192$.

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.15

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \,dx = \frac{bd(-2+p+q)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q x\operatorname{AppellF1}}{(-1+p+q)\left(-bd(-2+p+q)\operatorname{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax}{b},-\frac{cx}{d}\right)+x\left(adp\operatorname{AppellF1}\left(2\right.\right.\right.}$$

input
```
Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

output
```
(b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2
- p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*Appell
F1[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*Appel
lF1[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*Appel
lF1[2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

3.269.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {899, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{154}$$

$$-\left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \int \left(a+\frac{b}{x}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1} \left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

input
```
Int[(a + b/x)^p*(c + d/x)^q,x]
```

output
```
-((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
 b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d)
)^q))
```

3.269.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.269.4 Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{p} \left(c+\frac{d}{x}\right)^{q} dx$$

input
```
int((a+b/x)^p*(c+d/x)^q,x)
```

output
```
int((a+b/x)^p*(c+d/x)^q,x)
```

3.269.5 Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

3.269.6 Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)**p*(c+d/x)**q,x)
```

output
```
Integral((a + b/x)**p*(c + d/x)**q, x)
```

3.269.7 Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^p*(c + d/x)^q, x)
```

3.269.8 Giac [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x)^p*(c + d/x)^q, x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
int((a + b/x)^p*(c + d/x)^q,x)
```

output
```
int((a + b/x)^p*(c + d/x)^q, x)
```

3.270 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx$

3.270.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

output
```
a*x/c+(-a*d+b*c)*arctan(x*c^(1/2)/d^(1/2))/c^(3/2)/d^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx = \frac{ax}{c} - \frac{(-bc+ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

input
```
Integrate[(a + b/x^2)/(c + d/x^2),x]
```

output
```
(a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

3.270.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {898, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx$$

$$\Big\downarrow \text{ 898}$$

$$\int \frac{ax^2+b}{cx^2+d}dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(bc-ad)\int \frac{1}{cx^2+d}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

input
```
Int[(a + b/x^2)/(c + d/x^2),x]
```

output
```
(a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

3.270.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

3.270.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{ax}{c}+\frac{(-ad+bc)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{c\sqrt{cd}}$	34
risch	$\frac{ax}{c}-\frac{\ln\left(cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}+\frac{\ln\left(cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}+\frac{\ln\left(-cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}-\frac{\ln\left(-cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}$	106

input
```
int((a+b/x^2)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x/c+(-a*d+b*c)/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \left[\frac{2\,acdx+(bc-ad)\sqrt{-cd}\log\left(\frac{cx^2+2\,\sqrt{-cd}x-d}{cx^2+d}\right)}{2\,c^2d}, \frac{acdx+(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{c^2d}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/
(c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x
/d))/(c^2*d)]
```

3.270.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2),x)
```

output
```
a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 -
sqrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

output
```
a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
```

output
```
a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx=\frac{a\,x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

input
```
int((a + b/x^2)/(c + d/x^2),x)
```

output
```
(a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

3.271 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

3.271.1 Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}x}+\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{\sqrt{c}(bc+ad)\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

output
```
-2*d*(a+b/x^2)^(1/2)/x/(c+d/x^2)^(1/2)-(a*d+b*c)*(x^2*c/d/(1+x^2*c/d))^(1/
2)/x*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a
+b/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d
/(1+x^2*c/d))^(1/2)/x*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1
-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)
^(1/2)+x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
```

3.271.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x\left(\sqrt{\frac{a}{b}}(b+ax^2)\,(d+cx^2)+2iadx\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+i(bc-\right.}{\sqrt{\frac{a}{b}}\,(b+ax^2)\,(d+cx^2)}$$

input
```
Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

output
```
-((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) +
(2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c
*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b +
a*x^2)*(d + c*x^2)))
```

3.271.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {899, 375, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

$$\Big\downarrow \text{899}$$

$$-\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{375}$$

$$x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-2\int \frac{bc+ad+\frac{2bd}{x^2}}{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\int \frac{bc+ad+\frac{2bd}{x^2}}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow 406$$

$$-(ad+bc)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-2bd\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow 320$$

$$-2bd\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow 388$$

$$-2bd\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow 313$$

$$-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-2bd\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

input
```
Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

3.271. $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

output
```
Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x - 2*b*d*(Sqrt[a + b/x^2]/(b*Sqrt[c + d/x
^2]*x) - (Sqrt[c]*Sqrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
 d/x^2])) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/
(Sqrt[c]*x)], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/
x^2))]*Sqrt[c + d/x^2])
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.271. $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.271.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,x\sqrt{\frac{c\,x^2+d}{x^2}}\left(-\sqrt{-\frac{c}{d}}\,ac\,x^4+\sqrt{\frac{c\,x^2+d}{d}}\,\sqrt{\frac{a\,x^2+b}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)adx-cb\sqrt{\frac{c\,x^2+d}{d}}\,\sqrt{\frac{a\,x^2+b}{b}}\,xF\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)+2cb\sqrt{\frac{c\,x}{}}\right.}{(a\,x^4c+ad\,x^2+cb\,x^2+bd)\sqrt{-\frac{c}{d}}}$
risch	$-x\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{\frac{c\,x^2+d}{x^2}}+\frac{\left(\frac{ad\sqrt{1+\frac{c\,x^2}{d}}\,\sqrt{1+\frac{a\,x^2}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^4c+ad\,x^2+cb\,x^2+bd}}+\frac{bc\sqrt{1+\frac{c\,x^2}{d}}\,\sqrt{1+\frac{a\,x^2}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^4c+ad\,x^2+cb\,x^2+bd}}-\frac{2cb\sqrt{1}}{}\right.}{(a\,x}$

input
```
int((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*a*c*x^4+((c*x
^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2
))*a*d*x-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticF(x*(-c/d)^
(1/2),(a*d/b/c)^(1/2))+2*c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ell
ipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-(-c/d)^(1/2)*a*d*x^2-(-c/d)^(1/2)*b
*c*x^2-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

3.271.5 Fricas [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

3.271.6 Sympy [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

3.271.7 Maxima [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

3.271.8 Giac [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2),x)
```

output
```
int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.272.1 Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

output
```
-d*(a+b/x^2)^(1/2)/c/x/(c+d/x^2)^(1/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x*(1+
x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^
(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+(x^2*c/d/(1+x^2*c/
d))^(1/2)/x/c*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d
)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+x
*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)/c
```

3.272.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{d+cx^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{c}{d}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{b+ax^2}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*
d)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

3.272.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {899, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

$$\Big\downarrow \ 899$$

$$-\int \frac{\sqrt{a+\frac{b}{x^2}}x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \ 377$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{\int \frac{b\sqrt{c+\frac{d}{x^2}}}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\int \frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \textbf{324}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(c\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}\right)}{c}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}$$

$$\Big\downarrow \textbf{388}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)\right)}{c}$$

input
```
Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c - (b*(d*(Sqrt[a + b/x^2]/(b*Sqrt[c +
 d/x^2]*x) - (Sqrt[c]*Sqrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)
], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt
[c + d/x^2])) + (c^(3/2)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/(Sqrt[c]
*x)], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])))/c
```

3.272. $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.272.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.272. $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

3.272.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,E\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{a\,x^2+b}{b}}\,\sqrt{\frac{c\,x^2+d}{d}}\,b}{(a\,x^2+b)\sqrt{-\frac{c}{d}}\,\sqrt{\frac{c\,x^2+d}{x^2}}}$	94

input
```
int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*
((a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/((c*x^2+d)/x^2)^(1/
2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}} - \sqrt{ac}b\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}) + \sqrt{ac}(a+b)\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc})}{ac}$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2) - sqrt(a*c)*b*sqrt(-b/a)*
elliptic_e(arcsin(sqrt(-b/a)/x), a*d/(b*c)) + sqrt(a*c)*(a + b)*sqrt(-b/a)
*elliptic_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)))/(a*c)
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

3.272.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

3.272.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2),x)
```

output
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$

3.273.1 Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx = -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}x}-\frac{\sqrt{a+\frac{b}{x^2}}x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

output
```
-2*d*(a+b/x^2)^(1/2)/c^2/x/(c+d/x^2)^(1/2)-x*(a+b/x^2)^(1/2)/c/(c+d/x^2)^(
1/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^
2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2)
)^(1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c^2*d*(1+x^2*c/d)^
(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*
(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*x*(a+b/x^2)^(1/2)*(c+d/x^2)
^(1/2)/c^2
```

3.273.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{\frac{a}{b}}cx(b+ax^2)+2iad\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)\mid\frac{bc}{ad}\right)+i(bc-2ad)\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\right.}{\sqrt{\frac{a}{b}}c^2\sqrt{c+\frac{d}{x^2}}\,(b+ax^2)}$$

input
```
Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

output
```
-((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)
/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I
*(b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh
[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

3.273.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {899, 371, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^2}}x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{371}$$

$$\frac{\int -\frac{\left(2a+\frac{b}{x^2}\right)x^2}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 25

$$-\frac{\int \frac{\left(2a+\frac{b}{x^2}\right)x^2}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 445

$$-\frac{-\frac{\int -\frac{ab\left(c+\frac{2d}{x^2}\right)}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{ac}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{ab\left(c+\frac{2d}{x^2}\right)}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{ac}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 27

$$-\frac{\frac{b\int \frac{c+\frac{2d}{x^2}}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 406

$$-\frac{\frac{b\left(c\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+2d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$\downarrow$ 320

$$-\frac{\frac{b\left(2d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

3.273. $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$

$$\Big\downarrow \text{388}$$

$$-\frac{\frac{b\left(2d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{313}$$

$$-\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+2d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

output
```
-((Sqrt[a + b/x^2]*x)/(c*Sqrt[c + d/x^2])) - ((-2*Sqrt[a + b/x^2]*Sqrt[c +
 d/x^2]*x)/c + (b*(2*d*(Sqrt[a + b/x^2]/(b*Sqrt[c + d/x^2]*x) - (Sqrt[c]*S
qrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])) + (c^(3/2
)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1 - (b*c)/(a*d)])
/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])))/c)/c
```

3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.273. $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.273. $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.273.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(\sqrt{-\frac{c}{d}}\,a\,x^{3}+b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)-2b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,E\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)+\sqrt{-\frac{c}{d}}\,bx\right)(c\,x^{2}+d)}{x^{2}(a\,x^{2}+b)\sqrt{-\frac{c}{d}}\,c\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}}$

input
```
int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*((-c/d)^(1/2)*a*x^3+b*((c*x^2+d)/d)^(
1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-2*b*((c
*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1
/2))+(-c/d)^(1/2)*b*x)*(c*x^2+d)/(-c/d)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx =$$

$$-\frac{2\,(bcx^2+bd)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}) - ((a+2\,b)cx^2+(a+2\,b)d)\sqrt{ac}\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{a}{b}}{ac^3x^2+ac^2d}$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
-(2*(b*c*x^2 + b*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(sqrt(-b/a)/x),
a*d/(b*c)) - ((a + 2*b)*c*x^2 + (a + 2*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic
_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)) - (a*c*x^3 + 2*a*d*x)*sqrt((a*x^2 + b)
/x^2)*sqrt((c*x^2 + d)/x^2))/(a*c^3*x^2 + a*c^2*d)
```

3.273.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

output
```
Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

3.273.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

3.273.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)
```

output
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)
```

3.274 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$

3.274.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b
/a/x^2)^p)/((1+d/c/x^2)^q)
```

3.274.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-1+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

3.274.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {899, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{899}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -
(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

3.274.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.274.4 Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

3.274.5 Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

output
```
Timed out
```

3.274.7 Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.274.8 Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \,dx$$

input
```
int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.275 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

3.275.1 Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx=\frac{ax}{c}-\frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}$$

output
```
a*x/c+1/3*(-a*d+b*c)*ln(d^(1/3)+c^(1/3)*x)/c^(4/3)/d^(2/3)-1/6*(-a*d+b*c)*
ln(d^(2/3)-c^(1/3)*d^(1/3)*x+c^(2/3)*x^2)/c^(4/3)/d^(2/3)-1/3*(-a*d+b*c)*a
rctan(1/3*(d^(1/3)-2*c^(1/3)*x)/d^(1/3)*3^(1/2))/c^(4/3)/d^(2/3)*3^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$

$$=\frac{6a\sqrt[3]{c}d^{2/3}x-2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)-(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\right.}{6c^{4/3}d^{2/3}}$$

input
```
Integrate[(a + b/x^3)/(c + d/x^3),x]
```

output
```
(6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d
^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Lo
g[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

3.275.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {898, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$

$$\Big\downarrow \text{898}$$

$$\int \frac{ax^3+b}{cx^3+d}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{cx^3+d}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{750}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{c}x+\sqrt[3]{d}}dx}{3d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{c}\left(\sqrt[3]{d}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$\Big\downarrow$ 25

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{\int\frac{\sqrt[3]{c}\left(\sqrt[3]{d}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$\Big\downarrow$ 27

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$\Big\downarrow$ 1082

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$\Big\downarrow$ 217

3.275. $\int\frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \ 1103$$

$$\frac{(bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

input
```
Int[(a + b/x^3)/(c + d/x^3),x]
```

output
```
(a*x)/c + ((b*c - a*d)*(Log[d^(1/3) + c^(1/3)*x]/(3*c^(1/3)*d^(2/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*c^(1/3)*x)/d^(1/3))/Sqrt[3]])/c^(1/3)) - Log[d^(2/
3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/(3*d^(2/3))))/c
```

3.275.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.275. $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.275. $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}} \, dx$

3.275.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{ax}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{3}+d\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3c^{2}}$	42
default	$\frac{ax}{c}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{c}$	110

input
```
int((a+b/x^3)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

output
```
a*x/c+1/3/c^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*c+d))
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$
$$= \left[\frac{6\,acd^{2}x-3\,\sqrt{\frac{1}{3}}(bc^{2}d-acd^{2})\sqrt{\frac{(-cd^{2})^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^{3}+3\,(-cd^{2})^{\frac{1}{3}}dx-d^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+(-cd^{2})^{\frac{2}{3}}x+(-cd^{2})^{\frac{1}{3}}d\right)\sqrt{\frac{(-cd^{2})^{\frac{1}{3}}}{c}}}{cx^{3}+d}\right)}{6\,c^{2}}\right.$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

output
```
[1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)
*log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-
c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) -
(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)
*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2),
 1/6*(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c
)*arctan(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^
(1/3)/c)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x
- (-c*d^2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2
/3)))/(c^2*d^2)]
```

3.275.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

input
```
integrate((a+b/x**3)/(c+d/x**3),x)
```

output
```
a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2 - x\left(\frac{d}{c}\right)^{\frac{1}{3}} + \left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

output
```
a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/
c)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d
/c)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(
d/c)^(2/3))
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,\left(-c^2d\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,\left(-c^2d\right)^{\frac{2}{3}}} + \frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1
/3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2
/3))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d
/c)^(1/3)))/(c*d)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{a\,x}{c} - \frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,(a\,d-b\,c)}{3\,c^{4/3}\,d^{2/3}} + \frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,c^{4/3}\,d^{2/3}} - \frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,c^{4/3}\,d^{2/3}}$$

input
```
int((a + b/x^3)/(c + d/x^3),x)
```

output
```
(a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (lo
g(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
 - b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

3.275. $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

3.276 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

3.276.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

output
```
b*x/d+2*c*(-a*d+b*c)*ln(c+d*x^(1/2))/d^3-2*(-a*d+b*c)*x^(1/2)/d^2
```

3.276.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{\left(-2bc+2ad+bd\sqrt{x}\right)\sqrt{x}}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

input
```
Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

output
```
((-2*b*c + 2*a*d + b*d*Sqrt[x])*Sqrt[x])/d^2 + (2*c*(b*c - a*d)*Log[c + d*
Sqrt[x]])/d^3
```

3.276.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{ 900}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)\sqrt{x}}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 86}$$

$$2\int \left(\frac{\sqrt{x}b}{d}+\frac{ad-bc}{d^2}+\frac{c(bc-ad)}{d^2\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{\sqrt{x}(bc-ad)}{d^2}+\frac{bx}{2d}\right)$$

input
```
Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

output
```
2*(-(((b*c - a*d)*Sqrt[x])/d^2) + (b*x)/(2*d) + (c*(b*c - a*d)*Log[c + d*S
qrt[x]])/d^3)
```

3.276.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.276.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln(c+d\sqrt{x})}{d^3}$	48
default	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln(c+d\sqrt{x})}{d^3}$	48

input
```
int((a+b*x^(1/2))/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(1/2*b*d*x+a*d*x^(1/2)-b*c*x^(1/2))-2*(a*d-b*c)*c/d^3*ln(c+d*x^(1/2)
)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\frac{bd^2x+2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)-2\left(bcd-ad^2\right)\sqrt{x}}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
(b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x
))/d^3
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

output
```
Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c
/d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x*
*(3/2)/3)/c, True))
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2} + \frac{2\,(bc^2-acd)\log\left(d\sqrt{x}+c\right)}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
(b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)
/d^3
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{bdx-2\,bc\sqrt{x}+2\,ad\sqrt{x}}{d^2} + \frac{2\,(bc^2-acd)\log\left(\left|d\sqrt{x}+c\right|\right)}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
(b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*
sqrt(x) + c))/d^3
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \sqrt{x}\left(\frac{2\,a}{d}-\frac{2\,b\,c}{d^2}\right)+\frac{\ln\left(c+d\,\sqrt{x}\right)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{d^3}+\frac{b\,x}{d}$$

input
```
int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

output
```
x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))
/d^3 + (b*x)/d
```

3.277 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx$

3.277.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

output
```
6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

3.277.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

input
```
Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

output
```
6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

3.277.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {900, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}-1}{\sqrt[3]{x}+1}dx$$

$$\Big\downarrow \text{ 900}$$

$$3\int -\frac{\left(1-\sqrt[3]{x}\right)x^{2/3}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 25}$$

$$-3\int \frac{\left(1-\sqrt[3]{x}\right)x^{2/3}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 86}$$

$$-3\int \left(-x^{2/3}+2\sqrt[3]{x}+\frac{2}{\sqrt[3]{x}+1}-2\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-x^{2/3}+\frac{x}{3}+2\sqrt[3]{x}-2\log\left(\sqrt[3]{x}+1\right)\right)$$

input
```
Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

output
```
3*(2*x^(1/3) - x^(2/3) + x/3 - 2*Log[1 + x^(1/3)])
```

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
default	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
trager	$-1+x+6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	32
meijerg	$\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}-6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1+x^{\frac{1}{3}}\right)+\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}$	39

input
```
int((x^(1/3)-1)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = -3x^{\frac{2}{3}} + 6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right)$$

input
```
integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

output
```
-3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 6\,\ln\left(x^{1/3} + 1\right) + 6\,x^{1/3} - 3\,x^{2/3}$$

input
```
int((x^(1/3) - 1)/(x^(1/3) + 1),x)
```

output
```
x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)
```

3.278 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} dx$

3.278.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x-6\text{arctanh}\left(\sqrt[3]{x}\right)$$

output
```
6*x^(1/3)+x-6*arctanh(x^(1/3))
```

3.278.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x-6\text{arctanh}\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

output
```
6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

3.278.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {900, 25, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}+1}{x^{2/3}-1}dx$$

$$\Big\downarrow \text{900}$$

$$3\int -\frac{\left(x^{2/3}+1\right)x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{25}$$

$$-3\int \frac{\left(x^{2/3}+1\right)x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{363}$$

$$3\left(\frac{x}{3}-2\int \frac{x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{262}$$

$$3\left(\frac{x}{3}-2\left(\int \frac{1}{1-x^{2/3}}d\sqrt[3]{x}-\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$3\left(\frac{x}{3}-2\left(\text{arctanh}\left(\sqrt[3]{x}\right)-\sqrt[3]{x}\right)\right)$$

input
```
Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

output
```
3*(x/3 - 2*(-x^(1/3) + ArcTanh[x^(1/3)]))
```

3.278.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

3.278.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
default	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
trager	$-2+x+6x^{\frac{1}{3}}+3\ln\left(-\frac{2x^{\frac{2}{3}}-2x^{\frac{1}{3}}-x+1}{1+x}\right)$	34
meijerg	$-\frac{3i\left(2ix^{\frac{1}{3}}-2i\operatorname{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3i\left(-\frac{2ix^{\frac{1}{3}}\left(5x^{\frac{2}{3}}+15\right)}{15}+2i\operatorname{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}$	43

input
```
int((1+x^(2/3))/(-1+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(1+x^(1/3))
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

input
```
integrate((1+x**(2/3))/(-1+x**(2/3)),x)
```

output
```
6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x-6\,\mathrm{atanh}\left(x^{1/3}\right)+6\,x^{1/3}$$

input
```
int((x^(2/3) + 1)/(x^(2/3) - 1),x)
```

output
```
x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

3.279 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

3.279.1 Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)$$

output
```
-128*x^(1/4)+x+256/3*2^(1/3)*ln(2*2^(1/3)+x^(1/4))-128/3*2^(1/3)*ln(4*2^(2
/3)-2*2^(1/3)*x^(1/4)+x^(1/2))-256/3*2^(1/3)*arctan(1/6*(2^(1/3)-x^(1/4))*
2^(2/3)*3^(1/2))*3^(1/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\log\left(4+2^{2/3}\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(-8+2\ 2^{2/3}\sqrt[4]{x}-\sqrt[3]{2}\sqrt{x}\right)$$

input
```
Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

output
```
-128*x^(1/4) + x - (256*2^(1/3)*ArcTan[1/Sqrt[3] - x^(1/4)/(2^(1/3)*Sqrt[3
])])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (128*2^(1/3)*Log
[-8 + 2*2^(2/3)*x^(1/4) - 2^(1/3)*Sqrt[x]])/3
```

3.279.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {900, 25, 959, 843, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}-16}{x^{3/4}+16} \, dx$$

$$\Big\downarrow \text{900}$$

$$4\int -\frac{\left(16-x^{3/4}\right)x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\left(16-x^{3/4}\right)x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}$$

$$\Big\downarrow \text{959}$$

$$4\left(\frac{x}{4}-32\int \frac{x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{843}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\int \frac{1}{x^{3/4}+16}d\sqrt[4]{x}\right)\right)$$

$$\Big\downarrow \text{750}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{1}{\sqrt[4]{x}+2\sqrt[3]{2}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\int \frac{4\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{4\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$\downarrow$ 1142

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\sqrt[3]{2}\int \frac{1}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}-\frac{1}{2}\int -\frac{2\left(\sqrt[3]{2}-\sqrt[4]{x}\right)}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$\downarrow$ 27

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\sqrt[3]{2}\int \frac{1}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}+\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$\downarrow$ 1082

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\int \frac{1}{-\sqrt{x}-3}d\left(1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}\right)+\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$\downarrow$ 217

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}-\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}}{\sqrt{3}}\right)}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$\downarrow$ 1103

3.279. $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

input
```
Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

output
```
4*(x/4 - 32*(x^(1/4) - 16*(Log[2*2^(1/3) + x^(1/4)]/(12*2^(2/3)) + (-(Sqrt
[3]*ArcTan[(1 - x^(1/4)/2^(1/3))/Sqrt[3]]) - Log[4*2^(2/3) - 2*2^(1/3)*x^(
1/4) + Sqrt[x]]/2)/(12*2^(2/3)))))
```

3.279.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.279. $\int \frac{-16+x^{3/4}}{16+x^{3/4}} \, dx$

3.279.4 Maple [A] (verified)

Time = 56.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
derivativedivides	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right.}{3}$
default	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right.}{3}$
meijerg	$-\frac{128\,2^{\frac{1}{3}}\left(\frac{3\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}\left(\frac{2\,2^{\frac{1}{3}}\ln\left(1+\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}\right)}{x^{\frac{1}{4}}}-\frac{2^{\frac{1}{3}}\ln\left(1-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}+\frac{2^{\frac{1}{3}}\sqrt{x}}{8}\right)}{x^{\frac{1}{4}}}+\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{8-2^{\frac{2}{3}}x^{\frac{1}{4}}}\right)}{x^{\frac{1}{4}}}\right)}{4}\right)}{3}+\frac{128\,2^{\frac{1}{3}}}{}$
trager	Expression too large to display

input
```
int((-16+x^(3/4))/(16+x^(3/4)),x,method=_RETURNVERBOSE)
```

output
```
x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)
-16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8
*16^(2/3)*x^(1/4)-1))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx=\frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) -
128/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)
*log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

3.279.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x+\frac{256\cdot\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\cdot\sqrt[3]{2}\right)}{3} - \frac{128\cdot\sqrt[3]{2}\log\left(-2\cdot\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\cdot\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

output
```
-128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/
3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(
3)*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*
log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*
log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = x+\frac{256\,2^{1/3}\,\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}-6144\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

input
```
int((x^(3/4) - 16)/(x^(3/4) + 16),x)
```

output
```
x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128
*2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1
))/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(
1/2)*1i + 1))/3
```

3.280 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

3.280.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

output
```
-6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(-1+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

output
```
-6*x^(1/3) - 3*x^(2/3) - x - 6*Log[-1 + x^(1/3)]
```

3.280.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {898, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{1}{\sqrt[3]{x}}+1}{\frac{1}{\sqrt[3]{x}}-1}dx$$

$$\Big\downarrow \text{ 898}$$

$$\int \frac{\sqrt[3]{x}+1}{1-\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{ 900}$$

$$3\int \frac{\left(\sqrt[3]{x}+1\right)x^{2/3}}{1-\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 86}$$

$$3\int \left(-x^{2/3}-2\sqrt[3]{x}-\frac{2}{\sqrt[3]{x}-1}-2\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-x^{2/3}-\frac{x}{3}-2\sqrt[3]{x}-2\log\left(1-\sqrt[3]{x}\right)\right)$$

input
```
Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

output
```
3*(-2*x^(1/3) - x^(2/3) - x/3 - 2*Log[1 - x^(1/3)])
```

3.280.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.280.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
default	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
trager	$2-x-6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x+1\right)$	34
meijerg	$-\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}+6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1-x^{\frac{1}{3}}\right)-\frac{x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}+6\right)}{2}$	41

input
```
int((1+1/x^(1/3))/(-1+1/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

3.280.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -3x^{\frac{2}{3}}-6\sqrt[3]{x}-x-6\log\left(\sqrt[3]{x}-1\right)$$

input
```
integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

output
```
-3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-6\,\ln\left(x^{1/3}-1\right)-6\,x^{1/3}-3\,x^{2/3}$$

input
```
int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
```

output
```
- x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

3.281 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

3.281.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx = \frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2 x^{2n}}{a^2}}}$$

output
```
a^2*x*hypergeom([-3/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(
a+b*x^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx = \frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2 x^{2n}}{a^2}}}$$

input
```
Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

output
```
(a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1
+ 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

3.281.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$$

$$\Big\downarrow \text{785}$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \left(a^2-b^2x^{2n}\right)^{3/2}dx}{\sqrt{a^2-b^2x^{2n}}}$$

$$\Big\downarrow \text{779}$$

$$\frac{a^2\sqrt{a-bx^n}\sqrt{a+bx^n}\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

$$\Big\downarrow \text{778}$$

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

output
```
(a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

3.281.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

3.281.4 Maple [F]

$$\int (a-b\,x^{n})^{\frac{3}{2}} (a+b\,x^{n})^{\frac{3}{2}}\,dx$$

input
```
int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

output
```
int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

3.281.5 Fricas [F(-2)]

Exception generated.

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.281.6 Sympy [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (a-bx^n)^{\frac{3}{2}}(a+bx^n)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

output
```
Integral((a - b*x**n)**(3/2)*(a + b*x**n)**(3/2), x)
```

3.281.7 Maxima [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

3.281.8 Giac [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \int (a+b\,x^n)^{3/2}\,(a-b\,x^n)^{3/2}\,dx$$

input
```
int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2),x)
```

output
```
int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2), x)
```

3.282 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

3.282.1 Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

output
```
x*hypergeom([-1/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a+b*
x^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/
(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

3.282.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \ 785$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \sqrt{a^2-b^2x^{2n}}dx}{\sqrt{a^2-b^2x^{2n}}}$$

$$\Big\downarrow \ 779$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \sqrt{1-\frac{b^2x^{2n}}{a^2}}dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

$$\Big\downarrow \ 778$$

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n
^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

3.282.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

3.282.4 Maple [F]

$$\int \sqrt{a-b\,x^{n}}\,\sqrt{a+b\,x^{n}}dx$$

input
```
int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

output
```
int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

3.282.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.282.6 Sympy [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

input
```
integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

3.282.7 Maxima [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

3.282.8 Giac [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{a+b\,x^n}\,\sqrt{a-b\,x^n}\,dx$$

input
```
int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2),x)
```

output
```
int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2), x)
```

3.283 $\int (a-bx^n)^p (a+bx^n)^p\,dx$

3.283.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (a-bx^n)^p (a+bx^n)^p\,dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

output
```
x*(a-b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)
/((1-b^2*x^(2*n)/a^2)^p)
```

3.283.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^p (a+bx^n)^p\,dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

3.283.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^p (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{785}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p} \int \left(a^2-b^2x^{2n}\right)^p dx$$

$$\Big\downarrow \text{779}$$

$$(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

3.283.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

3.283.4 Maple [F]

$$\int (a - b\,x^n)^p\,(a + b\,x^n)^p\,dx$$

input
```
int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

3.283.5 Fricas [F]

$$\int (a - bx^n)^p\,(a + bx^n)^p\,dx = \int (bx^n + a)^p(-bx^n + a)^p\,dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.283.6 Sympy [F]

$$\int (a - bx^n)^p\,(a + bx^n)^p\,dx = \int (a - bx^n)^p\,(a + bx^n)^p\,dx$$

input
```
integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

3.283.7 Maxima [F]

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p(-bx^n+a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.283.8 Giac [F]

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p(-bx^n+a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = \int (a+b\,x^n)^p \,(a-b\,x^n)^p \, dx$$

input
```
int((a + b*x^n)^p*(a - b*x^n)^p,x)
```

output
```
int((a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.284 $\int (a+bx^n)(c+dx^n)^4 \, dx$

3.284.1 Optimal result

Integrand size = 17, antiderivative size = 132

$$\int (a+bx^n)(c+dx^n)^4 \, dx = ac^4x + \frac{c^3(bc+4ad)x^{1+n}}{1+n} + \frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n} + \frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n} + \frac{d^3(4bc+ad)x^{1+4n}}{1+4n} + \frac{bd^4x^{1+5n}}{1+5n}$$

output
```
a*c^4*x+c^3*(4*a*d+b*c)*x^(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^(1+2*n)/(1+2
*n)+2*c*d^2*(2*a*d+3*b*c)*x^(1+3*n)/(1+3*n)+d^3*(a*d+4*b*c)*x^(1+4*n)/(1+4
*n)+b*d^4*x^(1+5*n)/(1+5*n)
```

3.284.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \frac{bx(c+dx^n)^5 - (bc-ad(1+5n))x\left(c^4 + \frac{4c^3dx^n}{1+n} + \frac{6c^2d^2x^{2n}}{1+2n} + \frac{4cd^3x^{3n}}{1+3n} + \frac{d^4x^{4n}}{1+4n}\right)}{d+5dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

output
```
(b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n)
+ (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*
n))/(1 + 4*n)))/(d + 5*d*n)
```

3.284.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^4\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^3x^n(4ad+bc)+2c^2dx^{2n}(3ad+2bc)+d^3x^{4n}(ad+4bc)+2cd^2x^{3n}(2ad+3bc)+ac^4+bd^4x^{5n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)
*x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n)
+ (d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n
)
```

3.284.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.284.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^4x+\frac{b\,d^4x\,x^{5n}}{5n+1}+\frac{c^3(4ad+bc)x\,x^n}{1+n}+\frac{d^3(ad+4bc)x\,x^{4n}}{1+4n}+\frac{2c\,d^2(2ad+3bc)x\,x^{3n}}{1+3n}+\frac{2c^2d(3ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^4x+\frac{b\,d^4x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+\frac{c^3(4ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{d^3(ad+4bc)x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{2c\,d^2(2ad+3bc)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{2c^2d(3ad+2bc)}{1+2n}$
parallelrisch	$244x\,x^{4n}bc\,d^3n^3+354x\,x^{2n}a\,c^2d^2n^2+236x\,x^{2n}b\,c^3d\,n^2+164x\,x^{4n}bc\,d^3n^2+a\,c^4x+360x\,x^{2n}a\,c^2d^2n^4+240x\,x^{2n}b\,c^3d\,n^4+196x$

input
```
int((a+b*x^n)*(c+d*x^n)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+b*d^4/(5*n+1)*x*(x^n)^5+c^3*(4*a*d+b*c)/(1+n)*x*x^n+d^3*(a*d+4*b*c
)/(1+4*n)*x*(x^n)^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*(x^n)^3+2*c^2*d*(3*a*d
+2*b*c)/(1+2*n)*x*(x^n)^2
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(132) = 264.

Time = 0.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.99

$$\int (a+bx^n)(c+dx^n)^4\,dx$$
$$=\frac{(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4)xx^{5\,n}+(4\,bcd^3+ad^4+30\,(4\,bcd^3+ad^4)n^4+61\,(4\,bcd^3}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*
n) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d
^4)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n)
 + 2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b
*c^2*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2
*d^2 + 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*
d + 3*a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d +
 3*a*c^2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4
*a*c^3*d + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*
(b*c^4 + 4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5
 + 274*a*c^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(
120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

3.284.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2744 vs. $2(124) = 248$.

Time = 0.73 (sec) , antiderivative size = 2744, normalized size of antiderivative = 20.79

$$\int (a + bx^n) (c + dx^n)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

output
```
Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**
2 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 -
4*b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*
sqrt(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*
sqrt(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*
d**4/(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c*
*2*d**2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/
3)/2 + 12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3)
- 3*b*d**4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3
+ 12*a*c**2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4
*x**(3/4)/3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*lo
g(x) - 4*b*d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) +
10*a*c**2*d**2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c
**4*x**(4/5)/4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c
*d**3*x**(1/5) + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4...
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^4\,dx = ac^4x + \frac{bd^4x^{5n+1}}{5n+1} + \frac{4bcd^3x^{4n+1}}{4n+1} + \frac{ad^4x^{4n+1}}{4n+1} + \frac{6bc^2d^2x^{3n+1}}{3n+1} + \frac{4acd^3x^{3n+1}}{3n+1} + \frac{4bc^3dx^{2n+1}}{2n+1} + \frac{6ac^2d^2x^{2n+1}}{2n+1} + \frac{bc^4x^{n+1}}{n+1} + \frac{4ac^3dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) +
a*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^
3*x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1
)
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(132) = 264.

Time = 0.29 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.61

$$\int (a+bx^n)(c+dx^n)^4\,dx$$
$$= \frac{120\,ac^4n^5x + 24\,bd^4n^4xx^{5\,n} + 120\,bcd^3n^4xx^{4\,n} + 30\,ad^4n^4xx^{4\,n} + 240\,bc^2d^2n^4xx^{3\,n} + 160\,acd^3n^4xx^{3\,n} + 24}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

output
```
(120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30
*a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(
3*n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4
*n^4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5
*n) + 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n
^3*x*x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642
*a*c^2*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 2
25*a*c^4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a
*d^4*n^2*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*
n) + 236*b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^
2*x*x^n + 284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) +
44*b*c*d^3*n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) +
 48*a*c*d^3*n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n
) + 14*b*c^4*n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n)
 + 4*b*c*d^3*x*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d
^3*x*x^(3*n) + 4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n +
 4*a*c^3*d*x*x^n + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n +
 1)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (a+bx^n)(c+dx^n)^4\,dx = a\,c^4\,x + \frac{x\,x^n\,(b\,c^4+4\,a\,d\,c^3)}{n+1} + \frac{x\,x^{4\,n}\,(a\,d^4+4\,b\,c\,d^3)}{4\,n+1} + \frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1} + \frac{2\,c^2\,d\,x\,x^{2\,n}\,(3\,a\,d+2\,b\,c)}{2\,n+1} + \frac{2\,c\,d^2\,x\,x^{3\,n}\,(2\,a\,d+3\,b\,c)}{3\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^4,x)
```

output
```
a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*
d^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d
+ 2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

3.284. $\int (a+bx^n)(c+dx^n)^4 \, dx$

3.285 $\int (a+bx^n)(c+dx^n)^3\,dx$

3.285.1 Optimal result

Integrand size = 17, antiderivative size = 99

$$\int (a+bx^n)(c+dx^n)^3\,dx = ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}$$

output
```
a*c^3*x+c^2*(3*a*d+b*c)*x^(1+n)/(1+n)+3*c*d*(a*d+b*c)*x^(1+2*n)/(1+2*n)+d^
2*(a*d+3*b*c)*x^(1+3*n)/(1+3*n)+b*d^3*x^(1+4*n)/(1+4*n)
```

3.285.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int (a+bx^n)(c+dx^n)^3\,dx = \frac{bx(c+dx^n)^4-(bc-ad(1+4n))x\left(c^3+\frac{3c^2dx^n}{1+n}+\frac{3cd^2x^{2n}}{1+2n}+\frac{d^3x^{3n}}{1+3n}\right)}{d+4dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

output
```
(b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n)
+ (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

3.285.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^3\,dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(c^2x^n(3ad+bc)+d^2x^{3n}(ad+3bc)+3cdx^{2n}(ad+bc)+ac^3+bd^3x^{4n}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1
+ 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(
1 + 4*n))/(1 + 4*n)
```

3.285.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.285.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^{3}x+\frac{b\,d^{3}x\,x^{4n}}{1+4n}+\frac{c^{2}(3ad+bc)x\,x^{n}}{1+n}+\frac{d^{2}(ad+3bc)x\,x^{3n}}{1+3n}+\frac{3cd(ad+bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^{3}x+\frac{b\,d^{3}x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{c^{2}(3ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{d^{2}(ad+3bc)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{3cd(ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{21x\,x^{3n}bc\,d^{2}n+57x\,x^{2n}ac\,d^{2}n^{2}+a\,c^{3}x+9x\,x^{n}b\,c^{3}n+3x\,x^{n}a\,c^{2}d+78x\,x^{n}a\,c^{2}d\,n^{2}+27x\,x^{n}a\,c^{2}dn+24x\,x^{2n}ac\,d^{2}n+24x\,x^{n}b\,c^{3}n^{3}}{}$

input
```
int((a+b*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a*c^3*x+b*d^3/(1+4*n)*x*(x^n)^4+c^2*(3*a*d+b*c)/(1+n)*x*x^n+d^2*(a*d+3*b*c
)/(1+3*n)*x*(x^n)^3+3*c*d*(a*d+b*c)/(1+2*n)*x*(x^n)^2
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(99) = 198.

Time = 0.25 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.22

$$\int (a+bx^n)\,(c+dx^n)^3\;dx$$
$$=\frac{(6\,bd^3n^3+11\,bd^3n^2+6\,bd^3n+bd^3)xx^{4\,n}+(3\,bcd^2+ad^3+8\,(3\,bcd^2+ad^3)n^3+14\,(3\,bcd^2+ad^3)n^2+7\,(3}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
 a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c
*d^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)
*n^3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (
b*c^3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^
2 + 9*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c
^3*n^2 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

3.285.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(90) = 180$.

Time = 0.49 (sec) , antiderivative size = 1540, normalized size of antiderivative = 15.56

$$\int (a + bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
 b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(
n, -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqr
t(x) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/
x, Eq(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) +
 a*d**3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*lo
g(x) - 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) +
6*a*c*d**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*
sqrt(x) + 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n
**4*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 3
5*n**2 + 10*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + a*c**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x
*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 +
50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*
n**2 + 10*n + 1) + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 57*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 +
 10*n + 1) + 24*a*c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1) + 3*a*c*d**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +...
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int (a + bx^n) (c + dx^n)^3 \, dx = ac^3x + \frac{bd^3x^{4n+1}}{4n+1} + \frac{3bcd^2x^{3n+1}}{3n+1} + \frac{ad^3x^{3n+1}}{3n+1} + \frac{3bc^2dx^{2n+1}}{2n+1} + \frac{3acd^2x^{2n+1}}{2n+1} + \frac{bc^3x^{n+1}}{n+1} + \frac{3ac^2dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) +
a*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*
x^(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n +
 1)
```

3.285.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.55

$$\int (a + bx^n)(c + dx^n)^3\,dx = \frac{24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3xx}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d
^3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 2
4*b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x
*x^(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*
n^2*x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d
*n^2*x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n)
 + 7*a*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) +
 9*b*c^3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3
*b*c*d^2*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x
^(2*n) + b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^
2 + 10*n + 1)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(c+dx^n)^3\,dx = a\,c^3\,x + \frac{x\,x^n\,(b\,c^3+3\,a\,d\,c^2)}{n+1} + \frac{x\,x^{3\,n}\,(a\,d^3+3\,b\,c\,d^2)}{3\,n+1} + \frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1} + \frac{3\,c\,d\,x\,x^{2\,n}\,(a\,d+b\,c)}{2\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^3,x)
```

output
```
a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*
d^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*
c))/(2*n + 1)
```

3.286 $\int (a+bx^n)(c+dx^n)^2 \, dx$

3.286.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)(c+dx^n)^2 \, dx = ac^2x + \frac{c(bc+2ad)x^{1+n}}{1+n} + \frac{d(2bc+ad)x^{1+2n}}{1+2n} + \frac{bd^2x^{1+3n}}{1+3n}$$

output
```
a*c^2*x+c*(2*a*d+b*c)*x^(1+n)/(1+n)+d*(a*d+2*b*c)*x^(1+2*n)/(1+2*n)+b*d^2*
x^(1+3*n)/(1+3*n)
```

3.286.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(c+dx^n)^2 \, dx = \frac{bx(c+dx^n)^3 - (bc-ad(1+3n))x\left(c^2 + \frac{2cdx^n}{1+n} + \frac{d^2x^{2n}}{1+2n}\right)}{d+3dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

output
```
(b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) +
(d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

3.286.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^2\,dx$$

$$\Big\downarrow \ 897$$

$$\int \left(dx^{2n}(ad+2bc)+cx^n(2ad+bc)+ac^2+bd^2x^{3n}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*
n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

3.286.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.286.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^2 x+\frac{b\,d^2 x\,x^{3n}}{1+3n}+\frac{c(2ad+bc)x\,x^n}{1+n}+\frac{d(ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^2 x+\frac{b\,d^2 x\,e^{3n\ln(x)}}{1+3n}+\frac{c(2ad+bc)x\,e^{n\ln(x)}}{1+n}+\frac{d(ad+2bc)x\,e^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{2x\,x^{3n} b\,d^2 n^2+3x\,x^{3n} b\,d^2 n+3x\,x^{2n} a\,d^2 n^2+6x\,x^{2n} bcd\,n^2+b\,d^2 x\,x^{3n}+4x\,x^{2n} a\,d^2 n+8x\,x^{2n} bcdn+12x\,x^n acd\,n^2+6x\,x^n b\,c^2 n^2+}{(1+3n)(1+n)(1+2n}$

input
```
int((a+b*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a*c^2*x+b*d^2/(1+3*n)*x*(x^n)^3+c*(2*a*d+b*c)/(1+n)*x*x^n+d*(a*d+2*b*c)/(1
+2*n)*x*(x^n)^2
```

3.286.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(70) = 140$.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)(c+dx^n)^2\,dx = \frac{(2\,bd^2n^2+3\,bd^2n+bd^2)xx^{3\,n}+(2\,bcd+ad^2+3\,(2\,bcd+ad^2)n^2+4\,(2\,bcd+ad^2)n)xx^{2\,n}+(bc^2+2\,acd+}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c
*d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*
(b*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a
*c^2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

3.286.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.36 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a + bx^n)(c + dx^n)^2 \, dx$$

$$= \begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2\cdots}{6n^3+\cdots} \end{cases}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

output
```
Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/
x - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(
x) + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (
a*c**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*
b*c*d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 1
1*n**2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c*
*2*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1
) + 12*a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
 + 3*a*d**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x*
*(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2
+ 6*n + 1) + 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*
n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 +
6*n + 1) + 6*b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*
n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 3*b*d**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(
6*n**3 + 11*n**2 + 6*n + 1), True))
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)(c+dx^n)^2\,dx = ac^2x + \frac{bd^2x^{3n+1}}{3n+1} + \frac{2\,bcdx^{2n+1}}{2n+1} + \frac{ad^2x^{2n+1}}{2n+1} + \frac{bc^2x^{n+1}}{n+1} + \frac{2\,acdx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*
d^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n
 + 1)
```

3.286.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)(c+dx^n)^2\,dx$$
$$= \frac{6\,ac^2n^3x + 2\,bd^2n^2xx^{3n} + 6\,bcdn^2xx^{2n} + 3\,ad^2n^2xx^{2n} + 6\,bc^2n^2xx^n + 12\,acdn^2xx^n + 11\,ac^2n^2x + 3\,bd^2n\ldots}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n
^2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3
*b*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n
*x*x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2
*n) + a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11
*n^2 + 6*n + 1)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)(c+dx^n)^2\,dx = a\,c^2\,x+\frac{x\,x^{2\,n}\,\left(a\,d^2+2\,b\,c\,d\right)}{2\,n+1}+\frac{x\,x^n\,\left(b\,c^2+2\,a\,d\,c\right)}{n+1}+\frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^2,x)
```

output
```
a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*
d))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

3.287 $\int (a+bx^n)(c+dx^n)\,dx$

3.287.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int (a+bx^n)(c+dx^n)\,dx = acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}$$

output
```
a*c*x+(a*d+b*c)*x^(1+n)/(1+n)+b*d*x^(1+2*n)/(1+2*n)
```

3.287.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (a+bx^n)(c+dx^n)\,dx = x\left(ac+\frac{(bc+ad)x^n}{1+n}+\frac{bdx^{2n}}{1+2n}\right)$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n),x]
```

output
```
x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

3.287.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^n(ad+bc)+ac+bdx^{2n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n),x]
```

output
```
a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

3.287.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.287.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
risch	$acx+\frac{(ad+bc)x\,x^{n}}{1+n}+\frac{bdx\,x^{2n}}{1+2n}$	39
norman	$acx+\frac{(ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bdx\,\mathrm{e}^{2n\ln(x)}}{1+2n}$	43
parallelrisch	$\frac{x\,x^{2n}bdn+bdx\,x^{2n}+2x\,x^{n}adn+2x\,x^{n}bcn+2xac\,n^{2}+x\,x^{n}ad+x\,x^{n}bc+3xacn+acx}{(1+n)(1+2n)}$	84

input
```
int((a+b*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+(a*d+b*c)/(1+n)*x*x^n+b*d/(1+2*n)*x*(x^n)^2
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{(bdn+bd)xx^{2n}+(bc+ad+2(bc+ad)n)xx^n+(2acn^2+3acn+ac)x}{2n^2+3n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^
2 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

3.287.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.90

$$\int (a+bx^n)(c+dx^n)\,dx = \begin{cases} acx+ad\log(x)+bc\log(x)-\frac{bd}{x} \\ acx+2ad\sqrt{x}+2bc\sqrt{x}+bd\log(x) \\ \frac{2acn^2x}{2n^2+3n+1}+\frac{3acnx}{2n^2+3n+1}+\frac{acx}{2n^2+3n+1}+\frac{2adnxx^n}{2n^2+3n+1}+\frac{adxx^n}{2n^2+3n+1}+\frac{2bcnxx^n}{2n^2+3n+1}+\frac{bcxx^n}{2n^2+3n+1}+\frac{bdnxx^{2n}}{2n^2+3n+1}+\frac{bdxx^{2n}}{2n^2+3n+1} \end{cases}$$

input
```
integrate((a+b*x**n)*(c+d*x**n),x)
```

output
```
Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2
*a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*
n**2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1)
+ 2*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*
c*n*x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x*
*(2*n)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a+bx^n)(c+dx^n)\,dx = acx + \frac{bdx^{2n+1}}{2n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/
(n + 1)
```

3.287.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{2\,acn^2x + bdnxx^{2\,n} + 2\,bcnxx^n + 2\,adnxx^n + 3\,acnx + bdxx^{2\,n} + bcxx^n + adxx^n + acx}{2\,n^2 + 3\,n + 1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
 + b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (a+bx^n)(c+dx^n)\,dx = a\,c\,x + \frac{x\,x^n\,(a\,d+b\,c)}{n+1} + \frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n),x)
```

output
```
a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

3.288 $\int \frac{a+bx^n}{c+dx^n}\,dx$

3.288.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\frac{bx}{d}-\frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

output
```
b*x/d-(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d
```

3.288.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\frac{x\left(bc+(-bc+ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{cd}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n),x]
```

output
```
(x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n
)/c)]))/(c*d)
```

3.288.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{c+dx^n}\,dx$$

$$\Big\downarrow \ 913$$

$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^n+c}dx}{d}$$

$$\Big\downarrow \ 778$$

$$\frac{bx}{d}-\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

input
```
Int[(a + b*x^n)/(c + d*x^n),x]
```

output
```
(b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n
)/c)])/(c*d)
```

3.288.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.288.4 Maple [F]

$$\int \frac{a+b\,x^{n}}{c+d\,x^{n}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n),x)
```

output
```
int((a+b*x^n)/(c+d*x^n),x)
```

3.288.5 Fricas [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\int \frac{bx^n+a}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d*x^n + c), x)
```

3.288.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{ac^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{bc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)/(c+d*x**n),x)
```

output
```
a*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) - b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n)*x*ler
chphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(c*c**(
1/n)*n**2*gamma(1 + 1/n))
```

3.288.7 Maxima [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\int \frac{bx^n+a}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

3.288.8 Giac [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\int \frac{bx^n+a}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\int \frac{a+b\,x^n}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n),x)
```

output
```
int((a + b*x^n)/(c + d*x^n), x)
```

3.289 $\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$

3.289.1 Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = -\frac{(bc-ad)x}{cdn\,(c+dx^n)} + \frac{(bc-ad(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}$$

output
```
-(-a*d+b*c)*x/c/d/n/(c+d*x^n)+(b*c-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n]
,-d*x^n/c)/c^2/d/n
```

3.289.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \frac{x\left(\frac{b}{c+dx^n} - \frac{(bc+ad(-1+n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

output
```
(x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

3.289.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^2}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(bc-ad(1-n))\int \frac{1}{dx^n+c}dx}{cdn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(bc-ad(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

output
```
-(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

3.289.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.289.4 Maple [F]

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^2,x)
```

3.289.5 Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

3.289.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.15

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = a\left(\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}nx\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnxx^n\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dxx^n\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right)+b\left(\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}n^2x^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}+\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}nx^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}x^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{c^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}dnx^nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{c^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}dx^nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}\right)$$

input
```
integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

3.289. $\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$

output
```
a*(c*c**(1/n)*c**(-2 - 1/n)*n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)
*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) + c*c**(1
/n)*c**(-2 - 1/n)*n*x*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamm
a(1 + 1/n)) - c*c**(1/n)*c**(-2 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c
, 1, 1/n)*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))
+ c**(1/n)*c**(-2 - 1/n)*d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1,
1/n)*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) - c**
(1/n)*c**(-2 - 1/n)*d*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*ga
mma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + b*(c*c**(
-3 - 1/n)*c**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/
n) + d*n**3*x**n*gamma(2 + 1/n)) - c*c**(-3 - 1/n)*c**(1 + 1/n)*n*x**(n +
1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*g
amma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n)) + c*c**(-3 - 1/n)*c**(1 + 1/n)
*n*x**(n + 1)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2
+ 1/n)) - c*c**(-3 - 1/n)*c**(1 + 1/n)*x**(n + 1)*lerchphi(d*x**n*exp_pola
r(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n
*gamma(2 + 1/n)) - c**(-3 - 1/n)*c**(1 + 1/n)*d*n*x**n*x**(n + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/
n) + d*n**3*x**n*gamma(2 + 1/n)) - c**(-3 - 1/n)*c**(1 + 1/n)*d*x**n*x**(n
 + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*...
```

3.289.7 Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*
x/(c*d^2*n*x^n + c^2*d*n)
```

3.289.8 Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^2, x)
```

3.290 $\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$

3.290.1 Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=-\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad(1-2n))x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3dn}$$

output
```
-1/2*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^2+1/2*(b*c-a*d*(1-2*n))*x*hypergeom([2,
1/n],[1+1/n],-d*x^n/c)/c^3/d/n
```

3.290.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\frac{x\left(\frac{b}{(c+dx^n)^2}-\frac{(bc+ad(-1+2n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

output
```
(x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
 1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

3.290.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^3} \, dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(bc-ad(1-2n))\int \frac{1}{(dx^n+c)^2}dx}{2cdn} - \frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(bc-ad(1-2n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hype
rgeometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

3.290.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.290.4 Maple [F]

$$\int \frac{a+b\,x^n}{(c+d\,x^n)^3}dx$$

input
```
int((a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^3,x)
```

3.290.5 Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
x)
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

output
```
Timed out
```

3.290.7 Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3
*d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*
c^2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

3.290.8 Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{a+b\,x^n}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^3, x)
```

3.291 $\int \frac{a+bx^n}{(c+dx^n)^4} \, dx$

3.291.1 Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{a+bx^n}{(c+dx^n)^4} \, dx = -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3} + \frac{(bc-ad(1-3n))x \operatorname{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{3c^4dn}$$

output
```
-1/3*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^3+1/3*(b*c-a*d*(1-3*n))*x*hypergeom([3,
1/n],[1+1/n],-d*x^n/c)/c^4/d/n
```

3.291.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{a+bx^n}{(c+dx^n)^4} \, dx = \frac{x\left(\frac{b}{(c+dx^n)^3} - \frac{(bc+ad(-1+3n)) \operatorname{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

output
```
(x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1),
 1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

3.291.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^4} \, dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(bc-ad(1-3n)) \int \frac{1}{(dx^n+c)^3}dx}{3cdn} - \frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(bc-ad(1-3n)) \operatorname{Hypergeometric2F1}\left(3, \frac{1}{n}, 1+\frac{1}{n}, -\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

output
```
-1/3*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hype
rgeometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

3.291.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.291.4 Maple [F]

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{4}}d x$$

input
```
int((a+b*x^n)/(c+d*x^n)^4,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^4,x)
```

3.291.5 Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) +
4*c^3*d*x^n + c^4), x)
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

output
```
Timed out
```

3.291.7 Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^
3*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(
2*n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*
x^n - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n
^3*x^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

3.291.8 Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^4}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^4,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^4, x)
```

3.292 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

3.292.1 Optimal result

Integrand size = 19, antiderivative size = 158

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{ad^2(2bd+3ae)x^{1+n}}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{1+3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{1+4n}}{1+4n} + \frac{b^2e^3x^{1+5n}}{1+5n}$$

output
```
a^2*d^3*x+a*d^2*(3*a*e+2*b*d)*x^(1+n)/(1+n)+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2
)*x^(1+2*n)/(1+2*n)+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)*x^(1+3*n)/(1+3*n)+b*e^
2*(2*a*e+3*b*d)*x^(1+4*n)/(1+4*n)+b^2*e^3*x^(1+5*n)/(1+5*n)
```

3.292.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = x\left(a^2d^3 + \frac{ad^2(2bd+3ae)x^n}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{4n}}{1+4n} + \frac{b^2e^3x^{5n}}{1+5n}\right)$$

input
```
Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

output
```
x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
 + 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^
(3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^
(5*n))/(1 + 5*n))
```

3.292.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)+ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)+a^2d^3+ad^2x^n(3ae+2bd)+be^2x^{4n}(2ae+3bd)+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

input
```
Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

output
```
a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*
b*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^
2*e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4
*n) + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

3.292.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.292.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
risch	$x\,a^2 d^3+\frac{b^2 e^3 x\,x^{5n}}{5n+1}+\frac{d\left(3a^2e^2+6abde+b^2d^2\right)x\,x^{2n}}{1+2n}+\frac{e\left(a^2e^2+6abde+3b^2d^2\right)x\,x^{3n}}{1+3n}+\frac{a\,d^2(3ae+2bd)x\,x^n}{1+n}+\frac{e^2b(2ae-$
norman	$x\,a^2 d^3+\frac{b^2 e^3 x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+\frac{d\left(3a^2e^2+6abde+b^2d^2\right)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{e\left(a^2e^2+6abde+3b^2d^2\right)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{a\,d^2(3ae+2bd)x\,\mathrm{e}^{n}$
parallelrisch	$\frac{3x\,x^n a^2 d^2 e+2x\,x^n ab\,d^3+33x\,x^{4n} b^2 d\,e^2 n+6x\,x^{2n} ab\,d^2 e+22x\,x^{4n} ab\,e^3 n+82x\,x^{4n} ab\,e^3 n^2+180x\,x^{2n} a^2 d\,e^2 n^4+60x\,x^{4n} ab\,e^3 n^4}{}$

input
```
int((a+b*x^n)^2*(d+e*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
x*a^2*d^3+b^2*e^3/(5*n+1)*x*(x^n)^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/(1+2*n
)*x*(x^n)^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(1+3*n)*x*(x^n)^3+a*d^2*(3*a*e
+2*b*d)/(1+n)*x*x^n+e^2*b*(2*a*e+3*b*d)/(1+4*n)*x*(x^n)^4
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(158) = 316.

Time = 0.26 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.22

$$\int (a+bx^n)^2\,(d+ex^n)^3\;dx = \frac{(24\,b^2e^3n^4+50\,b^2e^3n^3+35\,b^2e^3n^2+10\,b^2e^3n+b^2e^3)xx^{5\,n}+(3\,b^2de^2+2\,abe^3+30\,(3\,b^2de^2+2\,abe^3)n^4+}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

output
```
((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^
3)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4
 + 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 1
1*(3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^
2*e^3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6
*a*b*d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 +
 12*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*
d^2*e + 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(
b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a
^2*d*e^2)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2
*a*b*d^3 + 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^
3 + 3*a^2*d^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 +
3*a^2*d^2*e)*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n
^3 + 85*a^2*d^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*
n^3 + 85*n^2 + 15*n + 1)
```

3.292.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3376 vs. $2(151) = 302$.

Time = 7.41 (sec) , antiderivative size = 3376, normalized size of antiderivative = 21.37

$$\int (a + bx^n)^2 (d + ex^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

output
```
Piecewise((a**2*d**3*x + 3*a**2*d**2*e*log(x) - 3*a**2*d*e**2/x - a**2*e**
3/(2*x**2) + 2*a*b*d**3*log(x) - 6*a*b*d**2*e/x - 3*a*b*d*e**2/x**2 - 2*a*
b*e**3/(3*x**3) - b**2*d**3/x - 3*b**2*d**2*e/(2*x**2) - b**2*d*e**2/x**3
- b**2*e**3/(4*x**4), Eq(n, -1)), (a**2*d**3*x + 6*a**2*d**2*e*sqrt(x) + 3
*a**2*d*e**2*log(x) - 2*a**2*e**3/sqrt(x) + 4*a*b*d**3*sqrt(x) + 6*a*b*d**
2*e*log(x) - 12*a*b*d*e**2/sqrt(x) - 2*a*b*e**3/x + b**2*d**3*log(x) - 6*b
**2*d**2*e/sqrt(x) - 3*b**2*d*e**2/x - 2*b**2*e**3/(3*x**(3/2)), Eq(n, -1/
2)), (a**2*d**3*x + 9*a**2*d**2*e*x**(2/3)/2 + 9*a**2*d*e**2*x**(1/3) + a*
*2*e**3*log(x) + 3*a*b*d**3*x**(2/3) + 18*a*b*d**2*e*x**(1/3) + 6*a*b*d*e*
*2*log(x) - 6*a*b*e**3/x**(1/3) + 3*b**2*d**3*x**(1/3) + 3*b**2*d**2*e*log
(x) - 9*b**2*d*e**2/x**(1/3) - 3*b**2*e**3/(2*x**(2/3)), Eq(n, -1/3)), (a*
*2*d**3*x + 4*a*d**2*x**(3/4)*(3*a*e + 2*b*d)/3 - 4*b**2*e**3/x**(1/4) + 4
*b*e**2*(2*a*e + 3*b*d)*log(x**(1/4)) + 2*d*sqrt(x)*(3*a**2*e**2 + 6*a*b*d
*e + b**2*d**2) + 4*e*x**(1/4)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2), Eq(n
, -1/4)), (a**2*d**3*x + 5*a*d**2*x**(4/5)*(3*a*e + 2*b*d)/4 + 5*b**2*e**3
*log(x**(1/5)) + 5*b*e**2*x**(1/5)*(2*a*e + 3*b*d) + 5*d*x**(3/5)*(3*a**2*
e**2 + 6*a*b*d*e + b**2*d**2)/3 + 5*e*x**(2/5)*(a**2*e**2 + 6*a*b*d*e + 3*
b**2*d**2)/2, Eq(n, -1/5)), (120*a**2*d**3*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a**2*d**3*n**4*x/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 225*a**2*d**3*n**3*x/(120*n**5 + 274...
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.53

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{3b^2de^2x^{4n+1}}{4n+1} + \frac{2abe^3x^{4n+1}}{4n+1} + \frac{3b^2d^2ex^{3n+1}}{3n+1} + \frac{6abde^2x^{3n+1}}{3n+1} + \frac{a^2e^3x^{3n+1}}{3n+1} + \frac{b^2d^3x^{2n+1}}{2n+1} + \frac{6abd^2ex^{2n+1}}{2n+1} + \frac{3a^2de^2x^{2n+1}}{2n+1} + \frac{2abd^3x^{n+1}}{n+1} + \frac{3a^2d^2ex^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

output
```
a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
 1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1)
+ 6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*
d^3*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^
2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n +
 1)/(n + 1)
```

3.292.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(158) = 316$.

Time = 0.30 (sec) , antiderivative size = 962, normalized size of antiderivative = 6.09

$$\int (a+bx^n)^2 (d+ex^n)^3\, dx$$
$$= \frac{120\, a^2d^3n^5x + 24\, b^2e^3n^4xx^{5\,n} + 90\, b^2de^2n^4xx^{4\,n} + 60\, abe^3n^4xx^{4\,n} + 120\, b^2d^2en^4xx^{3\,n} + 240\, abde^2n^4xx^{3\,n} -}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

output
```
(120*a^2*d^3*n^5*x + 24*b^2*e^3*n^4*x*x^(5*n) + 90*b^2*d*e^2*n^4*x*x^(4*n)
 + 60*a*b*e^3*n^4*x*x^(4*n) + 120*b^2*d^2*e*n^4*x*x^(3*n) + 240*a*b*d*e^2*
n^4*x*x^(3*n) + 40*a^2*e^3*n^4*x*x^(3*n) + 60*b^2*d^3*n^4*x*x^(2*n) + 360*
a*b*d^2*e*n^4*x*x^(2*n) + 180*a^2*d*e^2*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*
x^n + 360*a^2*d^2*e*n^4*x*x^n + 274*a^2*d^3*n^4*x + 50*b^2*e^3*n^3*x*x^(5*
n) + 183*b^2*d*e^2*n^3*x*x^(4*n) + 122*a*b*e^3*n^3*x*x^(4*n) + 234*b^2*d^2
*e*n^3*x*x^(3*n) + 468*a*b*d*e^2*n^3*x*x^(3*n) + 78*a^2*e^3*n^3*x*x^(3*n)
+ 107*b^2*d^3*n^3*x*x^(2*n) + 642*a*b*d^2*e*n^3*x*x^(2*n) + 321*a^2*d*e^2*
n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x^n + 462*a^2*d^2*e*n^3*x*x^n + 225*a^2*
d^3*n^3*x + 35*b^2*e^3*n^2*x*x^(5*n) + 123*b^2*d*e^2*n^2*x*x^(4*n) + 82*a*
b*e^3*n^2*x*x^(4*n) + 147*b^2*d^2*e*n^2*x*x^(3*n) + 294*a*b*d*e^2*n^2*x*x^
(3*n) + 49*a^2*e^3*n^2*x*x^(3*n) + 59*b^2*d^3*n^2*x*x^(2*n) + 354*a*b*d^2*
e*n^2*x*x^(2*n) + 177*a^2*d*e^2*n^2*x*x^(2*n) + 142*a*b*d^3*n^2*x*x^n + 21
3*a^2*d^2*e*n^2*x*x^n + 85*a^2*d^3*n^2*x + 10*b^2*e^3*n*x*x^(5*n) + 33*b^2
*d*e^2*n*x*x^(4*n) + 22*a*b*e^3*n*x*x^(4*n) + 36*b^2*d^2*e*n*x*x^(3*n) + 7
2*a*b*d*e^2*n*x*x^(3*n) + 12*a^2*e^3*n*x*x^(3*n) + 13*b^2*d^3*n*x*x^(2*n)
+ 78*a*b*d^2*e*n*x*x^(2*n) + 39*a^2*d*e^2*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n
 + 42*a^2*d^2*e*n*x*x^n + 15*a^2*d^3*n*x + b^2*e^3*x*x^(5*n) + 3*b^2*d*e^2
*x*x^(4*n) + 2*a*b*e^3*x*x^(4*n) + 3*b^2*d^2*e*x*x^(3*n) + 6*a*b*d*e^2*x*x
^(3*n) + a^2*e^3*x*x^(3*n) + b^2*d^3*x*x^(2*n) + 6*a*b*d^2*e*x*x^(2*n) ...
```

3.292.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2 d^3 x + \frac{x x^{2n} (3 a^2 d e^2 + 6 a b d^2 e + b^2 d^3)}{2n+1} + \frac{x x^{3n} (a^2 e^3 + 6 a b d e^2 + 3 b^2 d^2 e)}{3n+1} + \frac{b^2 e^3 x x^{5n}}{5n+1} + \frac{a d^2 x x^n (3 a e + 2 b d)}{n+1} + \frac{b e^2 x x^{4n} (2 a e + 3 b d)}{4n+1}$$

input
```
int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

output
```
a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) +
(x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x
^(5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*
n)*(2*a*e + 3*b*d))/(4*n + 1)
```

3.293 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

3.293.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{2ad(bd+ae)x^{1+n}}{1+n} + \frac{(b^2d^2+4abde+a^2e^2)\, x^{1+2n}}{1+2n} + \frac{2be(bd+ae)x^{1+3n}}{1+3n} + \frac{b^2e^2x^{1+4n}}{1+4n}$$

output
```
a^2*d^2*x+2*a*d*(a*e+b*d)*x^(1+n)/(1+n)+(a^2*e^2+4*a*b*d*e+b^2*d^2)*x^(1+2
*n)/(1+2*n)+2*b*e*(a*e+b*d)*x^(1+3*n)/(1+3*n)+b^2*e^2*x^(1+4*n)/(1+4*n)
```

3.293.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = x\left(a^2d^2 + \frac{2ad(bd+ae)x^n}{1+n} + \frac{(b^2d^2+4abde+a^2e^2)\, x^{2n}}{1+2n} + \frac{2be(bd+ae)x^{3n}}{1+3n} + \frac{b^2e^2x^{4n}}{1+4n}\right)$$

input
```
Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

output
```
x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2
*e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^
2*x^(4*n))/(1 + 4*n))
```

3.293.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(x^{2n}\left(a^2e^2+4abde+b^2d^2\right)+a^2d^2+2bex^{3n}(ae+bd)+2adx^n(ae+bd)+b^2e^2x^{4n}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

output
```
a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e
+ a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3
*n) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

3.293.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.293.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
risch	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,x^{2n}}{1+2n}+\frac{b^2e^2x\,x^{4n}}{1+4n}+\frac{2ad(ae+bd)x\,x^{n}}{1+n}+\frac{2be(ae+bd)x\,x^{3n}}{1+3n}$
norman	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2e^2x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{2ad(ae+bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{2be(ae+bd)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{28x\,x^{3n}b^2de\,n^2+14x\,x^{3n}ab\,e^2n+52x\,x^{n}a^2de\,n^2+35x\,a^2d^2n^2+10x\,a^2d^2n+b^2e^2x\,x^{4n}+18x\,x^{n}a^2den+4x\,x^{2n}abde+48x\,x^{n}a^2de}{}$

input
```
int((a+b*x^n)^2*(d+e*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*(x^n)^2+b^2*e^2/(1+4*n)*x*
(x^n)^4+2*a*d*(a*e+b*d)/(1+n)*x*x^n+2*b*e*(a*e+b*d)/(1+3*n)*x*(x^n)^3
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(112) = 224.

Time = 0.25 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.30

$$\int (a+bx^n)^2\,(d+ex^n)^2\;dx$$
$$=\frac{(6\,b^2e^2n^3+11\,b^2e^2n^2+6\,b^2e^2n+b^2e^2)xx^{4\,n}+2\,(b^2de+abe^2+8\,(b^2de+abe^2)n^3+14\,(b^2de+abe^2)n^2+}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b
^2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2
+ 7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12
*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*
n^2 + 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*
e + 24*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 +
 a^2*d*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 1
0*a^2*d^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

3.293.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. 2(104) = 208.

Time = 2.96 (sec) , antiderivative size = 1760, normalized size of antiderivative = 15.71

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(
x) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2
/(3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x
) + 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2
*log(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3
*a**2*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d
*e*x**(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x)
 - 3*b**2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e
+ b*d)/3 + 4*b**2*e**2*log(x**(1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - 2*sqrt
(x)*(-a**2*e**2 - 4*a*b*d*e - b**2*d**2), Eq(n, -1/4)), (24*a**2*d**2*n**4
*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d
*e*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**
3 + 35*n**2 + 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 3...
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{b^2e^2x^{4n+1}}{4n+1} + \frac{2b^2dex^{3n+1}}{3n+1} + \frac{2abe^2x^{3n+1}}{3n+1} + \frac{b^2d^2x^{2n+1}}{2n+1} + \frac{4abdex^{2n+1}}{2n+1} + \frac{a^2e^2x^{2n+1}}{2n+1} + \frac{2abd^2x^{n+1}}{n+1} + \frac{2a^2dex^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1
) + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*
b*d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^
(n + 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.81

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$
$$= \frac{24\,a^2d^2n^4x + 6\,b^2e^2n^3xx^{4\,n} + 16\,b^2den^3xx^{3\,n} + 16\,abe^2n^3xx^{3\,n} + 12\,b^2d^2n^3xx^{2\,n} + 48\,abden^3xx^{2\,n} + 12\,a^2e}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

output
```
(24*a^2*d^2*n^4*x + 6*b^2*e^2*n^3*x*x^(4*n) + 16*b^2*d*e*n^3*x*x^(3*n) + 1
6*a*b*e^2*n^3*x*x^(3*n) + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d*e*n^3*x*x^(2
*n) + 12*a^2*e^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 48*a^2*d*e*n^3*x*x
^n + 50*a^2*d^2*n^3*x + 11*b^2*e^2*n^2*x*x^(4*n) + 28*b^2*d*e*n^2*x*x^(3*n
) + 28*a*b*e^2*n^2*x*x^(3*n) + 19*b^2*d^2*n^2*x*x^(2*n) + 76*a*b*d*e*n^2*x
*x^(2*n) + 19*a^2*e^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 52*a^2*d*e*n^
2*x*x^n + 35*a^2*d^2*n^2*x + 6*b^2*e^2*n*x*x^(4*n) + 14*b^2*d*e*n*x*x^(3*n
) + 14*a*b*e^2*n*x*x^(3*n) + 8*b^2*d^2*n*x*x^(2*n) + 32*a*b*d*e*n*x*x^(2*n
) + 8*a^2*e^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 18*a^2*d*e*n*x*x^n + 10*a
^2*d^2*n*x + b^2*e^2*x*x^(4*n) + 2*b^2*d*e*x*x^(3*n) + 2*a*b*e^2*x*x^(3*n)
 + b^2*d^2*x*x^(2*n) + 4*a*b*d*e*x*x^(2*n) + a^2*e^2*x*x^(2*n) + 2*a*b*d^2
*x*x^n + 2*a^2*d*e*x*x^n + a^2*d^2*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1
)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2 d^2 x + \frac{x x^{2n} (a^2 e^2 + 4 a b d e + b^2 d^2)}{2n+1} + \frac{b^2 e^2 x x^{4n}}{4n+1} + \frac{2 b e x x^{3n} (a e + b d)}{3n+1} + \frac{2 a d x x^n (a e + b d)}{n+1}$$

input
```
int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

output
```
a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e
^2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d
*x*x^n*(a*e + b*d))/(n + 1)
```

3.294 $\int (a+bx^n)^2 (c+dx^n)\,dx$

3.294.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)^2 (c+dx^n)\,dx = a^2cx + \frac{a(2bc+ad)x^{1+n}}{1+n} + \frac{b(bc+2ad)x^{1+2n}}{1+2n} + \frac{b^2dx^{1+3n}}{1+3n}$$

output
```
a^2*c*x+a*(a*d+2*b*c)*x^(1+n)/(1+n)+b*(2*a*d+b*c)*x^(1+2*n)/(1+2*n)+b^2*d*
x^(1+3*n)/(1+3*n)
```

3.294.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2 (c+dx^n)\,dx = \frac{dx(a+bx^n)^3 - (ad-b(c+3cn))x\left(a^2+\frac{2abx^n}{1+n}+\frac{b^2x^{2n}}{1+2n}\right)}{b+3bn}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

output
```
(d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) +
(b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

3.294.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n) \, dx$$

$$\Big\downarrow \ 897$$

$$\int \left(a^2c+bx^{2n}(2ad+bc)+ax^n(ad+2bc)+b^2dx^{3n}\right) dx$$

$$\Big\downarrow \ 2009$$

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*
n))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

3.294.3.1 Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.294.4 Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a^2cx+\frac{a(ad+2bc)x\,x^n}{1+n}+\frac{b(2ad+bc)x\,x^{2n}}{1+2n}+\frac{b^2dx\,x^{3n}}{1+3n}$
norman	$a^2cx+\frac{a(ad+2bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{b(2ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2dx\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{2x\,x^{3n}b^2d\,n^2+3x\,x^{3n}b^2dn+6x\,x^{2n}abd\,n^2+3x\,x^{2n}b^2c\,n^2+b^2dx\,x^{3n}+8x\,x^{2n}abdn+4x\,x^{2n}b^2cn+6x\,x^na^2d\,n^2+12x\,x^nabc\,n^2+6x}{(1+n)(1+2n)(1+3n)}$

input
```
int((a+b*x^n)^2*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*x^n+b*(2*a*d+b*c)/(1+2*n)*x*(x^n)^2+b^2*d/(1
+3*n)*x*(x^n)^3
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)^2\,(c+dx^n)\;dx$$
$$=\frac{(2\,b^2dn^2+3\,b^2dn+b^2d)xx^{3\,n}+(b^2c+2\,abd+3\,(b^2c+2\,abd)n^2+4\,(b^2c+2\,abd)n)xx^{2\,n}+(2\,abc+a^2d+}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

output
```
((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
 + 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*
(2*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a
^2*c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

3.294.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a+bx^n)^2 (c+dx^n)\,dx$$
$$= \begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a}{6n^3+} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n),x)
```

output
```
Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/
x - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sq
rt(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (
a**2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b
**2*c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 1
1*n**2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2
*c*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1
) + 6*a**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
+ 12*a*b*c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6
*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) +
 6*a*b*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*
n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
*n + 1) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c
*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 3*b**2*d*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(
6*n**3 + 11*n**2 + 6*n + 1), True))
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)^2 (c+dx^n)\ dx = a^2cx + \frac{b^2dx^{3n+1}}{3n+1} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2\,abdx^{2n+1}}{2n+1} + \frac{2\,abcx^{n+1}}{n+1} + \frac{a^2dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*
b*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n
 + 1)
```

3.294.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)^2 (c+dx^n)\ dx$$
$$= \frac{6\,a^2cn^3x + 2\,b^2dn^2xx^{3n} + 3\,b^2cn^2xx^{2n} + 6\,abdn^2xx^{2n} + 12\,abcn^2xx^n + 6\,a^2dn^2xx^n + 11\,a^2cn^2x + 3\,b^2dnx}{}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

output
```
(6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n
^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3
*b^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*
n*x*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n
) + 2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11
*n^2 + 6*n + 1)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^2\,(c+dx^n)\ dx = a^2\,c\,x + \frac{x\,x^{2\,n}\,(c\,b^2+2\,a\,d\,b)}{2\,n+1} + \frac{x\,x^n\,(d\,a^2+2\,b\,c\,a)}{n+1} + \frac{b^2\,d\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((a + b*x^n)^2*(c + d*x^n),x)
```

output
```
a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*
c))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

3.295 $\int \frac{(a+bx^n)^2}{c+dx^n} dx$

3.295.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(a+bx^n)^2}{c+dx^n} dx = -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2 x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

output
```
-b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*
x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d^2
```

3.295.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2}{c+dx^n} dx = \frac{a^2x}{c} - \frac{(bc-ad)^2x}{cd^2} + \frac{b^2x^{1+n}}{d(1+n)} + \frac{(-bc+ad)^2 x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

output
```
(a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(
b*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c
*d^2)
```

3.295.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {933, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\frac{b(bc(n+1)-ad(2n+1))x^n+a(bc-ad(n+1))}{dx^n+c}dx}{d(n+1)}+\frac{bx(a+bx^n)}{d(n+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\int \frac{b(bc(n+1)-ad(2n+1))x^n+a(bc-ad(n+1))}{dx^n+c}dx}{d(n+1)}$$

$$\Big\downarrow \text{913}$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\frac{bx(bc(n+1)-ad(2n+1))}{d}-\frac{(n+1)(bc-ad)^2\int \frac{1}{dx^n+c}dx}{d}}{d(n+1)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\frac{bx(bc(n+1)-ad(2n+1))}{d}-\frac{(n+1)x(bc-ad)^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}}{d(n+1)}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n),x]
```

output
```
(b*x*(a + b*x^n))/(d*(1 + n)) - ((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/d - (
(b*c - a*d)^2*(1 + n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d))/(d*(1 + n))
```

3.295.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.295.4 Maple [F]

$$\int \frac{(a+b\,x^n)^2}{c+d\,x^n}dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n),x)
```

3.295.5 Fricas [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

3.295.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{a^2c^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{2abc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{2b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n),x)
```

output
```
a**2*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) - 2*a*b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n
)*x*lerchphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/
(c*c**(1/n)*n**2*gamma(1 + 1/n)) + 2*b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2
*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*g
amma(3 + 1/n)) + b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2*n + 1)*lerchphi(d*x
**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n))
```

3.295.7 Maxima [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d
*x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

3.295.8 Giac [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n), x)
```

3.296 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$

3.296.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = -\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{(bc-ad)(ad(1-n)-bc(1+n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2d^2n}$$

output
```
-b*(a*d-b*c*(1+n))*x/c/d^2/n-(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)+(-a*d+
b*c)*(a*d*(1-n)-b*c*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^2/d^2/
n
```

3.296.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \frac{x\left(\frac{c(-2abcd+a^2d^2+b^2c(c+cn+dnx^n))}{c+dx^n}-(bc-ad)(ad(-1+n)+bc(1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

output
```
(x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (
b*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

3.296.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {930, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(n+1))x^n}{dx^n+c}dx}{cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{cdn\left(c+dx^n\right)}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(bc-ad)(ad(1-n)-bc(n+1))\int \frac{1}{dx^n+c}dx}{d}-\frac{bx(ad-bc(n+1))}{d}}{cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{cdn\left(c+dx^n\right)}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}-\frac{bx(ad-bc(n+1))}{d}}{cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{cdn\left(c+dx^n\right)}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

output
```
-(((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n))) + (-((b*(a*d - b*c*(1 +
 n))*x)/d) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d))/(c*d*n)
```

3.296.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.296.4 Maple [F]

$$\int \frac{(a+b\,x^{n})^{2}}{(c+d\,x^{n})^{2}}dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

3.296.5 Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
x)
```

3.296.6 Sympy [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

output
```
Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

3.296.7 Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
-(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n
+ c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d
^2)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

3.296.8 Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^2, x)
```

3.297 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx$

3.297.1 Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx = -\frac{(bc-ad)x(a+bx^n)}{2cdn(c+dx^n)^2} + \frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2(c+dx^n)} - \frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2n^2))\, x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3d^2n^2}$$

output
```
-1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2+1/2*(-a*d+b*c)*(a*d*(1-2*n)-
b*c*(1+n))*x/c^2/d^2/n^2/(c+d*x^n)-1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*
d^2*(2*n^2-3*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/d^2/n^2
```

3.297.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx = \frac{x\left(\frac{c^2(bc-ad)^2n}{(c+dx^n)^2} - \frac{c(bc-ad)(ad(-1+2n)+b(c+2cn))}{c+dx^n} + (2abcd(-1+n)+b^2c^2(1+n)+a^2d^2(1-3n+2n^2))\,\text{Hyperge}\right.}{2c^3d^2n^2}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

output
```
(x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
 b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2
*d^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(2*c^3*d^2*n^2)
```

3.297.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {930, 910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(n+1))x^n}{(dx^n+c)^2}dx}{2cdn}-\frac{x(bc-ad)(a+bx^n)}{2cdn(c+dx^n)^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{cdn(c+dx^n)}-\frac{(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1))\int \frac{1}{dx^n+c}dx}{cdn}}{2cdn}-\frac{x(bc-ad)(a+bx^n)}{2cdn(c+dx^n)^2}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{cdn(c+dx^n)}-\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1))\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}}{2cdn}-\frac{x(bc-ad)(a+bx^n)}{2cdn(c+dx^n)^2}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

output
```
-1/2*((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n)^2) + (((b*c - a*d)*(a*
d*(1 - 2*n) - b*c*(1 + n))*x)/(c*d*n*(c + d*x^n)) - ((2*a*b*c*d*(1 - n) -
b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n))/(2*c*d*n)
```

3.297.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.297.4 Maple [F]

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

3.297.5 Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) +
3*c^2*d*x^n + c^3), x)
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

output
```
Timed out
```

3.297.7 Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrat
e(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^
2*d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n +
 1) - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c
^4*d^2*n^2)
```

3.297.8 Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx=\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^3, x)
```

3.298 $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

3.298.1 Optimal result

Integrand size = 19, antiderivative size = 310

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = -\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+24n^3)+ab^2c^2}{b^4(1+n)(1+2n)(1+3n)}$$
$$-\frac{d(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))\,x(c+dx^n)}{b^3(1+n)(1+2n)(1+3n)}$$
$$-\frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2\,(1+5n+6n^2)}+\frac{dx(c+dx^n)^3}{b(1+3n)}$$
$$+\frac{(bc-ad)^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^4}$$

output
```
-d*(a^3*d^3*(6*n^3+11*n^2+6*n+1)-b^3*c^3*(24*n^3+18*n^2+7*n+1)-a^2*b*c*d^2
*(24*n^3+38*n^2+19*n+3)+a*b^2*c^2*d*(36*n^3+45*n^2+20*n+3))*x/b^4/(6*n^3+1
1*n^2+6*n+1)-d*(2*a*b*c*d*(1+3*n)^2-a^2*d^2*(6*n^2+5*n+1)-b^2*c^2*(18*n^2+
7*n+1))*x*(c+d*x^n)/b^3/(6*n^3+11*n^2+6*n+1)-d*(a*d*(1+3*n)-b*(6*c*n+c))*x
*(c+d*x^n)^2/b^2/(6*n^2+5*n+1)+d*x*(c+d*x^n)^3/b/(1+3*n)+(-a*d+b*c)^4*x*hy
pergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^4
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{x\left(4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+d^4x^{4n}\Phi\left(-\frac{bx^n}{a},1,\right.\right.}{an}$$

input
```
Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

output
```
(x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x
^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurw
itzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((
b*x^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))
/(a*n)
```

3.298.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {933, 25, 1025, 25, 1025, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\frac{(dx^n+c)^2(d(ad(3n+1)-b(6nc+c))x^n+c(ad-b(3nc+c)))}{bx^n+a}dx}{b(3n+1)}+\frac{dx(c+dx^n)^3}{b(3n+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)}-\frac{\int \frac{(dx^n+c)^2(d(ad(3n+1)-b(6nc+c))x^n+c(ad-b(3nc+c)))}{bx^n+a}dx}{b(3n+1)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{\frac{\int -\frac{(dx^n+c)\left(c\left(b^2\left(6n^2+5n+1\right)c^2-2abd(4n+1)c+a^2d^2(3n+1)\right)-d\left(-b^2\left(18n^2+7n+1\right)c^2+2abd(3n+1)^2c-a^2d^2\left(6n^2+5n+1\right)\right)x^n\right)}{bx^n+a}dx}{b(2n+1)} + \frac{dx(c+dx^n)^2(ad(3n\cdots}{b(2n+\cdots}}{b(3n+1)}$$

$\downarrow$ 25

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{\int \frac{(dx^n+c)\left(c\left(b^2\left(6n^2+5n+1\right)c^2-2abd(4n+1)c+a^2d^2(3n+1)\right)-d\left(-b^2\left(18n^2+7n+1\right)c^2+2abd(3n+1)^2c-a^2d^2\left(6n^2+5\cdots\right.\right.\right.}{bx^n+a}}{b(2n+1)}}{b(3n+1)}$$

$\downarrow$ 1025

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{\frac{\int -\frac{d\left(-b^3\left(24n^3+18n^2+7n+1\right)c^3+ab^2d\left(36n^3+45n^2+20n+3\right)c^2-a^2bd^2\left(24n^3+38n^2+19n+3\right)c+a^3d^3\left(6n^3+11n^2+6\cdots\right.\right.}{bx^n\cdots}}{b(n+\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\frac{\int \frac{d\left(-b^3\left(24n^3+18n^2+7n+1\right)c^3+ab^2d\left(36n^3+45n^2+20n+3\right)c^2-a^2bd^2\left(24n^3+38n^2+19n+3\right)c+a^3d^3\left(6n^3+11n^2+6\cdots\right.\right.}{bx^n\cdots}}{b(n\cdots}}{\cdots}$$

$\downarrow$ 913

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\frac{\frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(24n^3+38n^2+19n+3\right)+ab^2c^2d\left(36n^3+45n^2+20n+3\right)-b^3c^3\left(24n^3+18n^2+7n\cdots\right.\right.}{b}}{b(n+1)}}{b(\cdots}}{b(3n+1)}$$

$\downarrow$ 778

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\frac{dx(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b(n+1)} - \frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(24\cdots\right.\right.}{\cdots}}{\cdots}}{b(3n+1\cdots}$$

3.298. $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

input
```
Int[(c + d*x^n)^4/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n)^3)/(b*(1 + 3*n)) - ((d*(a*d*(1 + 3*n) - b*(c + 6*c*n))*x*
(c + d*x^n)^2)/(b*(1 + 2*n)) - (-((d*(2*a*b*c*d*(1 + 3*n)^2 - a^2*d^2*(1 +
 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x^n))/(b*(1 + n))) -
((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 4
5*n^2 + 36*n^3))*x)/b - ((b*c - a*d)^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*Hyper
geometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(b*(1 + n)))/(b*
(1 + 2*n)))/(b*(1 + 3*n))
```

3.298.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.298.4 Maple [F]

$$\int \frac{(c+d\,x^{n})^{4}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^n)^4/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^4/(a+b*x^n),x)
```

3.298.5 Fricas [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n
+ c^4)/(b*x^n + a), x)
```

3.298.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{4a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n\Gamma\left(5+\frac{1}{n}\right)} + \frac{a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n^2\Gamma\left(5+\frac{1}{n}\right)} + \frac{12a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)} + \frac{4a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)} + \frac{12a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} - \frac{4a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**4/(a+b*x**n),x)
```

3.298. $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

output
```
a**(1/n)*a**(-1 - 1/n)*c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 4*a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n*ga
mma(5 + 1/n)) + a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n**2*gamma(5 + 1/n)) + 1
2*a**(-4 - 1/n)*a**(3 + 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*gamma(4 + 1/n)) + 4*a**(-4 - 1/n)*
a**(3 + 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 +
 1/n)*gamma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 12*a**(-3 - 1/n)*a**(2 + 1/n)
*c**2*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gam
ma(2 + 1/n)/(n*gamma(3 + 1/n)) + 6*a**(-3 - 1/n)*a**(2 + 1/n)*c**2*d**2*x*
*(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(
n**2*gamma(3 + 1/n)) - 4*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c**3*d*x*lerc
hphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1
/n)*n**2*gamma(1 + 1/n))
```

3.298.7 Maxima [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx=\int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*in
tegrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (
4*(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (
6*(6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2
 + 5*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6
*(6*n^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a
^2*b*c*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n
 + 1)*b^4)
```

3.298.8 Giac [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^4/(b*x^n + a), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^4}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^4/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^4/(a + b*x^n), x)
```

3.299 $\int \frac{(c+dx^n)^3}{a+bx^n}\,dx$

3.299.1 Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))\,x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{(bc-ad)^3x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^3}$$

output
```
d*(a^2*d^2*(2*n^2+3*n+1)+b^2*c^2*(6*n^2+4*n+1)-a*b*c*d*(6*n^2+7*n+2))*x/b^
3/(2*n^2+3*n+1)-d*(a*d*(1+2*n)-b*(4*c*n+c))*x*(c+d*x^n)/b^2/(2*n^2+3*n+1)+
d*x*(c+d*x^n)^2/b/(1+2*n)+(-a*d+b*c)^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n
/a)/a/b^3
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{x\left(3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

input
```
Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

output
```
(x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(
2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLer
chPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1,
n^(-1)]))/(a*n)
```

3.299.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {933, 25, 1025, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^3}{a+bx^n} \, dx$$

$$\Big\downarrow \textbf{933}$$

$$\frac{\int -\frac{(dx^n+c)(d(ad(2n+1)-b(4nc+c))x^n+c(ad-b(2nc+c)))}{bx^n+a}dx}{b(2n+1)}+\frac{dx(c+dx^n)^2}{b(2n+1)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\int \frac{(dx^n+c)(d(ad(2n+1)-b(4nc+c))x^n+c(ad-b(2nc+c)))}{bx^n+a}dx}{b(2n+1)}$$

$$\Big\downarrow \textbf{1025}$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{\int -\frac{d\left(b^2\left(6n^2+4n+1\right)c^2-abd\left(6n^2+7n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n+c\left(b^2\left(2n^2+3n+1\right)c^2-abd(5n+2)c+a^2d^2(2n+1)\right)}{bx^n+a}dx}{b(n+1)}+\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)}-\frac{\int \frac{d\left(b^2\left(6n^2+4n+1\right)c^2-abd\left(6n^2+7n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n+c\left(b^2\left(2n^2+3n+1\right)c^2-abd(5n+2)c+a^2d^2(2n+1)\right)}{bx^n+a}d}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \text{913}$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)} - \frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)} - \frac{\frac{(n+1)(2n+1)(bc-ad)^3\int\frac{1}{bx^n+a}dx}{b} + \frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b}}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \text{778}$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)} - \frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)} - \frac{\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b} + \frac{(n+1)(2n+1)x(bc-ad)^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\ldots\right.}{ab}}{b(n+1)}}{b(2n+1)}$$

input
```
Int[(c + d*x^n)^3/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n)^2)/(b*(1 + 2*n)) - ((d*(a*d*(1 + 2*n) - b*(c + 4*c*n))*x*
(c + d*x^n))/(b*(1 + n)) - ((d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4
*n + 6*n^2) - a*b*c*d*(2 + 7*n + 6*n^2))*x)/b + ((b*c - a*d)^3*(1 + n)*(1
+ 2*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(b
*(1 + n)))/(b*(1 + 2*n))
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.299.4 Maple [F]

$$\int \frac{(c+d\,x^n)^3}{a+b\,x^n}dx$$

input
```
int((c+d*x^n)^3/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^3/(a+b*x^n),x)
```

3.299.5 Fricas [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx=\int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a),
x)
```

3.299.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)} + \frac{a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)} + \frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{3a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} - \frac{3a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**3/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 3*a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*ga
mma(4 + 1/n)) + a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 6
*a**(-3 - 1/n)*a**(2 + 1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*gamma(3 + 1/n)) + 3*a**(-3 - 1/n)*a
**(2 + 1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 +
1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 3*a**(1 + 1/n)*b**(1/n)*b**(-1
 - 1/n)*c**2*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n
)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

3.299.7 Maxima [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
 a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^
3*(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*
a*b*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

3.299.8 Giac [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^3/(b*x^n + a), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^3}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^3/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^3/(a + b*x^n), x)
```

3.300 $\int \frac{(c+dx^n)^2}{a+bx^n} dx$

3.300.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)} + \frac{dx(c+dx^n)}{b(1+n)} + \frac{(bc-ad)^2 x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2}$$

output
```
-d*(a*d*(1+n)-b*(2*c*n+c))*x/b^2/(1+n)+d*x*(c+d*x^n)/b/(1+n)+(-a*d+b*c)^2*
x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^2
```

3.300.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = \frac{x\left(2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

output
```
(x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*H
urwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n
)/a), 1, n^(-1)]))/(a*n)
```

3.300.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {933, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{a+bx^n} \, dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{d(ad(n+1)-b(2nc+c))x^n+c(ad-bc(n+1))}{bx^n+a}dx}{b(n+1)}+\frac{dx(c+dx^n)}{b(n+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\int \frac{d(ad(n+1)-b(2nc+c))x^n+c(ad-bc(n+1))}{bx^n+a}dx}{b(n+1)}$$

$$\Big\downarrow \; 913$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\frac{dx(ad(n+1)-b(2cn+c))}{b}-\frac{(n+1)(bc-ad)^2\int \frac{1}{bx^n+a}dx}{b}}{b(n+1)}$$

$$\Big\downarrow \; 778$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\frac{dx(ad(n+1)-b(2cn+c))}{b}-\frac{(n+1)x(bc-ad)^2 \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}}{b(n+1)}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n))/(b*(1 + n)) - ((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/b - (
(b*c - a*d)^2*(1 + n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b))/(b*(1 + n))
```

3.300.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.300.4 Maple [F]

$$\int \frac{(c+d\,x^n)^2}{a+b\,x^n}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n),x)
```

3.300.5 Fricas [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

3.300.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\begin{aligned}\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = &\frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} \\ &+\frac{2a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} \\ &+\frac{a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} \\ &-\frac{2a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 2*a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*ga
mma(3 + 1/n)) + a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 2
*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c*d*x*lerchphi(a*exp_polar(I*pi)/(b*x
**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

3.300.7 Maxima [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2
*x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))
```

3.300.8 Giac [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^2}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n), x)
```

3.301 $\int \frac{c+dx^n}{a+bx^n}\,dx$

3.301.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\frac{dx}{b}+\frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

output
```
d*x/b+(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

3.301.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\frac{x\left(ad+(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{ab}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n),x]
```

output
```
(x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a
)]))/(a*b)
```

3.301.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{a+bx^n}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{bx^n+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

input
```
Int[(c + d*x^n)/(a + b*x^n),x]
```

output
```
(d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a*b)
```

3.301.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.301.4 Maple [F]

$$\int \frac{c+d\,x^n}{a+b\,x^n}dx$$

input
```
int((c+d*x^n)/(a+b*x^n),x)
```

output
```
int((c+d*x^n)/(a+b*x^n),x)
```

3.301.5 Fricas [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/(b*x^n + a), x)
```

3.301.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.62

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) - a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*d*x*ler
chphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(
1/n)*n**2*gamma(1 + 1/n))
```

3.301.7 Maxima [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

3.301.8 Giac [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{c+d\,x^n}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n),x)
```

output
```
int((c + d*x^n)/(a + b*x^n), x)
```

3.302 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

3.302.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

output
```
b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/
n],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

3.302.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{x\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d)
)
```

3.302.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {917, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{ 917}$$

$$\frac{b\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d
)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c
- a*d))
```

3.302.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

3.302.4 Maple [F]

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

3.302.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

3.302.6 Sympy [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

3.302.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.302.8 Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.303 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx$

3.303.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx = -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2 x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{d(bc(1-2n)-ad(1-n))x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^2 n}$$

output
```
-d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a
/(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^
n/c)/c^2/(-a*d+b*c)^2/n
```

3.303.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx = \frac{x\left(b^2c^2n(c+dx^n) \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right) + ad\left(c(-bc+ad) + (ad(-1+n)+b(c-2cn))\right.\right.}{ac^2(bc-ad)^2 n(c+dx^n)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
(x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^
n)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c -
a*d)^2*n*(c + d*x^n))
```

3.303.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{bd(1-n)x^n+bcn+a(d-dn)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{b^2cn\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d(ad(1-n)-b(c-2cn))\int \frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{b^2cnx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx(ad(1-n)-b(c-2cn))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + ((b^2*c*n*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)) - (d*(a*d*(1 - n) - b*(
c - 2*c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(c*(b*c - a*d)*n)
```

3.303.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.303.4 Maple [F]

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})^{2}}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

3.303.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2
*a*c*d)*x^n), x)
```

3.303.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.303.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b
^2*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n
+ a^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*
n)*x^n)
```

3.303.8 Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)\,\left(c+d\,x^n\right)^2}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^2), x)
```

3.304 $\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$

3.304.1 Optimal result

Integrand size = 19, antiderivative size = 210

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx = -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} - \frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^3} - \frac{d(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\right.}{2c^3(bc-ad)^3n^2}$$

output
```
-1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2
/(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(
-a*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(
6*n^2-5*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/n^2
```

3.304.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx = \frac{x\left(-ac^2d(bc-ad)^2n+acd(bc-ad)(ad(-1+2n)+b(c-4cn))\left(c+dx^n\right)+2b^3c^3n^2(c+dx^n)^2\,\text{Hypergeo}\right.}{}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

output
```
(x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c
- 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*
c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2
*(c + d*x^n)^2)
```

3.304.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {931, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} \, dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{bd(1-2n)x^n+2bcn+a(d-2dn)}{(bx^n+a)(dx^n+c)^2}dx}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)(c+dx^n)^2}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\int \frac{-bd(bc(1-4n)-ad(1-2n))(1-n)x^n+2b^2c^2n^2+a^2d^2\left(2n^2-3n+1\right)-abcd\left(4n^2-5n+1\right)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{dx(bc(1-4n)-ad(1-2n))}{cn(bc-ad)(c+dx^n)}}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)(c+dx^n)^2}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{\frac{2b^3c^2n^2\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\int \frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}+\frac{dx(bc(1-4n)-ad(1-2n))}{cn(bc-ad)(c+dx^n)}}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)(c+dx^n)^2}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{\frac{2b^3c^2n^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}+\frac{d}{}}{2cn(bc-ad)}$$

$$\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*n*(c + d*x^n)^2) + ((d*(b*c*(1 - 4*n) - a*d*(1 -
 2*n))*x)/(c*(b*c - a*d)*n*(c + d*x^n)) + ((2*b^3*c^2*n^2*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)) - (d*(a^2*d^2*(1
 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))
*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c - a*d))
)/(c*(b*c - a*d)*n))/(2*c*(b*c - a*d)*n)
```

3.304.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.304.4 Maple [F]

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})^{3}}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

3.304.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^3}\,dx=\int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2
*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

3.304.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^3}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.304.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
-b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
 (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^
2 - 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)
*a^2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^
2*n^2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c
^3*d^3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(
2*n - 1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2
- 2*a*b*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2
 + a^2*c^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c
^3*d^3*n^2)*x^n)
```

3.304.8 Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^3}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^3),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^3), x)
```

3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$

3.305.1 Optimal result

Integrand size = 19, antiderivative size = 341

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+a^2bcd^2(3+15n}{ab^4n(1+n)(1+2n)}$$
$$-\frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)}$$
$$+\frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)}+\frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)}$$
$$-\frac{(bc-ad)^3(bc(1-n)-ad(1+3n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^4n}$$

output
```
-d*(b^3*c^3*(2*n^2+3*n+1)-a^3*d^3*(6*n^3+11*n^2+6*n+1)-a*b^2*c^2*d*(12*n^3
+17*n^2+12*n+3)+a^2*b*c*d^2*(16*n^3+26*n^2+15*n+3))*x/a/b^4/n/(2*n^2+3*n+1
)-d*(b^2*c^2*(2*n^2+3*n+1)-2*a*b*c*d*(5*n^2+4*n+1)+a^2*d^2*(6*n^2+5*n+1))*
x*(c+d*x^n)/a/b^3/n/(2*n^2+3*n+1)-d*(-3*a*d*n+2*b*c*n-a*d+b*c)*x*(c+d*x^n)
^2/a/b^2/n/(1+2*n)+(-a*d+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(
b*c*(1-n)-a*d*(1+3*n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^4/n
```

3.305.2 Mathematica [A] (verified)

Time = 6.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$
$$= \frac{x\left(\frac{4ab^3c^3d-6a^2b^2c^2d^2+4a^3bcd^3-a^4d^4+b^4c^4(-1+n)}{a^2n}+\frac{(-bc+ad)^3(bc(-1+n)+ad(1+3n))}{a^2n}+\frac{2bd^3(2bc-ad)x^n}{1+n}+\frac{b^2d^4x^{2n}}{1+2n}+\frac{(bc-ad)^4}{an(a+bx^n)}\right.}{b^4}$$

input
```
Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

output
```
(x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4
*(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^
2*n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) +
 (b*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 +
 3*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b
^4
```

3.305.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {930, 1025, 1025, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$
$$\Big\downarrow \text{ 930}$$
$$\frac{\int \frac{(dx^n+c)^2(d(ad(3n+1)-b(2nc+c))x^n+c(ad-bc(1-n)))}{bx^n+a}dx}{abn}+\frac{x(bc-ad)\,(c+dx^n)^3}{abn\,(a+bx^n)}$$
$$\Big\downarrow \text{ 1025}$$

$$\frac{\frac{\int \frac{(dx^n+c)\left(c\left(-b^2\left(-2n^2+n+1\right)c^2+2abd(2n+1)c-a^2d^2(3n+1)\right)-d\left(b^2\left(2n^2+3n+1\right)c^2-2abd\left(5n^2+4n+1\right)c+a^2d^2\left(6n^2+5n+1\right)\right)x^n\right)}{bx^n+a}dx}{b(2n+1)}+\frac{dx(c+dx^n)^2(ad(3}{b(2n}}{abn}$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\Big\downarrow$ 1025

$$\frac{\frac{\int \frac{c\left(-b^3\left(-2n^3-n^2+2n+1\right)c^3+3ab^2d\left(2n^2+3n+1\right)c^2-a^2bd^2\left(13n^2+12n+3\right)c+a^3d^3\left(6n^2+5n+1\right)\right)-d\left(b^3\left(2n^2+3n+1\right)c^3-ab^2d\left(12n^3+17n^2+12n+3\right)c^2+a^2bd^2}{bx^n+a}}{b(n+1)}}{b(2n+1)}$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\Big\downarrow$ 913

$$\frac{\frac{-\frac{\left(2n^2+3n+1\right)(bc-ad)^3(bc(1-n)-ad(3n+1))\int \frac{1}{bx^n+a}dx}{b}-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c^3\left(\right.}{b}}{b(n+1)}}{b(2n+1)} \quad abn$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\Big\downarrow$ 778

$$\frac{\frac{-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c^3\left(2n^2+3n+1\right)\right)}{b}-\frac{\left(2n^2+3n+1\right)x(bc-ad)^3(bc(1-n)-ad(3n+}{a}}{b(n+1)}}{b(2n+1)} \quad ab$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) + ((d*(a*d*(1 + 3*n) - b
*(c + 2*c*n))*x*(c + d*x^n)^2)/(b*(1 + 2*n)) + (-((d*(b^2*c^2*(1 + 3*n + 2
*n^2) - 2*a*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*
x^n))/(b*(1 + n))) + (-((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n +
 11*n^2 + 6*n^3) - a*b^2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*
(3 + 15*n + 26*n^2 + 16*n^3))*x)/b) - ((b*c - a*d)^3*(1 + 3*n + 2*n^2)*(b*
c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/(a*b))/(b*(1 + n)))/(b*(1 + 2*n)))/(a*b*n)
```

3.305. $\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$

3.305.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.305.4 Maple [F]

$$\int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

3.305.5 Fricas [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n
+ c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.305.6 Sympy [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

input
```
integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**4/(a + b*x**n)**2, x)
```

3.305.7 Maxima [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1)
- b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n),
x) + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2
+ n)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*
n^3 + 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((
2*n^2 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*
n^2 + 4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 +
 (6*n^3 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (
2*n^3 + 3*n^2 + n)*a^2*b^4)
```

3.305.8 Giac [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^4/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^4/(a + b*x^n)^2, x)
```

3.306 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$

3.306.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx &= -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} \\ &\quad -\frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)}+\frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} \\ &\quad -\frac{(bc-ad)^2(bc(1-n)-ad(1+2n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^3n}\end{aligned}$$

output
```
-d*(b^2*c^2*(1+n)+a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(3*n^2+4*n+2))*x/a/b^3/n/(
1+n)-d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n)/a/b^2/n/(1+n)+(-a*d+b*c)*x*(c+d
*x^n)^2/a/b/n/(a+b*x^n)-(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([
1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^3/n
```

3.306.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 4.83 (sec) , antiderivative size = 2050, normalized size of antiderivative = 10.25

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

output
```
(x*(3*a*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + n)^4 + 3*c^2*d*(1
+ 4*n + 6*n^2 + 2*n^3 + n^4)*x^n + 3*c*d^2*(1 + n)^4*x^(2*n) + d^3*(1 + n)
^4*x^(3*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] - 3*a*(1 + 10*n +
 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + 2*n)^4 + 3*c^2*d*(1 + 2*n)^4*x^n + 3*
c*d^2*(1 + 8*n + 24*n^2 + 34*n^3 + 18*n^4)*x^(2*n) + d^3*(1 + 2*n)^4*x^(3*
n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + a*c^3*HurwitzLerchPhi[-
((b*x^n)/a), 1, 3 + n^(-1)] + 22*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1,
3 + n^(-1)] + 209*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
 1118*a*c^3*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3675*a*c^3*
n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7578*a*c^3*n^5*HurwitzL
erchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 9531*a*c^3*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, 3 + n^(-1)] + 6642*a*c^3*n^7*HurwitzLerchPhi[-((b*x^n)/a), 1,
 3 + n^(-1)] + 1944*a*c^3*n^8*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)]
 + 3*a*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c^2*d
*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c^2*d*n^2*x^n*
HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c^2*d*n^3*x^n*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 11025*a*c^2*d*n^4*x^n*HurwitzLerc
hPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 22734*a*c^2*d*n^5*x^n*HurwitzLerchPhi[
-((b*x^n)/a), 1, 3 + n^(-1)] + 28593*a*c^2*d*n^6*x^n*HurwitzLerchPhi[-((b*
x^n)/a), 1, 3 + n^(-1)] + 19926*a*c^2*d*n^7*x^n*HurwitzLerchPhi[-((b*x^...
```

3.306.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {930, 1025, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{(dx^n+c)(c(ad-bc(1-n))-d(bc(n+1)-ad(2n+1))x^n)}{bx^n+a}dx}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{\int \frac{c\left(-b^2\left(1-n^2\right)c^2+2abd(n+1)c-a^2d^2(2n+1)\right)-d\left(b^2(n+1)c^2-abd\left(3n^2+4n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n}{bx^n+a}dx}{b(n+1)}-\frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{b(n+1)}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{-\frac{(n+1)(bc-ad)^2(bc(1-n)-ad(2n+1))\int \frac{1}{bx^n+a}dx}{b}-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{b}}{b(n+1)}-\frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{b(n+1)}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{b}-\frac{(n+1)x(bc-ad)^2(bc(1-n)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}}{b(n+1)}-\frac{dx(c+dx^n)(bc(n\cdots}{b(n\cdots}}{abn}\ \frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) + (-((d*(b*c*(1 + n) - a
*d*(1 + 2*n))*x*(c + d*x^n))/(b*(1 + n))) + (-((d*(b^2*c^2*(1 + n) + a^2*d
^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^2))*x)/b) - ((b*c - a*d)^2*(
1 + n)*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/(a*b))/(b*(1 + n)))/(a*b*n)
```

3.306. $\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$

3.306.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.306.4 Maple [F]

$$\int \frac{(c + d\,x^n)^3}{(a + b\,x^n)^2} dx$$

input
```
int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

3.306.5 Fricas [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2} \, dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n)
+ 2*a*b*x^n + a^2), x)
```

3.306.6 Sympy [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx$$

input
```
integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**3/(a + b*x**n)**2, x)
```

3.306.7 Maxima [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2} \, dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2
*d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (
3*(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1
)*a^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*
d*(n + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

3.306.8 Giac [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx=\int \frac{(dx^n+c)^3}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx=\int \frac{(c+d\,x^n)^3}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^3/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^3/(a + b*x^n)^2, x)
```

3.307 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$

3.307.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx = -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x(c+dx^n)}{abn(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^2n}$$

output
```
-d*(b*c-a*d*(1+n))*x/a/b^2/n+(-a*d+b*c)*x*(c+d*x^n)/a/b/n/(a+b*x^n)-(-a*d+
b*c)*(b*c*(1-n)-a*d*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^2/
n
```

3.307.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.83 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.79

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx = \frac{x\left(-2a(1+6n+11n^2+6n^3)\left(c^2(1+n)^3+2cd(1+3n+4n^2+n^3)x^n+d^2(1+n)^3x^{2n}\right)\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right.}{}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

output
```
(x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^
2 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 +
 n^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)
^3*x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)
/a), 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a
*c^2*n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1
, n^(-1)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c
^2*n^5*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerc
hPhi[-((b*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*
a*c*d*n^2*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*
HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-
((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 11*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 6*a*d^2*n^3*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*
n^6*x^n*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*
x^n)/a)] - 4*b*c*d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1
, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2
, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 ...
```

3.307.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {930, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(n+1))x^n}{bx^n+a}dx}{abn}+\frac{x(bc-ad)\,(c+dx^n)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{913}$$

$$\frac{-\frac{(bc-ad)(bc(1-n)-ad(n+1))\int\frac{1}{bx^n+a}dx}{b}-\frac{dx(bc-ad(n+1))}{b}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \text{778}$$

$$\frac{-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}-\frac{dx(bc-ad(n+1))}{b}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n))/(a*b*n*(a + b*x^n)) + (-((d*(b*c - a*d*(1 + n)
)*x)/b) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(a*b*n)
```

3.307.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.307.4 Maple [F]

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

3.307.5 Fricas [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2),
x)
```

3.307.6 Sympy [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

3.307.7 Maxima [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n
+ a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c
*d)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

3.307.8 Giac [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^2, x)
```

3.308 $\int \frac{c+dx^n}{(a+bx^n)^2} dx$

3.308.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{c+dx^n}{(a+bx^n)^2} dx = \frac{(bc-ad)x}{abn(a+bx^n)} + \frac{(ad-bc(1-n))x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}$$

output
```
(-a*d+b*c)*x/a/b/n/(a+b*x^n)+(a*d-b*c*(1-n))*x*hypergeom([1, 1/n],[1+1/n],
-b*x^n/a)/a^2/b/n
```

3.308.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^n}{(a+bx^n)^2} dx = \frac{x\left(\frac{d}{a+bx^n} - \frac{(ad+bc(-1+n)) \operatorname{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
```

output
```
(x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

3.308.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad-bc(1-n))\int \frac{1}{bx^n+a}dx}{abn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \; 778$$

$$\frac{x(ad-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

input
```
Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

3.308.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.308.4 Maple [F]

$$\int \frac{c+d\,x^{n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^2,x)
```

3.308.5 Fricas [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.308.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.29

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = c\left(\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right) + d\left(\frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}n^2x^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)} - \frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)} + \frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}nx^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)} - \frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)} - \frac{a^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}bnx^nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)} - \frac{a^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}bx^nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}\right)$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

3.308. $\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$

output
```
c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)
*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1
/n)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamm
a(1 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
+ a**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**
(1/n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*(a*a**(
-3 - 1/n)*a**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*n*x**(n +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*g
amma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n)) + a*a**(-3 - 1/n)*a**(1 + 1/n)
*n*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2
+ 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*x**(n + 1)*lerchphi(b*x**n*exp_pola
r(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n
*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*n*x**n*x**(n + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*x**n*x**(n
 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*...
```

3.308.7 Maxima [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*
x/(a*b^2*n*x^n + a^2*b*n)
```

3.308.8 Giac [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{c+d\,x^n}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^2, x)
```

3.309 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

3.309.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n
],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c/(-a*d+b*c)^2
```

3.309.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1-2n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n} + \frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n)
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

3.309.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{bd(1-n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2n\int \frac{1}{dx^n+c}dx}{bc-ad}-\frac{b(ad(1-2n)-bc(1-n))\int \frac{1}{bx^n+a}dx}{bc-ad}}{an(bc-ad)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2nx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{bx(ad(1-2n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-((b*(a*d*(1 - 2*n) - b*c*(1 - n))*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)))
 - (a*d^2*n*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(a*(b*c - a*d)*n)
```

3.309.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.309.4 Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{2}\left(c+d\,x^{n}\right)}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

3.309.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

3.309.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.309.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)} dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a
^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n
+ a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n)
```

3.309.8 Giac [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)} dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{1}{(a+b\,x^n)^2 \, (c+d\,x^n)} \, dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.310 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2} \, dx$

3.310.1 Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2} \, dx = \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{b^2(ad(1-3n)-b(c-cn))x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^3n} - \frac{d^2(bc(1-3n)-ad(1-n))x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^3n}$$

output
```
d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c
+d*x^n)+b^2*(a*d*(1-3*n)-b*(-c*n+c))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a
)/a^2/(-a*d+b*c)^3/n-d^2*(b*c*(1-3*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1
/n],-d*x^n/c)/c^2/(-a*d+b*c)^3/n
```

3.310.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{x\left(\frac{b^2(bc-ad)}{a(a+bx^n)}+\frac{d^2(bc-ad)}{c(c+dx^n)}+\frac{b^2(ad(1-3n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2(-ad(-1+n)+bc(-1+3n))\,\text{Hypergeo}}{c^2}\right.}{(bc-ad)^3n}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output
```
(x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n))
+ (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeo
metric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

3.310.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {931, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\int \frac{bd(1-2n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)^2}dx}{an(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\frac{\int \frac{bd(bc+ad)(1-n)nx^n+n\left(b^2(1-n)c^2+2abdnc+a^2d^2(1-n)\right)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\frac{-\frac{b^2cn(ad(1-3n)-bc(1-n))\int\frac{1}{bx^n+a}dx}{bc-ad}-\frac{ad^2n(ad(1-n)-b(c-3cn))\int\frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

$$\Big\downarrow \; 778$$

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\frac{-\frac{b^2cnx(ad(1-3n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{ad^2nx(ad(1-n)-b(c-3cn))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)) - (-((d*(b*c + a*d)*x)/(c*
(b*c - a*d)*(c + d*x^n))) + (-((b^2*c*(a*d*(1 - 3*n) - b*c*(1 - n))*n*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d))) - (
a*d^2*n*(a*d*(1 - n) - b*(c - 3*c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), -((d*x^n)/c)])/(c*(b*c - a*d)))/(c*(b*c - a*d)*n))/(a*(b*c - a*d)*n
)
```

3.310.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.310.4 Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{2} \left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

3.310.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

3.310.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.310.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^
2*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n
 + 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*
(n - 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a
^3*c^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*
c*d^4*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a
^2*b^2*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^
2*c^2*d^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^
3*b*c^2*d^2*n + a^4*c*d^3*n)*x^n)
```

3.310.8 Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} dx = \int \frac{1}{(a+b x^n)^2 (c+d x^n)^2} dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

3.311 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

3.311.1 Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n)\,x}{2ac^2(bc-ad)^3n^2(c+dx^n)} + \frac{b^3(ad(1-4n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^4n} + \frac{d^2(a^2d^2(1-3n+2n^2)-2abcd(1-5n+4n^2)+b^2c^2(1-7n+12n^2))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{2c^3(bc-ad)^4n^2}$$

output
```
1/2*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b
*x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/
c^2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1,
 1/n],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-
2*a*b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n],[1+1/
n],-d*x^n/c)/c^3/(-a*d+b*c)^4/n^2
```

3.311.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3} dx$$
$$= \frac{x\left(\frac{2b^3(bc-ad)n}{a(a+bx^n)} + \frac{d^2(bc-ad)^2n}{c(c+dx^n)^2} + \frac{d^2(-bc+ad)(ad(-1+2n)+b(c-6cn))}{c^2(c+dx^n)} + \frac{2b^3(ad(1-4n)+bc(-1+n))n \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.}{a^2}\right.}{2(bc-ad)^4n^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output
```
(x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c +
d*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c
+ d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) -
2*a*b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

3.311.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {931, 1024, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3} dx$$
$$\Big\downarrow \text{ 931}$$
$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{bd(1-3n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)^3}dx}{an(bc-ad)}$$
$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2}-\frac{\frac{\int\frac{bd(2bc+ad)(1-2n)nx^n+n\left(2b^2(1-n)c^2+4abdnc+a^2d^2(1-2n)\right)}{(bx^n+a)(dx^n+c)^2}dx}{2cn(bc-ad)}-\frac{dx(ad+2bc)}{2c(bc-ad)(c+dx^n)^2}}{an(bc-ad)}}{}$$

$\downarrow$ 1024

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2}-\frac{\frac{\frac{\int\frac{n\left(2b^3(1-n)nc^3+6ab^2dn^2c^2-a^2bd^2\left(6n^2-7n+1\right)c+a^3d^3\left(2n^2-3n+1\right)\right)-bd(1-n)n\left(-2b^2nc^2+abd(1-6n)c-a^2d^2(1-2n)\right)x^n}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{dx\left(-a^2d^2(1-2n)+abcd(1-\right.}{c(bc-ad)(c+dx^n}}{2cn(bc-ad)}}{an(bc-ad)}$$

$\downarrow$ 1020

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2}-\frac{\frac{\frac{-\frac{ad^2n\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\int\frac{1}{dx^n+c}dx}{bc-ad}-\frac{2b^3c^2n^2(ad(1-4n)-bc(1-n))\int\frac{1}{bx^n+a}dx}{bc-ad}}{cn(bc-ad)}+\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)\right.}{c(bc-ad)(c+dx^n)}}{2cn(bc-ad)}}{an(bc-ad)}$$

$\downarrow$ 778

$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2}-\frac{\frac{\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n\right)}{c(bc-ad)(c+dx^n)}+\frac{-\frac{ad^2nx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{2b^3c}{}}{cn(bc-ad)}}{2cn(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (-1/2*(d*(2*b*c + a*d)
*x)/(c*(b*c - a*d)*(c + d*x^n)^2) + ((d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 -
2*n) - 2*b^2*c^2*n)*x)/(c*(b*c - a*d)*(c + d*x^n)) + ((-2*b^3*c^2*(a*d*(1
- 4*n) - b*c*(1 - n))*n^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*(b*c - a*d)) - (a*d^2*n*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*
d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c - a*d)))/(c*(b*c - a*d)*n))/(2*c
*(b*c - a*d)*n))/(a*(b*c - a*d)*n)
```

3.311. $\int\frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

3.311.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.311.4 Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{2}\left(c+d\,x^{n}\right)^{3}}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

3.311.5 Fricas [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n)
+ (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d +
 3*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

3.311.6 Sympy [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

output
```
Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

3.311.7 Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 -
 3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*
b^2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 -
 4*a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c
^2*d^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^
2*b^4*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n +
a^6*d^4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4
*b^2*c*d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b
*d^4*(2*n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a
^3*d^4*(2*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2
*(7*n - 1) - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^
3*b^2*c^6*d*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n
^2 - 3*a^2*b^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^
(3*n) + (2*a*b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2
 + a^4*b*c^3*d^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3
*c^6*d*n^2 - 3*a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n
^2)*x^n)
```

3.311.8 Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx=\int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx=\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^3}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

3.312 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

3.312.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

output
```
x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n,-p,-q,1+1/n,-b*x^n/a,-d*x^n/c)/((1+
b*x^n/a)^p)/((1+d*x^n/c)^q)
```

3.312.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(81) = 162$.

Time = 0.45 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \frac{ac(1+n)x(a+bx^n)^p (c+dx^n)^q \operatorname{AppellF1}\left(\frac{1}{n},-p,-q,1+\right.}{bcnpx^n \operatorname{AppellF1}\left(1+\frac{1}{n},1-p,-q,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+adnqx^n \operatorname{AppellF1}\left(1+\frac{1}{n},-p,1-q,2+\frac{1}{n},-\frac{bx^n}{a}\right.}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^
(-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, -q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n
^(-1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Ap
pellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

3.312.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{ 937}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{ 937}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \int \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 936}$$

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x
^n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

3.312.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.312.4 Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

3.312.5 Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

3.312.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.312.7 Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

3.312.8 Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^q, x)
```

3.313 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

3.313.1 Optimal result

Integrand size = 19, antiderivative size = 402

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+p)+n^2(11+6p+p^2)))\,x(a\ldots}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$
$$-\frac{d(ad(1+2n)-bc(1+n(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))}+\frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+3n+np)}$$
$$-\frac{(a^3d^3(1+3n+2n^2)-3a^2bcd^2(1+n)(1+n(3+p))+3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2))-b^3c\ldots}{b^3(1+\ldots}$$

output
```
d*(a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3
+p)+n^2*(p^2+6*p+11)))*x*(a+b*x^n)^(p+1)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3
+p))-d*(a*d*(1+2*n)-b*c*(1+n*(5+p)))*x*(a+b*x^n)^(p+1)*(c+d*x^n)/b^2/(1+n*
(2+p))/(1+n*(3+p))+d*x*(a+b*x^n)^(p+1)*(c+d*x^n)^2/b/(n*p+3*n+1)-(a^3*d^3*
(2*n^2+3*n+1)-3*a^2*b*c*d^2*(1+n)*(1+n*(3+p))+3*a*b^2*c^2*d*(1+n*(5+2*p)+n
^2*(p^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11)+n^3*(p^3+6*p^2+11
*p+6)))*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/
(1+n*(2+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

3.313.2 Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.42

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{3c^2dx^n \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+n} + \frac{3cd^2x^{2n} \text{Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+2n} + \frac{d^3x^{3n} \text{Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+3n} + c^3 \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1
), -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
 -p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1
[3 + n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric
2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

3.313.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^3 (a+bx^n)^p \, dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int -(bx^n+a)^p (dx^n+c) (d(ad(2n+1)-b(n(p+5)c+c))x^n+c(ad-b(n(p+3)c+c))) \, dx}{b(n(p+3)+1)} + \frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}$$

$\Big\downarrow$ 25

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-\frac{\int (bx^n+a)^p(dx^n+c)(d(ad(2n+1)-b(n(p+5)c+c))x^n+c(ad-b(n(p+3)c+c)))dx}{b(n(p+3)+1)}$$

$\Big\downarrow$ 1025

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{\int -(bx^n+a)^p(d(b^2((p^2+6p+11)n^2+2(p+3)n+1)c^2-abd((p+7)n^2+(2p+9)n+2)c+a^2d^2(2n^2+3n+1))x^n+c(b^2((p^2+5p+6)n^2+(2p+5)n+1)c^2-}{b(n(p+2)+1)}}{b(n(p+3)+1)}$$

$\Big\downarrow$ 25

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\int (bx^n+a)^p(d(b^2((p^2+6p+11)n^2+2(p+3)n+1)c^2-abd((p+7)n^2+(2p+9)n+2)c+a^2d^2(2n}{b(n(p-}}{b(n(p+3)+1)}$$

$\Big\downarrow$ 913

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\frac{dx(a+bx^n)^{p+1}(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3)-}{b(np+n+1)}}{}}{b(n}$$

$\Big\downarrow$ 779

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\frac{dx(a+bx^n)^{p+1}(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3)-}{b(np+n+1)}}{}}{}$$

$\Big\downarrow$ 778

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\frac{dx(a+bx^n)^{p+1}(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3)-}{b(np+n+1)}}{}}{}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

3.313. $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

output
```
(d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))) - ((d*(a*d*(1
+ 2*n) - b*(c + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n
*(9 + 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x
^n)^(1 + p))/(b*(1 + n + n*p)) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d
^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p
 + p^2)) - b^3*c^3*(1 + 3*n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 1
1*p + 6*p^2 + p^3)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(
-1), -((b*x^n)/a)])/(b*(1 + n + n*p)*(1 + (b*x^n)/a)^p))/(b*(1 + n*(2 + p)
)))/(b*(1 + n*(3 + p)))
```

3.313.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

3.313.4 Maple [F]

$$\int (a + b\,x^n)^p\,(c + d\,x^n)^3\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

3.313.5 Fricas [F]

$$\int (a + bx^n)^p\,(c + dx^n)^3\,dx = \int (dx^n + c)^3(bx^n + a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p
, x)
```

3.313.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.60

$$\int (a+bx^n)^p (c+dx^n)^3\,dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}c^3x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{n},-p\\ 1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}c^2dx^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}-p,1+\frac{1}{n}\\ 2+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{3a^{2+\frac{1}{n}}a^{p-2-\frac{1}{n}}cd^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}-p,2+\frac{1}{n}\\ 3+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{a^{3+\frac{1}{n}}a^{p-3-\frac{1}{n}}d^3x^{3n+1}\Gamma\left(3+\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}-p,3+\frac{1}{n}\\ 4+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
a**(1/n)*a**(p - 1/n)*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 3*a**(1 + 1/n)*a**(p - 1 - 1/n)*
c**2*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**(2 + 1/n)*a**(p - 2 - 1/n)*c*d
**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**(3 + 1/n)*a**(p - 3 - 1/n)*d**3*x
**(3*n + 1)*gamma(3 + 1/n)*hyper((-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(4 + 1/n))
```

3.313.7 Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^3\,dx = \int (dx^n+c)^3(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

3.313.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,3,4,3,0]%%%}+%%
%{6,[2,0,
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^3\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^3, x)
```

3.314 $\int (a+bx^n)^p (c+dx^n)^2 \, dx$

3.314.1 Optimal result

Integrand size = 19, antiderivative size = 202

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{d(ad(1+n)-bc(1+n(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+2n+np)} - \frac{(bc(1+n+np)(ad-bc(1+n(2+p)))-ad(ad(1+n)-bc(1+n(3+p))))x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{H}}{b^2(1+n+np)(1+n(2+p))}$$

output
```
-d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^(p+1)/b^2/(n*p+n+1)/(1+n*(2+p))
+d*x*(a+b*x^n)^(p+1)*(c+d*x^n)/b/(n*p+2*n+1)-(b*c*(n*p+n+1)*(a*d-b*c*(1+n*
(2+p)))-a*d*(a*d*(1+n)-b*c*(1+n*(3+p))))*x*(a+b*x^n)^p*hypergeom([-p, 1/n]
,[1+1/n],-b*x^n/a)/b^2/(n*p+n+1)/(1+n*(2+p))/((1+b*x^n/a)^p)
```

3.314.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.69

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(2cd(1+2n)x^n \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)+(1+n)\left(d^2x^{2n}\text{H}\right.\right.}{(1+n)(1}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2
+ n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1
), -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1),
 -p, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

3.314.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^2 (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -(bx^n+a)^p (d(ad(n+1)-b(n(p+3)c+c))x^n+c(ad-b(n(p+2)c+c)))dx}{b(n(p+2)+1)}+\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}-\frac{\int (bx^n+a)^p (d(ad(n+1)-b(n(p+3)c+c))x^n+c(ad-b(n(p+2)c+c)))dx}{b(n(p+2)+1)}$$

$$\Big\downarrow \text{913}$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}-\frac{\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}+acd-bc(cn(p+2)+c)\right)\int (bx^n+a)^p dx+\frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}}{b(n(p+2)+1)}$$

$$\Big\downarrow \text{779}$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}+acd-bc(cn(p+2)+c)\right)\int\left(\frac{bx^n}{a}+1\right)^p dx + \frac{dx(a+bx^n)^{p+1}(ad(n-}{b(np+}}{b(n(p+2)+1)}$$

$$\Big\downarrow 778$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}+acd-bc(cn(p+2)+c)\right)\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n}}{b(n(p+2)+1)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n*(2 + p))) - ((d*(a*d*(1 +
n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((a*c
*d - b*c*(c + c*n*(2 + p)) - (a*d*(a*d*(1 + n) - b*(c + c*n*(3 + p))))/(b*
(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
-((b*x^n)/a)])/(1 + (b*x^n)/a)^p)/(b*(1 + n*(2 + p)))
```

3.314.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.314.4 Maple [F]

$$\int (a+b\,x^{n})^{p} \left(c+d\,x^{n}\right)^{2}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

3.314.5 Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^2\,dx=\int (dx^n+c)^2(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

3.314.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}c^2x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}cdx^{n+1}\Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{a^{2+\frac{1}{n}}a^{p-2-\frac{1}{n}}d^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
a**(1/n)*a**(p - 1/n)*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 2*a**(1 + 1/n)*a**(p - 1 - 1/n)*
c*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**(2 + 1/n)*a**(p - 2 - 1/n)*d**2*x**
(2*n + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(3 + 1/n))
```

3.314.7 Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (dx^n+c)^2(bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

3.314.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,2,0]%%%}+
%%%{-3,[1
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^2\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^2, x)
```

3.315 $\int (a+bx^n)^p (c+dx^n)\, dx$

3.315.1 Optimal result

Integrand size = 17, antiderivative size = 98

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \frac{(ad-bc(1+n+np))x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(1+n+np)}$$

output
```
d*x*(a+b*x^n)^(p+1)/b/(n*p+n+1)-(a*d-b*c*(n*p+n+1))*x*(a+b*x^n)^p*hypergeo
m([-p, 1/n],[1+1/n],-b*x^n/a)/b/(n*p+n+1)/((1+b*x^n/a)^p)
```

3.315.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(d(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p + (-ad+bc(1+n+np))\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1-\right.\right.}{b(1+n+np)}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
 n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n
 + n*p)*(1 + (b*x^n)/a)^p)
```

3.315.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{bnp+bn+b}\right)\int (bx^n+a)^p dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\int \left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p
))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]
)/(1 + (b*x^n)/a)^p
```

3.315.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.315.4 Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

3.315.5 Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p, x)
```

3.315.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a+bx^n)^p (c+dx^n)\,dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(1/n)*a**(p - 1/n)*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(1 + 1/n)*a**(p - 1 - 1/n)*d*x**
(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 1/n))
```

3.315.7 Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)\,dx = \int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

3.315.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%
%{1,[0,0,
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)\,dx = \int (a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n), x)
```

3.316 $\int (a+bx^n)^p \, dx$

3.316.1 Optimal result

Integrand size = 9, antiderivative size = 46

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

3.316.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Integrate[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

3.316.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Int[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

3.316.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

3.316.4 Maple [F]

$$\int (a+b\,x^n)^p\,dx$$

input
```
int((a+b*x^n)^p,x)
```

output
```
int((a+b*x^n)^p,x)
```

3.316.5 Fricas [F]

$$\int (a+bx^n)^p\ dx = \int (bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p, x)
```

3.316.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p\ dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p,x)
```

output
```
a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

3.316.7 Maxima [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p, x)
```

3.316.8 Giac [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p, x)
```

3.316.9 Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int (a+bx^n)^p \, dx = \frac{x\,(a+b\,x^n)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

input
```
int((a + b*x^n)^p,x)
```

output
```
(x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1
)^p
```

3.317 $\int \frac{(a+bx^n)^p}{c+dx^n} dx$

3.317.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{c+dx^n} dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/((1+b*x^n/a)^p)
```

3.317.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{c+dx^n} dx$$

$$= \frac{ac(1+n)x(a+bx^n)^p \text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\right.}{(c+dx^n)\left(bcnpx^n \text{AppellF1}\left(1+\frac{1}{n},1-p,1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^n \text{AppellF1}\left(1+\frac{1}{n},-p,2,2+\frac{1}{n},\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1)
, -p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

3.317.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{c+dx^n}dx$$

$$\Big\downarrow \ 937$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \ 936$$

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c*(1 + (b*x^n)/a)^p)
```

3.317.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.317.4 Maple [F]

$$\int \frac{(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n),x)
```

3.317.5 Fricas [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c), x)
```

3.317.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.317.7 Maxima [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c), x)
```

3.317.8 Giac [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n), x)
```

3.318 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$

3.318.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx=\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/((1+b*x^n/a)^
p)
```

3.318.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$$
$$=\frac{ac(1+n)x(a+bx^n)^p\,\text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\right.}{(c+dx^n)^2\left(bcnpx^n\,\text{AppellF1}\left(1+\frac{1}{n},1-p,2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,3,2+\frac{1}{n}\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^
(-1), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF
1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

3.318.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

3.318.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.318.4 Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{2}}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

3.318.5 Fricas [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

3.318.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.318.7 Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

3.318.8 Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^2, x)
```

3.319 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$

3.319.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,3,1+1/n,-b*x^n/a,-d*x^n/c)/c^3/((1+b*x^n/a)^
p)
```

3.319.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$$
$$= \frac{ac(1+n)x(a+bx^n)^p \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\right.}{(c+dx^n)^3 \left(bcnpx^n \text{AppellF1}\left(1+\frac{1}{n},1-p,3,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^n \text{AppellF1}\left(1+\frac{1}{n},-p,4,2+\frac{1}{n}\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^
(-1), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF
1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

3.319.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \; 936$$

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

3.319.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.319.4 Maple [F]

$$\int \frac{(a + b\,x^{n})^{p}}{(c + d\,x^{n})^{3}}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

3.319.5 Fricas [F]

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3}\,dx = \int \frac{(bx^n + a)^p}{(dx^n + c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)
, x)
```

3.319.6 Sympy [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

3.319.7 Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

3.319.8 Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^3, x)
```

3.320 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx$

3.320.1 Optimal result

Integrand size = 28, antiderivative size = 93

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

output
```
x*(a+b*x^n)^p*(c+d*x^n)^(-1/n-p)*hypergeom([-p, 1/n],[1+1/n],-(-a*d+b*c)*x
^n/a/(c+d*x^n))/c/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

3.320.2 Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^{-\frac{1+np}{n}} \left(1+\frac{dx^n}{c}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{c}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

output
```
(x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1
), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)
^((1 + n*p)/n))
```

3.320.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p-1} dx$$

$$\Big\downarrow \text{ 905}$$

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

output
```
(x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(
c + d*x^n)))^p)
```

3.320.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

3.320.4 Maple [F]

$$\int (a+b\,x^n)^p (c+d\,x^n)^{-1-\frac{1}{n}-p} dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

3.320.5 Fricas [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

3.320.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-1-\frac{1}{n}-p} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.320.7 Maxima [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

3.320.8 Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-1}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+1}}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1), x)
```

3.321 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

3.321.1 Optimal result

Integrand size = 25, antiderivative size = 178

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^3(1+n)(1+2n)(1+3n)} + \frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(1+n)(1+2n)(1+3n)}$$

output
```
x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^(
-2-1/n)/c^2/(6*n^2+5*n+1)+6*a^2*n^2*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(6*
n^3+11*n^2+6*n+1)+6*a^3*n^3*x/c^4/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

3.321.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{x(c+dx^n)^{-3-\frac{1}{n}}(b^3c^3(1+3n+2n^2)x^{3n} + 3ab^2c^2(1+n)x^{2n}(c+3cn+dnx^n) + 3a^2bcx^n(c^2(1+5n+6n^2 \ldots}{c^4(\ldots}$$

input
```
Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*
c^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n +
6*n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n +
11*n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*
x^(2*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

3.321.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-4} \, dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3an\int (bx^n+a)^2 (dx^n+c)^{-3-\frac{1}{n}}dx}{c(3n+1)}+\frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{903}$$

$$\frac{3an\left(\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{903}$$

$$\frac{3an\left(\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{3an\left(\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

input
```
Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*((x*(a
+ b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*((x*(a + b*x^
n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c + d*x^
n)^n^(-1))))/(c*(1 + 2*n))))/(c*(1 + 3*n))
```

3.321.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(174) = 348.

Time = 5.02 (sec) , antiderivative size = 1288, normalized size of antiderivative = 7.24

method	result	size
parallelrisch	Expression too large to display	1288

input
```
int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

output
```
(6*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a^3*d^4*n^3+2*x*(x^n)^3*(c+d*x^n)^(-(1
+4*n)/n)*b^3*c^4*n^2+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d+3*x*(x^n)^
3*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^4*n+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b^
2*c^4+x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d+3*x*x^n*(c+d*x^n)^(-(1+4*n)/n
)*a^2*b*c^4+2*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d*n^2+24*x*(x^n)^3*
(c+d*x^n)^(-(1+4*n)/n)*a^3*c*d^3*n^3+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^
3*c^3*d*n+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^3*c*d^3*n^2+36*x*(x^n)^2*(c
+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d^2*n^3+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a
^3*c^2*d^2*n^2+9*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^4*n^2+24*x*x^n*(
c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*n^3+3*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b
^2*c^3*d+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d^2*n+12*x*(x^n)^2*(c+
d*x^n)^(-(1+4*n)/n)*a*b^2*c^4*n+26*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*
n^2+18*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^4*n^2+3*x*(x^n)^2*(c+d*x^n)^(-
(1+4*n)/n)*a^2*b*c^3*d+9*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*n+6*x*(x^n
)^4*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c*d^3*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)
/n)*a*b^2*c^2*d^2*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^2*d^2*n+2
4*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^2*d^2*n^2+12*x*(x^n)^3*(c+d*x^n
)^(-(1+4*n)/n)*a*b^2*c^3*d*n^2+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^
2*d^2*n+15*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^3*d*n+36*x*(x^n)^2*(c+
d*x^n)^(-(1+4*n)/n)*a^2*b*c^3*d*n^2+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)...
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(174) = 348$.

Time = 0.27 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.69

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$$
$$= \frac{(6\,a^3d^4n^3 + b^3c^3d + (2\,b^3c^3d + 3\,ab^2c^2d^2 + 6\,a^2bcd^3)n^2 + 3\,(b^3c^3d + ab^2c^2d^2)n)xx^{4\,n} + (24\,a^3cd^3n^3 + b^3c^4}{}$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

output
```
((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d
^3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 +
 b^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 +
 3*a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(
3*n) + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12
*a^2*b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2
*d^2)*n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^
2*b*c^4 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*
a^3*c^4*n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*
c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

3.321.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2822 vs. 2(160) = 320.

Time = 29.87 (sec) , antiderivative size = 2822, normalized size of antiderivative = 15.85

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

output
```
6*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
1)**(1/n)*gamma(4 + 1/n)) + 11*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma
(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*
d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**
(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)
*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**3*c**3*c**(1
/n)*c**(-4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)
*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamm
a(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma
(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
1/n)) + a**3*c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c
/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**3*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x*
*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3...
```

3.321.7 Maxima [F]

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} \, dx = \int (bx^n+a)^3 (dx^n+c)^{-\frac{1}{n}-4} \, dx$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

3.321.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%
{54,[2,0,
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \int \frac{(a+b\,x^n)^3}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

input
```
int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
```

output
```
int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

3.322 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx$

3.322.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx = \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(1+n)(1+2n)}$$

output
```
x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c/(1+2*n)+2*a*n*x*(a+b*x^n)*(c+d*x^n)^(-1
-1/n)/c^2/(2*n^2+3*n+1)+2*a^2*n^2*x/c^3/(2*n^2+3*n+1)/((c+d*x^n)^(1/n))
```

3.322.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx$$
$$= \frac{x(c+dx^n)^{-2-\frac{1}{n}} (b^2c^2(1+n)x^{2n} + 2abcx^n(c+2cn+dnx^n) + a^2(c^2(1+3n+2n^2) + 2cdn(1+2n)x^n + 2}{c^3(1+n)(1+2n)}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2
*c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d
^2*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

3.322.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-3} \, dx$$

$$\Big\downarrow \text{903}$$

$$\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

$$\Big\downarrow \text{903}$$

$$\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*((x*(a
+ b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c
+ d*x^n)^n^(-1))))/(c*(1 + 2*n))
```

3.322.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.322.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(116) = 232$.

Time = 4.51 (sec) , antiderivative size = 588, normalized size of antiderivative = 5.07

method	result
parallelrisch	$\frac{2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}a^{2}d^{3}n^{2}+2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}abc\,d^{2}n+x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}b^{2}c^{2}dn+6x\,x^{2n}(c+d\,x^{n})^{-\frac{1+3n}{n}}a^{2}c\,d^{2}n}{}$

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)
```

output
```
(2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a^2*d^3*n^2+2*x*(x^n)^3*(c+d*x^n)^(-(1
+3*n)/n)*a*b*c*d^2*n+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^2*d*n+6*x*(x^n
)^2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*
b^2*c^2*d+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n+6*x*(x^n)^2*(c+d*
x^n)^(-(1+3*n)/n)*a*b*c^2*d*n+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3*n+6
*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n
)/n)*a*b*c^2*d+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3+5*x*x^n*(c+d*x^n)^
(-(1+3*n)/n)*a^2*c^2*d*n+4*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*b*c^3*n+2*x*(c+d
*x^n)^(-(1+3*n)/n)*a^2*c^3*n^2+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d+2*x*
x^n*(c+d*x^n)^(-(1+3*n)/n)*a*b*c^3+3*x*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^3*n+x*
(c+d*x^n)^(-(1+3*n)/n)*a^2*c^3)/(1+n)/(1+2*n)/c^3
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} dx = \frac{(2a^2d^3n^2 + b^2c^2d + (b^2c^2d + 2abcd^2)n)xx^{3n} + (6a^2cd^2n^2 + b^2c^3 + 2abc^2d + (b^2c^3 + 6abc^2d + 2a^2cd^2)n)}{(2c^3n^2 + 3c^3n + c^3)(d}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

output
```
((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*
a^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d
^2)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 +
 5*a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^
3*n^2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

3.322.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(104) = 208.

Time = 12.92 (sec) , antiderivative size = 1035, normalized size of antiderivative = 8.92

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

output
```
2*a**2*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3
 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a**
2*c**2*c**(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1
/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)
+ d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a**2*c**2*c**
(1/n)*c**(-3 - 2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3
+ 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*
x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a**2*c*c**(1/n)*c**(-3
- 2/n)*d*n**2*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 +
 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x
**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c*c**(1/n)*c**(-3 -
 2/n)*d*n*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n
) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2
*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c**(1/n)*c**(-3 - 2/n)*
d**2*n**2*x*x**(2*n)*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 +
 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x
**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*b*c*c**(-3 - 1/n)*c**(
1 + 1/n)*n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**
2*gamma(3 + 1/n) + d*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + 2*a*b*c*c...
```

3.322.7 Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

3.322.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6
,[1,0,4,1
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+3}}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

3.323 $\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx$

3.323.1 Optimal result

Integrand size = 23, antiderivative size = 58

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \frac{x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

output
```
x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c/(1+n)+a*n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \frac{x(c+dx^n)^{-\frac{1+n}{n}} \left(bcx^n + a(1+n) (c+dx^n) \left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{c^2(1+n)}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2
F1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n
)^((1 + n)/n))
```

3.323.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^{-\frac{1}{n}-2}\,dx$$

$$\Big\downarrow \text{903}$$

$$\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 +
n)*(c + d*x^n)^n^(-1))
```

3.323.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.323.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(58) = 116$.

Time = 4.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.43

method	result
parallelrisch	$\frac{x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}a\,d^2n+x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}bcd+2x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}acdn+x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}acd+x\,x^n(c+d\,x^n)^{-}}{c^2(1+n)}$

input
```
int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x,method=_RETURNVERBOSE)
```

output
```
(x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)*a*d^2*n+x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)
*b*c*d+2*x*x^n*(c+d*x^n)^(-(1+2*n)/n)*a*c*d*n+x*x^n*(c+d*x^n)^(-(1+2*n)/n)
*a*c*d+x*x^n*(c+d*x^n)^(-(1+2*n)/n)*b*c^2+x*(c+d*x^n)^(-(1+2*n)/n)*a*c^2*n
+x*(c+d*x^n)^(-(1+2*n)/n)*a*c^2)/c^2/(1+n)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int (a+bx^n)\,(c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{(ad^2n+bcd)xx^{2\,n}+(2\,acdn+bc^2+acd)xx^n+(ac^2n+ac^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

output
```
((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*
n + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

3.323.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(48) = 96$.

Time = 2.21 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.36

$$\int (a+bx^n)\left(c+dx^n\right)^{-2-\frac{1}{n}}\,dx$$
$$=\frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}$$
$$+\frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}$$
$$+\frac{ac^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}$$
$$+\frac{bc^{-2-\frac{1}{n}}c^{1+\frac{1}{n}}d^{-1-\frac{1}{n}}\left(\frac{cx^{-n}}{d}+1\right)^{-1-\frac{1}{n}}\Gamma\left(1+\frac{1}{n}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

output
```
a*c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)*
*(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma
(2 + 1/n)) + a*c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*
x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**
(1/n)*gamma(2 + 1/n)) + a*c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d*
*(1/n)*n**2*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*
(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n)) + b*c**(-2 - 1/n)*c**(1 + 1/n)*d**
(-1 - 1/n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(n*gamma(2 + 1/n))
```

3.323.7 Maxima [F]

$$\int (a+bx^n)\left(c+dx^n\right)^{-2-\frac{1}{n}}\,dx=\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

3.323.8 Giac [F(-2)]

Exception generated.

$$\int (a + bx^n) (c + dx^n)^{-2-\frac{1}{n}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,
[0,0,2,1,
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int (a + bx^n) (c + dx^n)^{-2-\frac{1}{n}} dx = \int \frac{a + b x^n}{(c + d x^n)^{\frac{1}{n}+2}} dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

3.324 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

3.324.1 Optimal result

Integrand size = 15, antiderivative size = 18

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

output
```
x/c/((c+d*x^n)^(1/n))
```

3.324.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

input
```
Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

output
```
x/(c*(c + d*x^n)^n^(-1))
```

3.324.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^{-\frac{1}{n}-1}\,dx$$

$$\Big\downarrow \; 746$$

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

input
```
Int[(c + d*x^n)^(-1 - n^(-1)),x]
```

output
```
x/(c*(c + d*x^n)^n^(-1))
```

3.324.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

3.324.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 4.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

method	result	size
parallelrisch	$\frac{x\,x^n(c+d\,x^n)^{-\frac{1+n}{n}}d+x(c+d\,x^n)^{-\frac{1+n}{n}}c}{c}$	47
norman	$x\,\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}+\frac{dx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}}{c}$	53

input
```
int((c+d*x^n)^(-1-1/n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*(c+d*x^n)^(-(1+n)/n)*d+x*(c+d*x^n)^(-(1+n)/n)*c)/c
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \frac{dxx^n+cx}{(dx^n+c)^{\frac{n+1}{n}}c}$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

output
```
(d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

3.324.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \frac{c^{\frac{1}{n}}c^{-1-\frac{1}{n}}d^{-\frac{1}{n}}\left(\frac{cx^{-n}}{d}+1\right)^{-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**(-1-1/n),x)
```

output
```
c**(1/n)*c**(-1 - 1/n)*gamma(1/n)/(d**(1/n)*n*(c/(d*x**n) + 1)**(1/n)*gamm
a(1 + 1/n))
```

3.324.7 Maxima [F]

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-1}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 1), x)
```

3.324.8 Giac [F]

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-1}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 1), x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.17

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \frac{d\,x^{n+1}\left(\frac{c}{d\,x^n}-\left(\frac{c}{d\,x^n}+1\right)^{\frac{n+1}{n}}+1\right)}{c\,n\,\left(\frac{n+1}{n}-1\right)\,\left(c+d\,x^n\right)^{\frac{n+1}{n}}}$$

input
```
int(1/(c + d*x^n)^(1/n + 1),x)
```

output
```
(d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/
n - 1)*(c + d*x^n)^((n + 1)/n))
```

3.325 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx$

3.325.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx=\frac{x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

output
```
x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/((c+d*x^n)^(1/
n))
```

3.325.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx=\frac{x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

output
```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d
*x^n))])/(a*(c + d*x^n)^n^(-1))
```

3.325.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} \, dx$$

$$\Big\downarrow \; 904$$

$$\frac{x(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

output
```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*
x^n)))])/(a*(c + d*x^n)^n^(-1))
```

3.325.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

3.325.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{a+b\,x^n}dx$$

input
```
int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

3.325.5 Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

3.325.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.325.7 Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

3.325.8 Giac [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{1/n}}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n)), x)
```

3.326 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$

3.326.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

output
```
c*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/((c+d*x^n)
^(1/n))
```

3.326.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

input
```
Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

output
```
(c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c +
 d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

3.326.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 904$$

$$\frac{cx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

input
```
Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

output
```
(c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
d*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

3.326.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

3.326.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

3.326.5 Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.326.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.326.7 Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

3.326.8 Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2, x)
```

3.327 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

3.327.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \frac{c^2x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

output
```
c^2*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^
n)^(1/n))
```

3.327.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \frac{c^2x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

input
```
Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
 + d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

3.327.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$$
$$\Big\downarrow \; 904$$
$$\frac{c^2x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

input
```
Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c
+ d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

3.327.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

3.327.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}dx$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

3.327.5 Fricas [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*
b*x^n + a^3), x)
```

3.327.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.327.7 Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

3.327.8 Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

input
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3,x)
```

output
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3, x)
```

3.328 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$

3.328.1 Optimal result

Integrand size = 28, antiderivative size = 193

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{(bc+(bc-ad)n(1+p))x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-1-p}(c+dx^n)^{-1-\frac{1}{n}-p}\,\text{Hypergeometric2F1}\left(\frac{1}{n},-1-p\right.}{ac(bc-ad)n(1+p)}$$

output
```
-b*x*(a+b*x^n)^(p+1)*(c+d*x^n)^(-1-1/n-p)/a/(-a*d+b*c)/n/(p+1)+(b*c+(-a*d+
b*c)*n*(p+1))*x*(a+b*x^n)^(p+1)*(c*(a+b*x^n)/a/(c+d*x^n))^(-1-p)*(c+d*x^n)
^(-1-1/n-p)*hypergeom([1/n, -1-p],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/c
/(-a*d+b*c)/n/(p+1)
```

3.328.2 Mathematica [F]

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = \int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

output
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p), x]
```

3.328.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {907, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p-2}\,dx$$

$$\Big\downarrow \text{907}$$

$$\frac{\left(\frac{bc}{n(p+1)(bc-ad)}+1\right)\int (bx^n+a)^{p+1}(dx^n+c)^{-p-\frac{1}{n}-2}\,dx}{a}-\frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

$$\Big\downarrow \text{905}$$

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{bc}{n(p+1)(bc-ad)}+1\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{n},-p-1,1+\frac{1}{n},-\frac{(bc-\cdots}{a(d\cdots}\right.}{ac}$$
$$\frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

output
```
-((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n
*(1 + p))) + ((1 + (b*c)/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((
c*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyp
ergeometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*c)
```

3.328.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.328.4 Maple [F]

$$\int (a+b\,x^{n})^{p} (c+d\,x^{n})^{-2-\frac{1}{n}-p}\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

3.328.5 Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = \int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-2}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

3.328.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.328.7 Maxima [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

3.328.8 Giac [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+2}} \, dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2), x)
```

3.329 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx$

3.329.1 Optimal result

Integrand size = 69, antiderivative size = 57

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

output
```
x*(c+d*x^n)^(a*d/(-a*d+b*c)/n)/a/c/((a+b*x^n)^(b*c/(-a*d+b*c)/n))
```

3.329.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx = \frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}} (c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

input
```
Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^
((a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

output
```
(x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a
*d*n)))
```

3.329.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{\frac{adn-bc(n+1)}{n(bc-ad)}}(c+dx^n)^{\frac{adn+ad-bcn}{bcn-adn}}\,dx$$

$$\Big\downarrow \; 906$$

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

input
```
Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d
- b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

output
```
(x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*
d)*n)))
```

3.329.3.1 Defintions of rubi rules used

rule 906
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

3.329.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(58) = 116.

Time = 8.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 5.86

method	result
parallelrisch	$\frac{x\,x^{2n}(a+b\,x^n)^{\frac{-adn+bc(1+n)}{(ad-bc)n}}(c+d\,x^n)^{-\frac{adn-bcn+ad}{(ad-bc)n}}b^2d^2+x\,x^n(a+b\,x^n)^{\frac{-adn+bc(1+n)}{(ad-bc)n}}(c+d\,x^n)^{-\frac{adn-bcn+ad}{(ad-bc)n}}ab\,d^2+x\,x^n(a+}{abcd}$

input
```
int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d
)/(-a*d*n+b*c*n)),x,method=_RETURNVERBOSE)
```

output
```
(x*(x^n)^2*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b
*c*n+a*d)/(a*d-b*c)/n)*b^2*d^2+x*x^n*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*
c)/n)*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/(a*d-b*c)/n)*a*b*d^2+x*x^n*(a+b*x^n)^(
(-a*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/(a*d-b*c)/n)
*b^2*c*d+x*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b
*c*n+a*d)/(a*d-b*c)/n)*a*b*c*d)/a/b/c/d
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\, dx = \frac{(bdxx^{2n}+acx+(bc+ad)xx^n)(dx^n+c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n+a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

output
```
(b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d
)*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))
*a*c)
```

3.329.6 Sympy [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\, dx = \int (a+bx^n)^{\frac{adn-bc(n+1)}{n(-ad+bc)}} (c+dx^n)^{\frac{adn+ad-bcn}{-adn+bcn}}\, dx$$

input
```
integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n
-b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

output
```
Integral((a + b*x**n)**((a*d*n - b*c*(n + 1))/(n*(-a*d + b*c)))*(c + d*x**
n)**((a*d*n + a*d - b*c*n)/(-a*d*n + b*c*n)), x)
```

3.329.7 Maxima [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n +
c)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

3.329.8 Giac [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n +
c)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(n+1)}{n\,(a\,d-b\,c)}}\,(c+d\,x^n)^{\frac{a\,d+a\,d\,n-b\,c\,n}{a\,d\,n-b\,c\,n}}}\,dx$$

input
```
int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a
*d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
```

output
```
int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a
*d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

3.330 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx$

3.330.1 Optimal result

Integrand size = 25, antiderivative size = 327

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx = -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} - \frac{(3adn-b(c+3cn))x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(bc-ad)n(1+3n)} - \frac{(3adn-b(c+3cn))x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(bc-ad)(1+5n+6n^2)} - \frac{2an(3adn-b(c+3cn))x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^3(bc-ad)(1+n)(1+2n)(1+3n)} - \frac{2a^2n^2(3adn-b(c+3cn))x(c+dx^n)^{-1/n}}{c^4(bc-ad)(1+n)(1+2n)(1+3n)}$$

output
```
-1/3*b*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/(-a*d+b*c)/n-1/3*(3*a*d*n-b*(3*c
*n+c))*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/c/(-a*d+b*c)/n/(1+3*n)-(3*a*d*n-
b*(3*c*n+c))*x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c^2/(-a*d+b*c)/(6*n^2+5*n+1)
-2*a*n*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(-a*d+b*c)
/(6*n^3+11*n^2+6*n+1)-2*a^2*n^2*(3*a*d*n-b*(3*c*n+c))*x/c^4/(-a*d+b*c)/(6*
n^3+11*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

3.330.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.42

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx$$
$$=\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(b^2c^2\,\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)-(bc-ad)\left(2bc\,\text{Hypergeom}\right.\right.}{c^4d^2}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
 + n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1)
, n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 +
 n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

3.330.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {907, 903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-4}\,dx$$
$$\Big\downarrow \ 907$$
$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\int (bx^n+a)^3 (dx^n+c)^{-4-\frac{1}{n}}dx}{3a}-\frac{bx(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$
$$\Big\downarrow \ 903$$
$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\int (bx^n+a)^2(dx^n+c)^{-3-\frac{1}{n}}dx}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$
$$\Big\downarrow \ 903$$

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$\Big\downarrow$ 903

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$\Big\downarrow$ 746

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
-1/3*(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(a*(b*c - a*d)*n) + ((3
 + (b*c)/(b*c*n - a*d*n))*((x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*
(1 + 3*n)) + (3*a*n*((x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2
*n)) + (2*a*n*((x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*
n*x)/(c^2*(1 + n)*(c + d*x^n)^n^(-1))))/(c*(1 + 2*n))))/(c*(1 + 3*n))))/(3
*a)
```

3.330. $\int (a+bx^n)^2 \left(c+dx^n\right)^{-4-\frac{1}{n}}\,dx$

3.330.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.330.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(319) = 638$.

Time = 4.52 (sec) , antiderivative size = 1059, normalized size of antiderivative = 3.24

method	result	size
parallelrisch	Expression too large to display	1059

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

output
```
(24*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n^3+3*x*(x^n)^2*(c+d*x^n)^(-(1+
4*n)/n)*a^2*c^2*d^2*n+26*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n^2+12*x*x
^n*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^4*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a
*b*c^3*d+9*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n+10*x*x^n*(c+d*x^n)^(-(
1+4*n)/n)*a*b*c^4*n+16*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n^2+4*
x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b*c*d^3*n^2+24*x*(x^n)^3*(c+d*x^n)^(-(1
+4*n)/n)*a^2*c*d^3*n^3+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^2*d^2*n+6*x*
(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*c*d^3*n^2+4*x*(x^n)^3*(c+d*x^n)^(-(1+4*
n)/n)*b^2*c^3*d*n^2+4*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n+24*x*
(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d*n^2+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)
/n)*b^2*c^2*d^2*n^2+14*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d*n+36*x*(
x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n^3+5*x*(x^n)^3*(c+d*x^n)^(-(1+4
*n)/n)*b^2*c^3*d*n+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n^2+6*x
*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a^2*d^4*n^3+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n
)/n)*b^2*c^4*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^3*d+4*x*(x^n)^2*(c
+d*x^n)^(-(1+4*n)/n)*b^2*c^4*n+x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d+2*x*
x^n*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^4+x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4+6*x*(c
+d*x^n)^(-(1+4*n)/n)*a^2*c^4*n^3+x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^4+
11*x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4*n^2+6*x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4
*n)/(2*n^2+3*n+1)/(1+3*n)/c^4
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.22

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\, dx$$
$$= \frac{(6\,a^2d^4n^3 + b^2c^2d^2n + (b^2c^2d^2 + 4\,abcd^3)n^2)xx^{4\,n} + (24\,a^2cd^3n^3 + b^2c^3d + 2\,(2\,b^2c^3d + 8\,abc^2d^2 + 3\,a^2cd^3)}{}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

output
```
((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*
n) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^
2*c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^
2*n^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*
n^2 + (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^
3*d*n^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a
*b*c^4 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c
^4*n + a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^(
(4*n + 1)/n))
```

3.330.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. 2(282) = 564.

Time = 13.32 (sec) , antiderivative size = 2746, normalized size of antiderivative = 8.40

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

output
```
6*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
1)**(1/n)*gamma(4 + 1/n)) + 11*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma
(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*
d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**
(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)
*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**2*c**3*c**(1
/n)*c**(-4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)
*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamm
a(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma
(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
1/n)) + a**2*c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c
/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**2*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x*
*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3...
```

3.330.7 Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-4}\,dx$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

3.330.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{
9,[1,0,4,
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

3.331 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

3.331.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}$$

output
```
-(-a*d+b*c)*x*(c+d*x^n)^(-2-1/n)/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^(-1
-1/n)/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)
^(1/n))
```

3.331.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$$
$$= \frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(bc\,\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)+(-bc+ad)\,\text{Hypergeometric}\right.}{c^3d}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n
^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1)
, 1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

3.331.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {910, 777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^{-\frac{1}{n}-3}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(2adn+bc)\int (dx^n+c)^{-2-\frac{1}{n}}dx}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

$$\Big\downarrow \; 777$$

$$\frac{(2adn+bc)\left(\frac{n\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

$$\Big\downarrow \; 746$$

$$\frac{\left(\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)(2adn+bc)}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
-(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a
*d*n)*((x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c +
 d*x^n)^n^(-1))))/(c*d*(1 + 2*n))
```

3.331.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

3.331.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(127) = 254.

Time = 4.40 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.42

method	result
parallelrisch	$2x\,x^{3n}(c+d\,x^n)^{-\frac{1+3n}{n}}\,a\,d^3n^2+x\,x^{3n}(c+d\,x^n)^{-\frac{1+3n}{n}}\,bc\,d^2n+6x\,x^{2n}(c+d\,x^n)^{-\frac{1+3n}{n}}\,ac\,d^2n^2+2x\,x^{2n}(c+d\,x^n)^{-\frac{1+3n}{n}}\,ac\,d^2n+3$

input
```
int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)
```

output
```
(2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a*d^3*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n
)/n)*b*c*d^2*n+6*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n^2+2*x*(x^n)^2*
(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n+3*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b*c^2*
d*n+6*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n^2+x*(x^n)^2*(c+d*x^n)^(-(1+3*
n)/n)*b*c^2*d+5*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n+2*x*x^n*(c+d*x^n)^(
-(1+3*n)/n)*b*c^3*n+2*x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3*n^2+x*x^n*(c+d*x^n)^(
-(1+3*n)/n)*a*c^2*d+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*b*c^3+3*x*(c+d*x^n)^(-(1+
3*n)/n)*a*c^3*n+x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3)/c^3/(2*n^2+3*n+1)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$$
$$= \frac{(2\,ad^3n^2+bcd^2n)xx^{3\,n}+(6\,acd^2n^2+bc^2d+(3\,bc^2d+2\,acd^2)n)xx^{2\,n}+(6\,ac^2dn^2+bc^3+ac^2d+(2\,bc^3+}{(2\,c^3n^2+3\,c^3n+c^3)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

output
```
((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2
*d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3
 + 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2
+ 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

3.331.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. 2(105) = 210.

Time = 2.48 (sec) , antiderivative size = 959, normalized size of antiderivative = 7.55

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

output
```
2*a*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c
)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 +
1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a*c**2
*c**(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*ga
mma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2
*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a*c**2*c**(1/n)*c**
(-3 - 2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) +
2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*
(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*c*c**(1/n)*c**(-3 - 2/n)*d*n**
2*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*
d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 +
 d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a*c*c**(1/n)*c**(-3 - 2/n)*d*n*x*x**
n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*
x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n
/c)**(1/n)*gamma(3 + 1/n)) + 2*a*c**(1/n)*c**(-3 - 2/n)*d**2*n**2*x*x**(2*
n)*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3
*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n)) + 2*b*c*c**(-3 - 1/n)*c**(1 + 1/n)*n*(c/(d*x**
n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) + d
*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + b*c*c**(-3 - 1/n)*c**(1 + 1/n...
```

3.331.7 Maxima [F]

$$\int (a+bx^n)\,(c+dx^n)^{-3-\frac{1}{n}}\,dx=\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

3.331.8 Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,
[0,0,2,1,
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^{\frac{1}{n}+3}}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)
```

3.332 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

3.332.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

output
```
x*(c+d*x^n)^(-1-1/n)/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

3.332.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}$$

input
```
Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1)
, -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

3.332.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^{-\frac{1}{n}-2} \, dx$$

$$\Big\downarrow \text{777}$$

$$\frac{n\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

input
```
Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)
^n^(-1))
```

3.332.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

3.332.4 Maple [F]

$$\int (c+d\,x^n)^{-2-\frac{1}{n}}\,dx$$

input
```
int((c+d*x^n)^(-2-1/n),x)
```

output
```
int((c+d*x^n)^(-2-1/n),x)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{d^2nxx^{2\,n}+(2\,cdn+cd)xx^n+(c^2n+c^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

output
```
(d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)
*(d*x^n + c)^((2*n + 1)/n))
```

3.332.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(39) = 78.

Time = 0.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.00

$$\begin{aligned}\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = & \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} \\ & + \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} \\ & + \frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((c+d*x**n)**(-2-1/n),x)
```

output
```
c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)**(
1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(2
 + 1/n)) + c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n
) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n
)*gamma(2 + 1/n)) + c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d**(1/n)
*n**2*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*
x**n) + 1)**(1/n)*gamma(2 + 1/n))
```

3.332.7 Maxima [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 2), x)
```

3.332.8 Giac [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 2), x)
```

3.332.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = -\frac{x^{1-2\,n}\,\left(\frac{c}{d\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,\left(c+d\,x^n\right)^{1/n}}$$

input
```
int(1/(c + d*x^n)^(1/n + 2),x)
```

output
```
-(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n))
)/(2*d^2*n*(c + d*x^n)^(1/n))
```

3.332. $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

3.333 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx$

3.333.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{bx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}$$

output
```
-d*x/c/(-a*d+b*c)/((c+d*x^n)^(1/n))+b*x*hypergeom([1, 1/n],[1+1/n],-(-a*d+
b*c)*x^n/a/(c+d*x^n))/a/(-a*d+b*c)/((c+d*x^n)^(1/n))
```

3.333.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 6.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(\frac{a(c+dx^n)}{c(a+bx^n)}+\frac{bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)}{a}+\frac{b(-bc+ad)nx^{2n}\,\text{Hypergeometric2F1}\left(2,2+\frac{1}{n},3+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2(1+2n)(c+dx^n)}\right)}{a}$$

input
```
Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

output
```
(x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*
d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*H
ypergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d
*x^n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

3.333.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-\frac{1}{n}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{b\int \frac{(dx^n+c)^{-1/n}}{bx^n+a}dx}{bc-ad}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

$$\Big\downarrow \text{904}$$

$$\frac{bx(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

input
```
Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c
 + d*x^n)^n^(-1))
```

3.333.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.333.4 Maple [F]

$$\int \frac{(c+d\,x^{n})^{-1-\frac{1}{n}}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

3.333.5 Fricas [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

3.333.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.333.7 Maxima [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

3.333.8 Giac [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx=\int \frac{1}{(a+b\,x^n)\,\left(c+d\,x^n\right)^{\frac{1}{n}+1}}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

3.334 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$

3.334.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)/((c+d*x^n)^((1-n)/n))-(b*c*(1-n)+a*d*n)*x*hyp
ergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/(-a*d+b*c)/n/((c+
d*x^n)^(1/n))
```

3.334.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \frac{x(c+dx^n)^{\frac{-1+n}{n}}\left(bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+(an+b(-1+n)x^n)\,\Phi\left(\frac{(-bc\cdots}{a(c\cdots}\right.\right.}{n\,(a+bx^n)\left(-b(bc-ad)x^{2n}\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+a\,(c+dx^n)\left(n\,(a+bx^n)-bx\cdots\right.\right.}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

output
```
(x*(c + d*x^n)^((-1 + n)/n)*(b*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a
*(c + d*x^n)), 1, 1 + n^(-1)] + (a*n + b*(-1 + n)*x^n)*HurwitzLerchPhi[((-
(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)]))/(n*(a + b*x^n)*(-(b*(b*c -
 a*d)*x^(2*n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 +
 n^(-1)]) + a*(c + d*x^n)*(n*(a + b*x^n) - b*x^n*HurwitzLerchPhi[((-(b*c)
+ a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

3.334.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)}-\frac{(adn+bc(1-n))\int \frac{(dx^n+c)^{-1/n}}{bx^n+a}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{ 904}$$

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)}-\frac{x(c+dx^n)^{-1/n}(adn+bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n
) + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/
(a*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

3.334.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.334.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{(a+b\,x^n)^2}dx$$

input
```
int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

3.334.5 Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

3.334.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.334.7 Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

3.334.8 Giac [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^{1/n}}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)
```

3.335 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx$

3.335.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}$$

output
```
1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^2-1/2*c*(b*c*(1-2*n)+2*
a*d*n)*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d
+b*c)/n/((c+d*x^n)^(1/n))
```

3.335.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.80 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.54

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx = -\frac{x(c+dx^n)^{2-\frac{1}{n}}\left((2an+b(-1+2n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)-bx^n \cdots\right.}{2n(a+bx^n)^2\left(-a(c+dx^n)(an+b(-1+n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)+x^n\left(-b(bc-ad)x^n\Phi\left(\cdots\right.\right.\right.}$$

input
```
Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
-1/2*(x*(c + d*x^n)^(2 - n^(-1))*((2*a*n + b*(-1 + 2*n)*x^n)*HurwitzLerchP
hi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)] - b*x^n*HurwitzLe
rchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)] - 2*(a*n + b*(
-1 + n)*x^n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-
1)]))/(n*(a + b*x^n)^2*(-(a*(c + d*x^n)*(a*n + b*(-1 + n)*x^n)*HurwitzLerc
hPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)]) + x^n*(-(b*(b*
c - a*d)*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 +
n^(-1)]) + (a^2*d*n - b^2*c*(-1 + n)*x^n + a*b*(-(c*(1 + n)) + d*(-2 + n)*
x^n))*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

3.335.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2}-\frac{(2adn+bc(1-2n))\int \frac{(dx^n+c)^{-\frac{1-n}{n}}}{(bx^n+a)^2}dx}{2an(bc-ad)}$$

$$\Big\downarrow \text{ 904}$$

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2}-\frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n))\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

input
```
Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c
*(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

3.335.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.335.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^3}dx$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

3.335.5 Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*
x^n + a^3), x)
```

3.335.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.335.7 Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

3.335.8 Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

input
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3,x)
```

output
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3, x)
```

3.336 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$

3.336.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx = \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}$$

output
```
1/3*b*x*(c+d*x^n)^(3-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^3-1/3*c^2*(b*c*(1-3*n)+
3*a*d*n)*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^4/(-a
*d+b*c)/n/((c+d*x^n)^(1/n))
```

3.336.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 36.96 (sec) , antiderivative size = 6405, normalized size of antiderivative = 48.16

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

output
```
Result too large to show
```

3.336.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{(3adn+bc(1-3n))\int \frac{(dx^n+c)^{2-\frac{1}{n}}}{(bx^n+a)^3}dx}{3an(bc-ad)}$$

$$\Big\downarrow \text{ 904}$$

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

input
```
Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

output
```
(b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b
*c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c
 - a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

3.336.3.1 Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

3.336.4 Maple [F]

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}dx$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

output
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

3.336.5 Fricas [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*
b^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

3.336.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.336.7 Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

3.336.8 Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

input
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4,x)
```

output
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4, x)
```

3.337 $\int x^5\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.337.1 Optimal result

Integrand size = 31, antiderivative size = 152

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{c^4(bc^2+ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^8} + \frac{c^2(3bc^2+2ad^2)(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^8} + \frac{(3bc^2+ad^2)(-c+dx)^{7/2}(c+dx)^{7/2}}{7d^8} + \frac{b(-c+dx)^{9/2}(c+dx)^{9/2}}{9d^8}$$

output
```
1/3*c^4*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^8+1/5*c^2*(2*a*d^2+3*b
*c^2)*(d*x-c)^(5/2)*(d*x+c)^(5/2)/d^8+1/7*(a*d^2+3*b*c^2)*(d*x-c)^(7/2)*(d
*x+c)^(7/2)/d^8+1/9*b*(d*x-c)^(9/2)*(d*x+c)^(9/2)/d^8
```

3.337.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{(-c+dx)^{3/2}(c+dx)^{3/2}\left(3ad^2(8c^4+12c^2d^2x^2+15d^4x^4)+b(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6)\right)}{315d^8}$$

input
```
Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^
4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x^6)))/(315*
d^8)
```

3.337.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {960, 111, 27, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right) \sqrt{dx-c}\sqrt{c+dx} \, dx$$

$$\Big\downarrow \; 960$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{dx-c}\sqrt{c+dx}dx+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

$$\Big\downarrow \; 111$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\left(\frac{\int 4c^2x^3\sqrt{dx-c}\sqrt{c+dx}dx}{7d^2}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}\right)+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\left(\frac{4c^2\int x^3\sqrt{dx-c}\sqrt{c+dx}dx}{7d^2}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}\right)+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

$$\Big\downarrow \; 111$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\left(\frac{4c^2\left(\frac{\int 2c^2x\sqrt{dx-c}\sqrt{c+dx}dx}{5d^2}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)}{7d^2}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}\right)+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\left(\frac{4c^2\left(\frac{2c^2\int x\sqrt{dx-c}\sqrt{c+dx}dx}{5d^2}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)}{7d^2}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}\right)+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{3}\left(\frac{4c^2\left(\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)}{7d^2}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}\right)\left(3a+\frac{2bc^2}{d^2}\right)+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

input
```
Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(b*x^6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2) + ((3*a + (2*b*c^2)/d^2)*
((x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2) + (4*c^2*((2*c^2*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))
/(5*d^2)))/(7*d^2)))/3
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.337.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)}{315d^8}$	92
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)}{315d^8}$	104
risch	$\frac{\sqrt{dx+c}\,(-35b\,x^8d^8-45a\,d^8x^6+5b\,c^2d^6x^6+9a\,c^2d^6x^4+6b\,c^4d^4x^4+12a\,c^4d^4x^2+8b\,c^6d^2x^2+24a\,c^6d^2+16b\,c^8)(-dx+c)}{315\sqrt{dx-c}\,d^8}$	122

input
```
int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/315/d^8*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*
d^4*x^4+36*a*c^2*d^4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$=\frac{(35\,bd^8x^8-16\,bc^8-24\,ac^6d^2-5\,(bc^2d^6-9\,ad^8)x^6-3\,(2\,bc^4d^4+3\,ac^2d^6)x^4-4\,(2\,bc^6d^2+3\,ac^4d^4)x^2)\sqrt{d}}{315\,d^8}$$

input
```
integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^
6 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)
*sqrt(d*x + c)*sqrt(d*x - c)/d^8
```

3.337.6 Sympy [F]

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\int x^5\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^6}{315\,d^8}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^4}{105\,d^6}$$

input
```
integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4
/d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b
*c^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2
- c^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(128) = 256$.

Time = 0.45 (sec) , antiderivative size = 621, normalized size of antiderivative = 4.09

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$= \frac{168\left(\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{5\,(dx+c)}{d^5}-\frac{31\,c}{d^5}\right)+\frac{321\,c^2}{d^5}\right)-\frac{451\,c^3}{d^5}\right)(dx+c)+\frac{745\,c^4}{d^5}\right)(dx+c)-\frac{405\,c^5}{d^5}\right)\sqrt{dx}}{}$$

input
```
integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/40320*(168*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 3
21*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^
5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x
 - c)))/d^5)*a*c + 3*(((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x + c)/d^7 -
 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^4/d^7)*(d*
x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23205*c^7/d
^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c) + sqrt(d
*x - c)))/d^7)*b*c + 24*(((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 -
 43*c/d^6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x +
 c) - 6335*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c)
+ 1050*c^7*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*a*d + (((2*((4*(5
*(2*(d*x + c)*(7*(d*x + c)*(8*(d*x + c)/d^8 - 73*c/d^8) + 2073*c^2/d^8) -
9833*c^3/d^8)*(d*x + c) + 75293*c^4/d^8)*(d*x + c) - 310203*c^5/d^8)*(d*x
+ c) + 216993*c^6/d^8)*(d*x + c) - 205275*c^7/d^8)*(d*x + c) + 69615*c^8/d
^8)*sqrt(d*x + c)*sqrt(d*x - c) + 22050*c^9*log(abs(-sqrt(d*x + c) + sqrt(
d*x - c)))/d^8)*b*d)/d
```

3.337.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = -\sqrt{d\,x-c}\left(\frac{\left(16\,b\,c^8+24\,a\,c^6\,d^2\right)\sqrt{c+d\,x}}{315\,d^8}-\frac{b\,x^8\,\sqrt{c+d\,x}}{9}+\frac{x^4\,\left(6\,b\,c^4\,d^4+9\,a\,c^2\,d^6\right)\sqrt{c+d\,x}}{315\,d^8}+\frac{x^2\,\left(8\,b\,c^6\,d^2+12\,a\,c^4\,d^4\right)\sqrt{c+d\,x}}{315\,d^8}-\frac{x^6\,\left(45\,a\,d^8-5\,b\,c^2\,d^6\right)\sqrt{c+d\,x}}{315\,d^8}\right)$$

input
```
int(x^5*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
-(d*x - c)^(1/2)*(((16*b*c^8 + 24*a*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8) -
(b*x^8*(c + d*x)^(1/2))/9 + (x^4*(9*a*c^2*d^6 + 6*b*c^4*d^4)*(c + d*x)^(1/
2))/(315*d^8) + (x^2*(12*a*c^4*d^4 + 8*b*c^6*d^2)*(c + d*x)^(1/2))/(315*d^
8) - (x^6*(45*a*d^8 - 5*b*c^2*d^6)*(c + d*x)^(1/2))/(315*d^8))
```

3.337. $\int x^5\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.338 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.338.1 Optimal result

Integrand size = 31, antiderivative size = 109

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx=\frac{c^2(bc^2+ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^6}+\frac{(2bc^2+ad^2)(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^6}+\frac{b(-c+dx)^{7/2}(c+dx)^{7/2}}{7d^6}$$

output
```
1/3*c^2*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/5*(a*d^2+2*b*c^2)*
(d*x-c)^(5/2)*(d*x+c)^(5/2)/d^6+1/7*b*(d*x-c)^(7/2)*(d*x+c)^(7/2)/d^6
```

3.338.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx=\frac{(-c+dx)^{3/2}(c+dx)^{3/2}(7ad^2(2c^2+3d^2x^2)+b(8c^4+12c^2d^2x^2+15d^4x^4))}{105d^6}$$

input
```
Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2) + b*(8*c^4
+ 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

3.338.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {960, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2) \sqrt{dx-c}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{7}\left(7a+\frac{4bc^2}{d^2}\right)\int x^3\sqrt{dx-c}\sqrt{c+dx}dx+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{7}\left(7a+\frac{4bc^2}{d^2}\right)\left(\frac{\int 2c^2x\sqrt{dx-c}\sqrt{c+dx}dx}{5d^2}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(7a+\frac{4bc^2}{d^2}\right)\left(\frac{2c^2\int x\sqrt{dx-c}\sqrt{c+dx}dx}{5d^2}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{7}\left(\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}\right)\left(7a+\frac{4bc^2}{d^2}\right)+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

input
```
Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(b*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2) + ((7*a + (4*b*c^2)/d^2)*
((2*c^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (x^2*(-c + d*x)^(3/2)
*(c + d*x)^(3/2))/(5*d^2)))/7
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.338.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4)}{105d^6}$	68
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4)}{105d^6}$	80
risch	$\frac{\sqrt{dx+c}\,(-15b\,x^6d^6-21a\,d^6x^4+3b\,c^2d^4x^4+7a\,c^2d^4x^2+4b\,c^4d^2x^2+14a\,c^4d^2+8b\,c^6)(-dx+c)}{105\sqrt{dx-c}\,d^6}$	98

input
```
int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105/d^6*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*
d^2*x^2+14*a*c^2*d^2+8*b*c^4)
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{(15\,bd^6x^6-8\,bc^6-14\,ac^4d^2-3\,(bc^2d^4-7\,ad^6)x^4-(4\,bc^4d^2+7\,ac^2d^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{105\,d^6}$$

input
```
integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
 - (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

3.338.6 Sympy [F]

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \int x^3\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^4}{7\,d^2}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^2}{35\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^2}{5\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4}{105\,d^6}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2}{15\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2
/d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b
*c^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(91) = 182.

Time = 0.38 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.54

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$

$$=\frac{70\left(\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)ac\ \cdots}{\cdots}$$

input
```
integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/1680*(70*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/
d^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x
+ c) + sqrt(d*x - c)))/d^3)*a*c + 7*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x +
 c)/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)
*(d*x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-s
qrt(d*x + c) + sqrt(d*x - c)))/d^5)*b*c + 14*(((2*(d*x + c)*(3*(d*x + c)*(
4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*
c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqr
t(d*x - c)))/d^4)*a*d + (((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 -
 43*c/d^6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x +
 c) - 6335*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c)
+ 1050*c^7*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*b*d)/d
```

3.338.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = -\sqrt{d\,x-c}\left(\frac{\left(8\,b\,c^6+14\,a\,c^4\,d^2\right)\,\sqrt{c+d\,x}}{105\,d^6} - \frac{b\,x^6\,\sqrt{c+d\,x}}{7} + \frac{x^2\,\left(4\,b\,c^4\,d^2+7\,a\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6} - \frac{x^4\,\left(21\,a\,d^6-3\,b\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}\right)$$

input
```
int(x^3*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
-(d*x - c)^(1/2)*(((8*b*c^6 + 14*a*c^4*d^2)*(c + d*x)^(1/2))/(105*d^6) - (
b*x^6*(c + d*x)^(1/2))/7 + (x^2*(7*a*c^2*d^4 + 4*b*c^4*d^2)*(c + d*x)^(1/2
))/(105*d^6) - (x^4*(21*a*d^6 - 3*b*c^2*d^4)*(c + d*x)^(1/2))/(105*d^6))
```

3.339 $\int x\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\ dx$

3.339.1 Optimal result

Integrand size = 29, antiderivative size = 67

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\ dx=\frac{(bc^2+ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^4}+\frac{b(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^4}$$

output
```
1/3*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/5*b*(d*x-c)^(5/2)*(d*x
+c)^(5/2)/d^4
```

3.339.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\ dx=\frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(2bc^2+5ad^2+3bd^2x^2)}{15d^4}$$

input
```
Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2*x^2))/(15*d
^4)
```

3.339.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{5}\left(5a+\frac{2bc^2}{d^2}\right)\int x\sqrt{dx-c}\sqrt{c+dx}dx+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(5a+\frac{2bc^2}{d^2}\right)}{15d^2}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

input
```
Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
((5*a + (2*b*c^2)/d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^2) + (b*x^2
*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

3.339.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.339.4 Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(3b\,d^{2}x^{2}+5a\,d^{2}+2b\,c^{2})}{15d^{4}}$	44
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^{2}x^{2}+c^{2})(3b\,d^{2}x^{2}+5a\,d^{2}+2b\,c^{2})}{15d^{4}}$	56
risch	$\frac{\sqrt{dx+c}\,(-3b\,d^{4}x^{4}-5a\,d^{4}x^{2}+b\,c^{2}d^{2}x^{2}+5a\,c^{2}d^{2}+2b\,c^{4})(-dx+c)}{15\sqrt{dx-c}\,d^{4}}$	73

input
```
int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d^4*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-(bc^2d^2-5\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^4}$$

input
```
integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqr
t(d*x + c)*sqrt(d*x - c)/d^4
```

3.339.6 Sympy [F]

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \int x\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^2}{5\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2}{15\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,d^2}$$

input
```
integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
 + 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

3.339.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(55) = 110.

Time = 0.35 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.39

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$

$$=\frac{5\left(\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bc+\cdots}{\cdots}$$

input
```
integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/120*(5*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^
3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c)))/d^3)*b*c + 20*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)
*(2*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) +
 sqrt(d*x - c)))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 -
 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x
 + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)
*b*d - 60*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) - sqrt(d*x + c)*
sqrt(d*x - c)*(d*x - 2*c))*a*c/d)/d
```

3.339.9 Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\sqrt{d\,x-c}\left(\frac{b\,x^4\,\sqrt{c+d\,x}}{5}-\frac{\left(2\,b\,c^4+5\,a\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}+\frac{x^2\,\left(5\,a\,d^4-b\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}\right)$$

input
```
int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
(d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c +
 d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^
4))
```

3.340 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$

3.340.1 Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)$$

output
```
1/3*b*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-a*c*arctan((d*x-c)^(1/2)*(d*x+c)^(1/
2)/c)+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(-bc^2+3ad^2+bd^2x^2)}{3d^2} - 2ac\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

input
```
Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 3*a*d^2 + b*d^2*x^2))/(3*d^2) -
2*a*c*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

3.340.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {960, 112, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}}{x}dx$$

$$\Big\downarrow \text{960}$$

$$a\int \frac{\sqrt{dx-c}\sqrt{c+dx}}{x}dx+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

$$\Big\downarrow \text{112}$$

$$a\left(\sqrt{dx-c}\sqrt{c+dx}-\int \frac{c^2}{x\sqrt{dx-c}\sqrt{c+dx}}dx\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

$$\Big\downarrow \text{27}$$

$$a\left(\sqrt{dx-c}\sqrt{c+dx}-c^2\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

$$\Big\downarrow \text{103}$$

$$a\left(\sqrt{dx-c}\sqrt{c+dx}-c^2d\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

$$\Big\downarrow \text{218}$$

$$a\left(\sqrt{dx-c}\sqrt{c+dx}-c\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

input
```
Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

output
```
(b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^2) + a*(Sqrt[-c + d*x]*Sqrt[c +
d*x] - c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])
```

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.340.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(66) = 132.

Time = 4.02 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

method	result
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(b\,d^2x^2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}+3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2d^2+3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2-b\,c^2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{3\sqrt{d^2x^2-c^2}\,d^2\sqrt{-c^2}}$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*d^2*x^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2
)+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(-c^2)^(1/
2)*(d^2*x^2-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*
x^2-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = -\frac{6\,acd^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(bd^2x^2-bc^2+3\,ad^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^2}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
-1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^
2 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

3.340.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = \int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

output
```
Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = ac\arcsin\left(\frac{c}{d|x|}\right)+\sqrt{d^2x^2-c^2}a+\frac{(d^2x^2-c^2)^{\frac{3}{2}}b}{3\,d^2}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
a*c*arcsin(c/(d*abs(x))) + sqrt(d^2*x^2 - c^2)*a + 1/3*(d^2*x^2 - c^2)^(3/
2)*b/d^2
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$$
$$=2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)$$
$$+\frac{1}{3}\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^2}-\frac{2\,bc}{d^2}\right)+3\,a\right)$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

output
```
2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/3*sqrt(d*x + c)*
sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^2 - 2*b*c/d^2) + 3*a)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$$
$$= a\sqrt{-c}\sqrt{c}\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)$$
$$-a\sqrt{-c}\sqrt{c}\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)-\frac{b\,(c^2-d^2\,x^2)\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{3\,d^2}$$
$$-\frac{8\,a\,\sqrt{-c}\,\sqrt{c}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2\,\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}$$

input
```
int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x,x)
```

output
```
a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
- c)^(1/2))^2 + 1) - a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/
((-c)^(1/2) - (d*x - c)^(1/2))) - (b*(c^2 - d^2*x^2)*(c + d*x)^(1/2)*(d*x
- c)^(1/2))/(3*d^2) - (8*a*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^
2)/(((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^
(1/2) - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2)
 - (d*x - c)^(1/2))^2 + 1))
```

3.341 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx$

3.341.1 Optimal result

Integrand size = 31, antiderivative size = 96

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx = b\sqrt{-c+dx}\sqrt{c+dx} - \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2x^2} - \frac{(2bc^2-ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c}$$

output
```
-1/2*(-a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1
/2)*(d*x+c)^(1/2)-1/2*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2
```

3.341.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(-a+2bx^2)}{2x^2} + \left(-2bc+\frac{ad^2}{c}\right)\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

input
```
Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(-a + 2*b*x^2))/(2*x^2) + (-2*b*c + (a*d^2)/
c)*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

3.341.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {956, 112, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}}{x^3}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int \frac{\sqrt{dx-c}\sqrt{c+dx}}{x}dx+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

$$\Big\downarrow \text{112}$$

$$\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\left(\sqrt{dx-c}\sqrt{c+dx}-\int \frac{c^2}{x\sqrt{dx-c}\sqrt{c+dx}}dx\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\left(\sqrt{dx-c}\sqrt{c+dx}-c^2\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\left(\sqrt{dx-c}\sqrt{c+dx}-c^2d\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\left(\sqrt{dx-c}\sqrt{c+dx}-c\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

input
```
Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

output
```
(a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(2*c^2*x^2) + ((2*b - (a*d^2)/c^2)*(S
qrt[-c + d*x]*Sqrt[c + d*x] - c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]))
/2
```

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.341. $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx$

3.341.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

method	result
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2x^2\sqrt{dx-c}}-\frac{\left(-b\sqrt{(dx-c)(dx+c)}+\frac{\left(a\,d^2-2b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{2\sqrt{-c^2}}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2-2b\,x^2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}+\sqrt{-c^2}\,\sqrt{}}{2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(-d*x+c)*(d*x+c)^(1/2)/x^2/(d*x-c)^(1/2)-(-b*((d*x-c)*(d*x+c))^(1/2)
+1/2*(a*d^2-2*b*c^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^
(1/2))/x))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int\frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx$$
$$=-\frac{2\,(2\,bc^2-ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(2\,bcx^2-ac)\sqrt{dx+c}\sqrt{dx-c}}{2\,cx^2}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/
c) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

3.341.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = \int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x^3}\,dx$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

output
```
Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = bc\arcsin\left(\frac{c}{d|x|}\right) - \frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c} + \sqrt{d^2x^2-c^2}b - \frac{\sqrt{d^2x^2-c^2}ad^2}{2\,c^2} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{2\,c^2x^2}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
b*c*arcsin(c/(d*abs(x))) - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2
 - c^2)*b - 1/2*sqrt(d^2*x^2 - c^2)*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^(3/2)*
a/(c^2*x^2)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}bd + \frac{(2\,bc^2d-ad^3)\arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2\,c}\right)}{c} + \frac{2\left(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6-4\,ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2\right)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^4+4\,c^2\right)^2}}{d}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
(sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*
x + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^
6 - 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(
d*x - c))^4 + 4*c^2)^2)/d
```

3.341.9 Mupad [B] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.08

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = b\sqrt{-c}\sqrt{c}\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)$$
$$-\frac{\frac{a\sqrt{-c}\,d^2}{32\,c^{3/2}}+\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}$$
$$-b\sqrt{-c}\sqrt{c}\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)+\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{3/2}}-\frac{a}{32}$$

input
```
int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^3,x)
```

output
```
b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
- c)^(1/2))^2 + 1) - ((a*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (a*(-c)^(1/2)*d^2*
((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^
2) - (15*a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)
^(1/2) - (d*x - c)^(1/2))^4))/((((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) -
 (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x
 - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/
2))^6) - b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2)
- (d*x - c)^(1/2))) + (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((
-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^2*log(((c + d
*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2))
- (a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2)
 - (d*x - c)^(1/2))^2) - (8*b*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2
))^2)/(((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-
c)^(1/2) - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1
/2) - (d*x - c)^(1/2))^2 + 1))
```

3.341. $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx$

3.342 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$

3.342.1 Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = -\frac{(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^2x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{d^2(4bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^3}$$

output
```
1/4*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^4+1/8*d^2*(a*d^2+4*b*c^2)*arctan((
d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3-1/8*(a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c
)^(1/2)/c^2/x^2
```

3.342.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = \frac{c\sqrt{-c+dx}\sqrt{c+dx}(-2ac^2-4bc^2x^2+ad^2x^2)+2d^2(4bc^2+ad^2)\,x^4\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^3x^4}$$

input
```
Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

output
```
(c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a*c^2 - 4*b*c^2*x^2 + a*d^2*x^2) + 2*d
^2*(4*b*c^2 + a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^3*x^4)
```

3.342.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {956, 105, 105, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}}{x^5}dx$$

$$\Big\downarrow \ 956$$

$$\frac{1}{4}\left(\frac{ad^2}{c^2}+4b\right)\int \frac{\sqrt{dx-c}\sqrt{c+dx}}{x^3}dx+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

$$\Big\downarrow \ 105$$

$$\frac{1}{4}\left(\frac{ad^2}{c^2}+4b\right)\left(\frac{1}{2}d\int \frac{\sqrt{c+dx}}{x^2\sqrt{dx-c}}dx-\frac{\sqrt{dx-c}(c+dx)^{3/2}}{2cx^2}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

$$\Big\downarrow \ 105$$

$$\frac{1}{4}\left(\frac{ad^2}{c^2}+4b\right)\left(\frac{1}{2}d\left(d\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{\sqrt{dx-c}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{dx-c}(c+dx)^{3/2}}{2cx^2}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

$$\Big\downarrow \ 103$$

$$\frac{1}{4}\left(\frac{ad^2}{c^2}+4b\right)\left(\frac{1}{2}d\left(d^2\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)+\frac{\sqrt{dx-c}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{dx-c}(c+dx)}{2cx^2}\right.$$
$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

$$\Big\downarrow \ 218$$

$$\frac{1}{4}\left(\frac{ad^2}{c^2}+4b\right)\left(\frac{1}{2}d\left(\frac{d\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{\sqrt{dx-c}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{dx-c}(c+dx)^{3/2}}{2cx^2}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

input
```
Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

output
```
(a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*c^2*x^4) + ((4*b + (a*d^2)/c^2)*(-
1/2*(Sqrt[-c + d*x]*(c + d*x)^(3/2))/(c*x^2) + (d*((Sqrt[-c + d*x]*Sqrt[c
+ d*x])/(c*x) + (d*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/c))/2))/4
```

3.342.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.342.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(-a\,d^{2}x^{2}+4b\,c^{2}x^{2}+2c^{2}a\right)}{8x^{4}c^{2}\sqrt{dx-c}}-\frac{d^{2}\left(a\,d^{2}+4b\,c^{2}\right)\ln\left(\frac{-2c^{2}+2\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}}{x}\right)\sqrt{(dx-c)(dx+c)}}{8c^{2}\sqrt{-c^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)a\,d^{4}x^{4}+4\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)b\,c^{2}d^{2}x^{4}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{2}x^{2}+4\sqrt{\cdots}}{8c^{2}\sqrt{d^{2}x^{2}-c^{2}}\,x^{4}\sqrt{-c^{2}}}$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x+c)^(1/2)*(-d*x+c)*(-a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/x^4/c^2/(d*x-c
)^(1/2)-1/8*d^2*(a*d^2+4*b*c^2)/c^2/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/
2)
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$$
$$=\frac{2\,(4\,bc^2d^2+ad^4)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(2\,ac^3+(4\,bc^3-acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^3x^4}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c
))/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^3*x^4)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = -\frac{bd^2 \arcsin\left(\frac{c}{d|x|}\right)}{2\,c} - \frac{ad^4 \arcsin\left(\frac{c}{d|x|}\right)}{8\,c^3} - \frac{\sqrt{d^2x^2-c^2}bd^2}{2\,c^2} - \frac{\sqrt{d^2x^2-c^2}ad^4}{8\,c^4} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}b}{2\,c^2x^2} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}ad^2}{8\,c^4x^2} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{4\,c^2x^4}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/2*b*d^2*arcsin(c/(d*abs(x)))/c - 1/8*a*d^4*arcsin(c/(d*abs(x)))/c^3 - 1
/2*sqrt(d^2*x^2 - c^2)*b*d^2/c^2 - 1/8*sqrt(d^2*x^2 - c^2)*a*d^4/c^4 + 1/2
*(d^2*x^2 - c^2)^(3/2)*b/(c^2*x^2) + 1/8*(d^2*x^2 - c^2)^(3/2)*a*d^2/(c^4*
x^2) + 1/4*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^4)
```

3.342.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(103) = 206.

Time = 0.36 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = -\frac{\frac{(4\,bc^2d^3+ad^5)\arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2\,c}\right)}{c^3} - \frac{2\left(4\,bc^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^{14} - ad^5(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 16\,bc^4d^3(\sqrt{dx+c}-\sqrt{dx-c})^{10} + 28\ldots}{\ldots}}{\ldots}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c
)/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*
x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^
10 + 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(
d*x + c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c)
)^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt
(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)
^4*c^2))/d
```

3.342.9 Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 1004, normalized size of antiderivative = 8.30

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$$
$$= \frac{\frac{a\sqrt{-c}\,d^4}{1024\,c^{7/2}} + \frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} + \frac{11\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{7\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6} - \frac{239\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{1024\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8} + \cdots}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6} + \frac{6\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8} + \frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}} + \frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}}$$
$$- \frac{\frac{b\sqrt{-c}\,d^2}{32\,c^{3/2}} + \frac{b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} - \frac{15\,b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} + \frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}} + \frac{a\sqrt{-c}\,d^4\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{7/2}}$$
$$+ \frac{b\sqrt{-c}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{3/2}} - \frac{a\sqrt{-c}\,d^4\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{8\,c^{7/2}}$$
$$- \frac{b\sqrt{-c}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{3/2}} + \frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{256\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$
$$+ \frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{1024\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} - \frac{b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

input
```
int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^5,x)
```

output
```
((a*(-c)^(1/2)*d^4)/(1024*c^(7/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) -
c^(1/2))^2)/(128*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (11*a*(-c)^(1
/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(7/2)*((-c)^(1/2) - (d*x - c
)^(1/2))^4) + (7*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(7
/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) - (239*a*(-c)^(1/2)*d^4*((c + d*x)^(
1/2) - c^(1/2))^8)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (a*(-
c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(7/2)*((-c)^(1/2) - (d
*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)
^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2
))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
+ (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (
(c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - ((b*(-c
)^(1/2)*d^2)/(32*c^(3/2)) + (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^
2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(1/2)*d^2*((
c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4)
)/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c
 + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^
(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*(-c)^(1/2)*d^4*l
og(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(7/2)
) + (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*...
```

3.342. $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$

3.343 $\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$

3.343.1 Optimal result

Integrand size = 31, antiderivative size = 208

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{c^6(5bc^2+8ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{64d^7}$$

output
```
1/64*c^2*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/48*(8*a*d^2
+5*b*c^2)*x^3*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/8*b*x^5*(d*x-c)^(3/2)*(d*x
+c)^(3/2)/d^2-1/64*c^6*(8*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/
2))/d^7+1/128*c^4*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

3.343.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$=\frac{dx\sqrt{-c+dx}\sqrt{c+dx}(8ad^2(-3c^4-2c^2d^2x^2+8d^4x^4)-b(15c^6+10c^4d^2x^2+8c^2d^4x^4-48d^6x^6))-6c^6(5b}{384d^7}$$

input
```
Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(8*a*d^2*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4
*x^4) - b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4 - 48*d^6*x^6)) - 6*c^6*
(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(384*d^7)
```

3.343.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {960, 111, 27, 101, 27, 40, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2\right)\sqrt{dx-c}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\int x^4\sqrt{dx-c}\sqrt{c+dx}dx+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{\int 3c^2x^2\sqrt{dx-c}\sqrt{c+dx}dx}{6d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\int x^2\sqrt{dx-c}\sqrt{c+dx}dx}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$\downarrow$ 101

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{\int c^2\sqrt{dx-c}\sqrt{c+dx}dx}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$\downarrow$ 27

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{c^2\int \sqrt{dx-c}\sqrt{c+dx}dx}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$\downarrow$ 40

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{1}{2}c^2\int\frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$\downarrow$ 45

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-c^2\int\frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

$\downarrow$ 221

3.343. $\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

$$\frac{1}{8}\left(8a+\frac{5bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)}{2d^2}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}\right)+$$

$$\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

input
```
Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(b*x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) + ((8*a + (5*b*c^2)/d^2)*
((x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(6*d^2) + (c^2*((x*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(4*d^2) + (c^2*((x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 - (c
^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d))/(4*d^2)))/(2*d^2)))/8
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.343.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x\left(-48b\,x^{6}d^{6}-64a\,d^{6}x^{4}+8b\,c^{2}d^{4}x^{4}+16a\,c^{2}d^{4}x^{2}+10b\,c^{4}d^{2}x^{2}+24a\,c^{4}d^{2}+15b\,c^{6}\right)\left(-dx+c\right)\sqrt{dx+c}}{384d^{6}\sqrt{dx-c}}-\frac{c^{6}\left(8a\,d^{2}+5b\,c^{2}\right)\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}\cdots}\right.}{128d^{6}\sqrt{d^{2}}\,\sqrt{dx-\cdots}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-48\,\mathrm{csgn}(d)b\,d^{7}x^{7}\sqrt{d^{2}x^{2}-c^{2}}-64\,\mathrm{csgn}(d)a\,d^{7}x^{5}\sqrt{d^{2}x^{2}-c^{2}}+8\,\mathrm{csgn}(d)b\,c^{2}d^{5}x^{5}\sqrt{d^{2}x^{2}-c^{2}}+16\,\mathrm{csgn}(d)a\,c^{2}d^{5}x^{3}\cdots\right.}{\cdots}$

input
```
int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*x*(-48*b*d^6*x^6-64*a*d^6*x^4+8*b*c^2*d^4*x^4+16*a*c^2*d^4*x^2+10*b*
c^4*d^2*x^2+24*a*c^4*d^2+15*b*c^6)*(-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2
)-1/128*c^6*(8*a*d^2+5*b*c^2)/d^6*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2)
)/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$=\frac{(48\,bd^7x^7-8\,(bc^2d^5-8\,ad^7)x^5-2\,(5\,bc^4d^3+8\,ac^2d^5)x^3-3\,(5\,bc^6d+8\,ac^4d^3)x)\sqrt{dx+c}\sqrt{dx-c}+3\,(5\,b}{384\,d^7}$$

input
```
integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*
c^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c)
+ 3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

3.343.6 Sympy [F]

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx=\int x^4(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.18

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{128\,d^7}-\frac{ac^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^5}+\frac{5\,\sqrt{d^2x^2-c^2}bc^6x}{128\,d^6}+\frac{\sqrt{d^2x^2-c^2}ac^4x}{16\,d^4}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x}{64\,d^6}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x}{8\,d^4}$$

input
```
integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3
/d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d)/d^7 - 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)
*d)/d^5 + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)
*a*c^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2
)^(3/2)*a*c^2*x/d^4
```

3.343.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(178) = 356.

Time = 0.40 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.68

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{112\left(\left(\left(2\left(dx+c\right)\left(3\left(dx+c\right)\left(\frac{4\,(dx+c)}{d^4}-\frac{21\,c}{d^4}\right)+\frac{133\,c^2}{d^4}\right)-\frac{295\,c^3}{d^4}\right)(dx+c)+\frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{90\,c^5}{\cdots}}{\cdots}$$

input
```
integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/13440*(112*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 13
3*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x
- c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*a*c + 8*(((2*(
(4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6) + 661*c^2/d^6) - 45
51*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335*c^5/d^6)*(d*x + c)
+ 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*log(abs(-sqrt(d*x +
 c) + sqrt(d*x - c)))/d^6)*b*c + 56*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x +
 c)/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)
*(d*x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-s
qrt(d*x + c) + sqrt(d*x - c)))/d^5)*a*d + (((2*((4*(5*(d*x + c)*(6*(d*x +
c)*(7*(d*x + c)/d^7 - 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c)
 + 64233*c^4/d^7)*(d*x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d
*x + c) - 23205*c^7/d^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-s
qrt(d*x + c) + sqrt(d*x - c)))/d^7)*b*d)/d
```

3.343.9 Mupad [B] (verification not implemented)

Time = 49.01 (sec) , antiderivative size = 2314, normalized size of antiderivative = 11.12

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \text{Too large to display}$$

input
```
int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
((35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) - (a*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/
2))) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) -
(d*x - c)^(1/2))^7) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)
^(1/2) - (d*x - c)^(1/2))^9) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^11
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*a*c^6*((c + d*x)^(1/2) -
c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*a*c^6*((c + d*
x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*a*c^
6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) +
(757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2
))^19) + (35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x
- c)^(1/2))^21) - (a*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) -
(d*x - c)^(1/2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^
(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^
(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)
^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c
)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((
-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)
/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2...
```

3.343. $\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.344 $\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.344.1 Optimal result

Integrand size = 31, antiderivative size = 159

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx=\frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}-\frac{c^4(bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}$$

output
```
1/8*(2*a*d^2+b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/6*b*x^3*(d*x-c)^(3
/2)*(d*x+c)^(3/2)/d^2-1/8*c^4*(2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c
)^(1/2))/d^5+1/16*c^2*(2*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4
```

3.344.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$$
$$=\frac{dx\sqrt{-c+dx}\sqrt{c+dx}(-6ad^2(c^2-2d^2x^2)+b(-3c^4-2c^2d^2x^2+8d^4x^4))-6c^4(bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+}}{\sqrt{c+}}\right.}{48d^5}$$

input
```
Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-6*a*d^2*(c^2 - 2*d^2*x^2) + b*(-3*c^4
- 2*c^2*d^2*x^2 + 8*d^4*x^4)) - 6*c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c +
d*x]/Sqrt[c + d*x]])/(48*d^5)
```

3.344.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {960, 101, 27, 40, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right) \sqrt{dx-c}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \ 960$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{dx-c}\sqrt{c+dx}dx+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

$$\Big\downarrow \ 101$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\left(\frac{\int c^2\sqrt{dx-c}\sqrt{c+dx}dx}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\left(\frac{c^2\int\sqrt{dx-c}\sqrt{c+dx}dx}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

$$\Big\downarrow \ 40$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{1}{2}c^2\int\frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

$$\Big\downarrow \ 45$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-c^2\int\frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\left(\frac{c^2\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}\right)}{4d^2}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}\right)+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

input
```
Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(b*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(6*d^2) + ((2*a + (b*c^2)/d^2)*((
x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*d^2) + (c^2*((x*Sqrt[-c + d*x]*Sqrt
[c + d*x])/2 - (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d))/(4*d^2)))/2
```

3.344.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.344.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x\left(-8b\,d^4x^4-12a\,d^4x^2+2b\,c^2d^2x^2+6a\,c^2d^2+3b\,c^4\right)\left(-dx+c\right)\sqrt{dx+c}}{48d^4\sqrt{dx-c}}-\frac{c^4\left(2a\,d^2+b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{16d^4\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-8\,\mathrm{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}-12\,\mathrm{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+2\,\mathrm{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}+6\,\mathrm{csgn}(d)d^3\sqrt{d^2x^2-c^2}\right.}{48\sqrt{}}$

input
```
int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(-8*b*d^4*x^4-12*a*d^4*x^2+2*b*c^2*d^2*x^2+6*a*c^2*d^2+3*b*c^4)*(-d
*x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)-1/16*c^4*(2*a*d^2+b*c^2)/d^4*ln(x*d^
2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*
x-c)^(1/2)/(d*x+c)^(1/2)
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$=\frac{(8\,bd^5x^5-2\,(bc^2d^3-6\,ad^5)x^3-3\,(bc^4d+2\,ac^2d^3)x)\sqrt{dx+c}\sqrt{dx-c}+3\,(bc^6+2\,ac^4d^2)\log\left(-dx+\sqrt{dx\right.}{48\,d^5}$$

input
```
integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^
3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqr
t(d*x + c)*sqrt(d*x - c)))/d^5
```

3.344.6 Sympy [F]

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\int x^2\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.21

$$\begin{aligned}\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=&\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^5}\\&-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^3}\\&+\frac{\sqrt{d^2x^2-c^2}bc^4x}{16\,d^4}+\frac{\sqrt{d^2x^2-c^2}ac^2x}{8\,d^2}\\&+\frac{(d^2x^2-c^2)^{\frac{3}{2}}bc^2x}{8\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax}{4\,d^2}\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*
x^2 - c^2)*d)/d^5 - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 +
 1/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^
2 + 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/
d^2
```

3.344.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(135) = 270$.

Time = 0.38 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.72

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$$
$$= \frac{40\left(\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)ac+2\left(\left(\left(2\,(dx+c)\left(3\,(c\right.\right.\right.\right.}{}$$

input
```
integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/240*(40*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d
^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)*a*c
 + 2*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^
4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 9
0*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*c + 10*(((d*x + c)*(
2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*sqrt(
d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d
^3)*a*d + (((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 321*
c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)*
sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^5)*b*d)/d
```

3.344.9 Mupad [B] (verification not implemented)

Time = 55.45 (sec) , antiderivative size = 1681, normalized size of antiderivative = 10.57

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \text{Too large to display}$$

input
```
int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/
2))) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) -
(d*x - c)^(1/2))^7) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)
^(1/2) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*b*c^6*((c + d*x)^(1/2) -
c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*
x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*b*c^
6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) +
(757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2
))^19) + (35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x
- c)^(1/2))^21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) -
(d*x - c)^(1/2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^
(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^
(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)
^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c
)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((
-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)
/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2...
```

3.345 $\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.345.1 Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2(bc^2+4ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}$$

output
```
1/4*b*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-1/4*c^2*(4*a*d^2+b*c^2)*arctanh((d
*x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/8*(4*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)
^(1/2)/d^2
```

3.345.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{dx\sqrt{-c+dx}\sqrt{c+dx}(-bc^2+4ad^2+2bd^2x^2) - 2c^2(bc^2+4ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^3}$$

input
```
Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 4*a*d^2 + 2*b*d^2*x^2) - 2*c
^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^3)
```

3.345.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {646, 40, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\sqrt{dx-c}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{646}$$

$$\frac{(4ad^2+bc^2)\int \sqrt{dx-c}\sqrt{c+dx}dx}{4d^2}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{40}$$

$$\frac{(4ad^2+bc^2)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{1}{2}c^2\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx\right)}{4d^2}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{45}$$

$$\frac{(4ad^2+bc^2)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-c^2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^2}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{(4ad^2+bc^2)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}\right)}{4d^2}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

input
```
Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

output
```
(b*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*d^2) + ((b*c^2 + 4*a*d^2)*((x*Sq
rt[-c + d*x]*Sqrt[c + d*x])/2 - (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
)/d))/(4*d^2)
```

3.345.3.1 Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

3.345.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{x\left(2b\,d^{2}x^{2}+4a\,d^{2}-b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{8d^{2}\sqrt{dx-c}}-\frac{c^{2}\left(4a\,d^{2}+b\,c^{2}\right)\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{8d^{2}\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\,\mathrm{csgn}(d)b\,d^{3}x^{3}\sqrt{d^{2}x^{2}-c^{2}}+4\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)d^{3}ax-\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)db\,c^{2}x-4\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)+dx\right)\mathrm{c}\right.\right.}{8\sqrt{d^{2}x^{2}-c^{2}}\,d^{3}}$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*b*d^2*x^2+4*a*d^2-b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^2/(d*x-c)^(1/2
)-1/8*c^2*(4*a*d^2+b*c^2)/d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d
^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$=\frac{(2\,bd^3x^3-(bc^2d-4\,ad^3)x)\sqrt{dx+c}\sqrt{dx-c}+(bc^4+4\,ac^2d^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^3}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (
b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

3.345.6 Sympy [F]

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\int \left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=-\frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^3}-\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d}+\frac{1}{2}\sqrt{d^2x^2-c^2}ax+\frac{\sqrt{d^2x^2-c^2}bc^2x}{8\,d^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx}{4\,d^2}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 - 1/2*a*c^2*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(
d^2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2
```

3.345.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(96) = 192$.

Time = 0.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.53

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$= \frac{24\left(2\,c\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)+\sqrt{dx+c}\sqrt{dx-c}\right)ac+4\left(\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right.\right.\right.}{}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*(24*(2*c*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) + sqrt(d*x + c)*sqr
t(d*x - c))*a*c + 4*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d
^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))
)/d^2)*b*c + (((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^
2/d^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*
x + c) + sqrt(d*x - c)))/d^3)*b*d - 12*(2*c^2*log(abs(-sqrt(d*x + c) + sqr
t(d*x - c))) - sqrt(d*x + c)*sqrt(d*x - c)*(d*x - 2*c))*a)/d
```

3.345.9 Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 734, normalized size of antiderivative = 6.44

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{a\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}$$
$$-\frac{\frac{b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{35\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{273\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{715\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{715\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{273\,b}{2\,(}}{d^3-\frac{8\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28}{}}$$
$$-\frac{a\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d}+\frac{b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^3}$$

input
```
int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

output
```
(a*x*(c + d*x)^(1/2)*(d*x - c)^(1/2))/2 - ((b*c^4*((c + d*x)^(1/2) - c^(1/
2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*b*c^4*((c + d*x)^(1/2) - c^(
1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*b*c^4*((c + d*x)^(1/2
) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*b*c^4*((c + d*
x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*b*c^4*(
(c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*
b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11
) + (35*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(
1/2))^13) + (b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x -
 c)^(1/2))^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) -
 (d*x - c)^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) -
 (d*x - c)^(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) -
 (d*x - c)^(1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) -
 (d*x - c)^(1/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2)
- (d*x - c)^(1/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2
) - (d*x - c)^(1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/
2) - (d*x - c)^(1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2
) - (d*x - c)^(1/2))^16) - (a*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(1/2
)))/(2*d) + (b*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x -
c)^(1/2))))/(2*d^3)
```

3.345. $\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

3.346 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx$

3.346.1 Optimal result

Integrand size = 31, antiderivative size = 104

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{(bc^2-2ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

output
```
a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x-(-2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)
/(d*x+c)^(1/2))/d+1/2*(b-2*a*d^2/c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(-2a+bx^2)}{2x} + \left(-\frac{bc^2}{d}+2ad\right)\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

input
```
Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a + b*x^2))/(2*x) + (-((b*c^2)/d) + 2*a*
d)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

3.346.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {956, 40, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}}{x^2}dx$$

$$\Big\downarrow \text{956}$$

$$\left(b-\frac{2ad^2}{c^2}\right)\int \sqrt{dx-c}\sqrt{c+dx}dx+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

$$\Big\downarrow \text{40}$$

$$\left(b-\frac{2ad^2}{c^2}\right)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{1}{2}c^2\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

$$\Big\downarrow \text{45}$$

$$\left(b-\frac{2ad^2}{c^2}\right)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-c^2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

$$\Big\downarrow \text{221}$$

$$\left(b-\frac{2ad^2}{c^2}\right)\left(\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

input
```
Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

output
```
(a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(c^2*x) + (b - (2*a*d^2)/c^2)*((x*Sqr
t[-c + d*x]*Sqrt[c + d*x])/2 - (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])
/d)
```

3.346.3.1 Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.346.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(-b\,x^{2}+2a\right)}{2x\sqrt{dx-c}}-\frac{\left(-a\,d^{2}+\frac{b\,c^{2}}{2}\right)\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-\operatorname{csgn}(d)bd\,x^{2}\sqrt{d^{2}x^{2}-c^{2}}-2\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^{2}x+\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right.\right.}{2\sqrt{d^{2}x^{2}-c^{2}}\,xd}$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x+c)^(1/2)*(-d*x+c)*(-b*x^2+2*a)/x/(d*x-c)^(1/2)-(-a*d^2+1/2*b*c^2)
*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(
1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{2\,ad^2x-(bc^2-2\,ad^2)x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-(bdx^2-2\,ad)\sqrt{dx+c}\sqrt{dx-c}}{2\,dx}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x -
c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

3.346.6 Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

output
```
Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d} + ad\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right) + \frac{1}{2}\,\sqrt{d^2x^2-c^2}bx - \frac{\sqrt{d^2x^2-c^2}a}{x}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + a*d*log(2*d^2*x + 2*
sqrt(d^2*x^2 - c^2)*d) + 1/2*sqrt(d^2*x^2 - c^2)*b*x - sqrt(d^2*x^2 - c^2)
*a/x
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{\frac{32\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - 2\left((dx+c)b-bc\right)\sqrt{dx+c}\sqrt{dx-c} - (bc^2-2\,ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{4\,d}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/4*(32*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x +
 c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

3.346.9 Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \frac{a\,d+\frac{5\,a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}} - 4\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) + \frac{b\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2} - \frac{b\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d} + \frac{a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

input
```
int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)
```

output
```
(a*d + (5*a*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2)
)^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*
((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*a*d*a
tanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) + (b*x*(c
 + d*x)^(1/2)*(d*x - c)^(1/2))/2 - (b*c^2*log(d*x + (c + d*x)^(1/2)*(d*x -
 c)^(1/2)))/(2*d) + (a*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*
x - c)^(1/2)))
```

3.347 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx$

3.347.1 Optimal result

Integrand size = 31, antiderivative size = 84

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

output
```
1/3*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^3+2*b*d*arctanh((d*x-c)^(1/2)/(d*x
+c)^(1/2))-b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x
```

3.347.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{\sqrt{-c+dx}\sqrt{c+dx}(3bc^2x^2+a(c^2-d^2x^2))}{3c^2x^3} + 2bd\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

input
```
Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

output
```
-1/3*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 - d^2*x^2)))/(c^2
*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

3.347.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {956, 108, 27, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{dx-c}\sqrt{c+dx}}{x^4}dx$$

$$\Big\downarrow \text{956}$$

$$b\int \frac{\sqrt{dx-c}\sqrt{c+dx}}{x^2}dx+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}$$

$$\Big\downarrow \text{108}$$

$$b\left(\int \frac{d^2}{\sqrt{dx-c}\sqrt{c+dx}}dx-\frac{\sqrt{dx-c}\sqrt{c+dx}}{x}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}$$

$$\Big\downarrow \text{27}$$

$$b\left(d^2\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx-\frac{\sqrt{dx-c}\sqrt{c+dx}}{x}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}$$

$$\Big\downarrow \text{45}$$

$$b\left(2d^2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}-\frac{\sqrt{dx-c}\sqrt{c+dx}}{x}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\left(2d\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)-\frac{\sqrt{dx-c}\sqrt{c+dx}}{x}\right)$$

input
```
Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

output
```
(a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*c^2*x^3) + b*(-((Sqrt[-c + d*x]*Sq
rt[c + d*x])/x) + 2*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.347.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.48

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(-a\,d^{2}x^{2}+3b\,c^{2}x^{2}+c^{2}a\right)}{3x^{3}c^{2}\sqrt{dx-c}}+\frac{b\,d^{2}\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-3\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^{2}d\,x^{3}-\operatorname{csgn}(d)a\,d^{2}x^{2}\sqrt{d^{2}x^{2}-c^{2}}+3\,\operatorname{csgn}(d)b\,c^{2}x^{2}\sqrt{d^{2}x^{2}-c^{2}}+\operatorname{csgn}(\right.}{3\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x^{3}}$

3.347. $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx$

input
```
int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x+c)^(1/2)*(-d*x+c)*(-a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^2/(d*x-c)^
(1/2)+b*d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)
*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)+(3\,bc^2d-ad^3)x^3+(ac^2+(3\,bc^2-ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^2x^3}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
 a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))
/(c^2*x^3)
```

3.347.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = \text{Exception raised: MellinTransformStripError}$$

input
```
integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

output
```
Exception raised: MellinTransformStripError >> Pole inside critical strip?
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = bd\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right) - \frac{\sqrt{d^2x^2-c^2}b}{x} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{3\,c^2x^3}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
b*d*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d) - sqrt(d^2*x^2 - c^2)*b/x + 1/3
*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^3)
```

3.347.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(70) = 140.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{3\,bd^2\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+\cdots\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

input
```
integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sq
rt(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8
 + 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^
4*d^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

3.347.9 Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = \frac{b\,d+\frac{5\,b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}} - 4\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) - \frac{\left(\frac{a\,\sqrt{c+d\,x}}{3}-\frac{a\,d^2\,x^2\,\sqrt{c+d\,x}}{3\,c^2}\right)\,\sqrt{d\,x-c}}{x^3} + \frac{b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

input
```
int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^4,x)
```

output
```
(b*d + (5*b*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2)
)^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*
((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*b*d*a
tanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) - (((a*(c
 + d*x)^(1/2))/3 - (a*d^2*x^2*(c + d*x)^(1/2))/(3*c^2))*(d*x - c)^(1/2))/x
^3 + (b*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
```

3.348 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.348.1 Optimal result

Integrand size = 29, antiderivative size = 125

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\operatorname{arccosh}(cx)}{16c^7}$$

output
```
1/16*(6*a*c^2+5*b)*arccosh(c*x)/c^7+1/16*(6*a*c^2+5*b)*x*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^6+1/24*(6*a*c^2+5*b)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/6*
b*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

3.348.2 Mathematica [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{cx\sqrt{-1+cx}\sqrt{1+cx}(6ac^2(3+2c^2x^2)+b(15+10c^2x^2+8c^4x^4))+6(5b+6ac^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{48c^7}$$

input
```
Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^
2*x^2 + 8*c^4*x^4)) + 6*(5*b + 6*a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]
])/(48*c^7)
```

3.348.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {960, 111, 27, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^2)}{\sqrt{cx-1}\sqrt{cx+1}}\,dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{6}\left(6a+\frac{5b}{c^2}\right)\int \frac{x^4}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{6}\left(6a+\frac{5b}{c^2}\right)\left(\frac{\int \frac{3x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(6a+\frac{5b}{c^2}\right)\left(\frac{3\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{6}\left(6a+\frac{5b}{c^2}\right)\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{6}\left(6a+\frac{5b}{c^2}\right)\left(\frac{3\left(\frac{\text{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}\right)+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

input
```
Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

3.348. $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

output
```
(b*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^2) + ((6*a + (5*b)/c^2)*((x^3*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + (3*((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/(4*c^2)))/6
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.348. $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.348.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
risch	$\frac{x\left(8b\,x^{4}c^{4}+12a\,c^{4}x^{2}+10b\,c^{2}x^{2}+18c^{2}a+15b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{48c^{6}}+\frac{\left(6c^{2}a+5b\right)\ln\left(\frac{c^{2}x}{\sqrt{c^{2}}}+\sqrt{c^{2}x^{2}-1}\right)\sqrt{\left(cx-1\right)\left(cx+1\right)}}{16c^{6}\sqrt{c^{2}}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8\,\mathrm{csgn}(c)b\,c^{5}x^{5}\sqrt{c^{2}x^{2}-1}+12\,\mathrm{csgn}(c)a\,c^{5}x^{3}\sqrt{c^{2}x^{2}-1}+10\,\mathrm{csgn}(c)c^{3}\sqrt{c^{2}x^{2}-1}\,b\,x^{3}+18\,\mathrm{csgn}(c)c^{3}\sqrt{c^{2}x^{2}-1}\,ax+15\,\mathrm{cs}}{48c^{7}\sqrt{c^{2}x^{2}-1}}$

input
```
int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b*c^4*x^4+12*a*c^4*x^2+10*b*c^2*x^2+18*a*c^2+15*b)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^6+1/16*(6*a*c^2+5*b)/c^6*ln(c^2*x/(c^2)^(1/2)+(c^2*x^2-1)
^(1/2))/(c^2)^(1/2)*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(8\,bc^5x^5+2\,(6\,ac^5+5\,bc^3)x^3+3\,(6\,ac^3+5\,bc)x)\sqrt{cx+1}\sqrt{cx-1}-3\,(6\,ac^2+5\,b)\log\left(-cx+\sqrt{cx+1}\sqrt{c}\right.}{48\,c^7}$$

input
```
integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sq
rt(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqr
t(c*x - 1)))/c^7
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
Timed out
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^5}{6\,c^2} + \frac{\sqrt{c^2x^2-1}ax^3}{4\,c^2} + \frac{5\,\sqrt{c^2x^2-1}bx^3}{24\,c^4} + \frac{3\,\sqrt{c^2x^2-1}ax}{8\,c^4} + \frac{3\,a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{8\,c^5} + \frac{5\,\sqrt{c^2x^2-1}bx}{16\,c^6} + \frac{5\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{16\,c^7}$$

input
```
integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*s
qrt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c
^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5 + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*
b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left(\left(2\left(\left(cx+1\right)\left(4\left(cx+1\right)\left(\frac{(cx+1)b}{c^6}-\frac{5\,b}{c^6}\right)+\frac{3\,(2\,ac^{38}+15\,bc^{36})}{c^{42}}\right)-\frac{18\,ac^{38}+55\,bc^{36}}{c^{42}}\right)(cx+1)+\frac{54\,ac^{38}+85\,bc^{36}}{c^{42}}\right)(cx+\right.}{48\,c}$$

input
```
integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/48*(((2*((c*x + 1)*(4*(c*x + 1)*((c*x + 1)*b/c^6 - 5*b/c^6) + 3*(2*a*c^3
8 + 15*b*c^36)/c^42) - (18*a*c^38 + 55*b*c^36)/c^42)*(c*x + 1) + (54*a*c^3
8 + 85*b*c^36)/c^42)*(c*x + 1) - 3*(10*a*c^38 + 11*b*c^36)/c^42)*sqrt(c*x
+ 1)*sqrt(c*x - 1) - 6*(6*a*c^2 + 5*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/
c^6)/c
```

3.348.9 Mupad [B] (verification not implemented)

Time = 45.44 (sec) , antiderivative size = 1154, normalized size of antiderivative = 9.23

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Too large to display}$$

input
```
int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((23*a*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*a*((c*
x - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*a*((c*x - 1)^(1/2
) - 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*a*((c*x - 1)^(1/2) - 1i)^9)/
(2*((c*x + 1)^(1/2) - 1)^9) + (333*a*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x +
 1)^(1/2) - 1)^11) + (23*a*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2)
- 1)^13) - (3*a*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) -
(3*a*((c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*
x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) -
 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x
+ 1)^(1/2) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
)^8 - (56*c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^
5*((c*x - 1)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)
/((c*x + 1)^(1/2) - 1)^16) - ((311*b*((c*x - 1)^(1/2) - 1i)^5)/(4*((c*x +
1)^(1/2) - 1)^5) - (175*b*((c*x - 1)^(1/2) - 1i)^3)/(12*((c*x + 1)^(1/2) -
 1)^3) + (8361*b*((c*x - 1)^(1/2) - 1i)^7)/(4*((c*x + 1)^(1/2) - 1)^7) + (
42259*b*((c*x - 1)^(1/2) - 1i)^9)/(6*((c*x + 1)^(1/2) - 1)^9) + (25295*b*(
(c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^(1/2) - 1)^11) + (25295*b*((c*x -
1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^13) + (42259*b*((c*x - 1)^(1/2
) - 1i)^15)/(6*((c*x + 1)^(1/2) - 1)^15) + (8361*b*((c*x - 1)^(1/2) - 1...
```

3.349 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.349.1 Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2(4b+5ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6} + \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}$$

output
```
2/15*(5*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6+1/15*(5*a*c^2+4*b)*x^2*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/5*b*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

3.349.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(5ac^2(2+c^2x^2)+b(8+4c^2x^2+3c^4x^4))}{15c^6}$$

input
```
Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 +
3*c^4*x^4)))/(15*c^6)
```

3.349.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {960, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \textbf{960}$$

$$\frac{1}{5}\left(5a+\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

$$\Big\downarrow \textbf{111}$$

$$\frac{1}{5}\left(5a+\frac{4b}{c^2}\right)\left(\frac{\int \frac{2x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\left(5a+\frac{4b}{c^2}\right)\left(\frac{2\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx}{3c^2}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

$$\Big\downarrow \textbf{83}$$

$$\frac{1}{5}\left(\frac{2\sqrt{cx-1}\sqrt{cx+1}}{3c^4}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}\right)\left(5a+\frac{4b}{c^2}\right)+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

input
```
Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2) + ((5*a + (4*b)/c^2)*((2*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*
c^2)))/5
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.349.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57

input
```
int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a
*c^2+8*b)/c^6
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(3\,bc^4x^4 + 10\,ac^2 + (5\,ac^4 + 4\,bc^2)x^2 + 8\,b)\sqrt{cx+1}\sqrt{cx-1}}{15\,c^6}$$

input
```
integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1
)*sqrt(c*x - 1)/c^6
```

3.349.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.10

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 & \\ -\frac{11}{4}, -\frac{9}{4} & -3, -\frac{5}{2}, -\frac{5}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6}$$

input
```
integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2
, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi
)/(c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2
, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*
c**6) + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4),
(-3, -5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^4}{5\,c^2} + \frac{\sqrt{c^2x^2-1}ax^2}{3\,c^2} + \frac{4\sqrt{c^2x^2-1}bx^2}{15\,c^4} + \frac{2\sqrt{c^2x^2-1}a}{3\,c^4} + \frac{8\sqrt{c^2x^2-1}b}{15\,c^6}$$

input
```
integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*s
qrt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x
^2 - 1)*b/c^6
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left(\left(\left((cx+1)\left(3\,(cx+1)\left(\frac{(cx+1)b}{c^5}-\frac{4\,b}{c^5}\right)+\frac{5\,ac^{27}+22\,bc^{25}}{c^{30}}\right)-\frac{10\,(ac^{27}+2\,bc^{25})}{c^{30}}\right)(cx+1)+\frac{15\,(ac^{27}+bc^{25})}{c^{30}}\right)\sqrt{cx+1}\sqrt{c}}{15\,c}$$

input
```
integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/15*(((c*x + 1)*(3*(c*x + 1)*((c*x + 1)*b/c^5 - 4*b/c^5) + (5*a*c^27 + 22
*b*c^25)/c^30) - 10*(a*c^27 + 2*b*c^25)/c^30)*(c*x + 1) + 15*(a*c^27 + b*c
^25)/c^30)*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

3.349.9 Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{\sqrt{c\,x-1}\,\left(\frac{10\,a\,c^2+8\,b}{15\,c^6}+\frac{b\,x^5}{5\,c}+\frac{b\,x^4}{5\,c^2}+\frac{x^2\,\left(5\,a\,c^4+4\,b\,c^2\right)}{15\,c^6}+\frac{x^3\,\left(5\,a\,c^5+4\,b\,c^3\right)}{15\,c^6}+\frac{x\,\left(10\,a\,c^3+8\,b\,c\right)}{15\,c^6}\right)}{\sqrt{c\,x+1}}$$

input
```
int((x^3*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((c*x - 1)^(1/2)*((8*b + 10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c
^2) + (x^2*(5*a*c^4 + 4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5 + 4*b*c^3))/(15*c
^6) + (x*(8*b*c + 10*a*c^3))/(15*c^6)))/(c*x + 1)^(1/2)
```

3.350 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.350.1 Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\operatorname{arccosh}(cx)}{8c^5}$$

output
```
1/8*(4*a*c^2+3*b)*arccosh(c*x)/c^5+1/8*(4*a*c^2+3*b)*x*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^4+1/4*b*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

3.350.2 Mathematica [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{cx\sqrt{-1+cx}\sqrt{1+cx}(4ac^2+b(3+2c^2x^2)) + (6b+8ac^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{8c^5}$$

input
```
Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (6*b + 8
*a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(8*c^5)
```

3.350.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {960, 101, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{4}\left(4a+\frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{4}\left(4a+\frac{3b}{c^2}\right)\left(\frac{\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(4a+\frac{3b}{c^2}\right)\left(\frac{\operatorname{arccosh}(cx)}{2c^3}+\frac{x\sqrt{cx-1}\sqrt{cx+1}}{2c^2}\right)+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

input
```
Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + ((4*a + (3*b)/c^2)*((x*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ArcCosh[c*x]/(2*c^3)))/4
```

3.350.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.350.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x\left(2b\,c^2x^2+4c^2a+3b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{8c^4}+\frac{\left(4c^2a+3b\right)\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx-1)(cx+1)}}{8c^4\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\Big(2\,\operatorname{csgn}(c)c^3\sqrt{c^2x^2-1}\,b\,x^3+4\,\operatorname{csgn}(c)c^3\sqrt{c^2x^2-1}\,ax+3\,\operatorname{csgn}(c)c\sqrt{c^2x^2-1}\,bx+4\ln\Big(\Big(\sqrt{c^2x^2-1}\,\operatorname{csgn}(c)+cx\Big)\operatorname{csgn}(c)}{8c^5\sqrt{c^2x^2-1}}$

input
```
int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*c^2*x^2+4*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/8*(4*a*c
^2+3*b)/c^4*ln(c^2*x/(c^2)^(1/2)+(c^2*x^2-1)^(1/2))/(c^2)^(1/2)*((c*x-1)*(
c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(2\,bc^3x^3+(4\,ac^3+3\,bc)x)\sqrt{cx+1}\sqrt{cx-1}-(4\,ac^2+3\,b)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{8\,c^5}$$

input
```
integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*
a*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
Timed out
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^3}{4\,c^2}+\frac{\sqrt{c^2x^2-1}ax}{2\,c^2}+\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{2\,c^3} + \frac{3\,\sqrt{c^2x^2-1}bx}{8\,c^4}+\frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{8\,c^5}$$

input
```
integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*lo
g(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3
/8*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left((cx+1)\left(2\,(cx+1)\left(\frac{(cx+1)b}{c^4}-\frac{3\,b}{c^4}\right)+\frac{4\,ac^{18}+9\,bc^{16}}{c^{20}}\right)-\frac{4\,ac^{18}+5\,bc^{16}}{c^{20}}\right)\sqrt{cx+1}\sqrt{cx-1}-\frac{2\,(4\,ac^2+3\,b)\log(\sqrt{cx+1}-}{c^4}}{8\,c}$$

input
```
integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/8*(((c*x + 1)*(2*(c*x + 1)*((c*x + 1)*b/c^4 - 3*b/c^4) + (4*a*c^18 + 9*b
*c^16)/c^20) - (4*a*c^18 + 5*b*c^16)/c^20)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2
*(4*a*c^2 + 3*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^4)/c
```

3.350.9 Mupad [B] (verification not implemented)

Time = 29.41 (sec) , antiderivative size = 720, normalized size of antiderivative = 8.28

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{23\,b\,(\sqrt{cx-1}-\mathrm{i})^3}{2\,(\sqrt{cx+1}-1)^3}+\frac{333\,b\,(\sqrt{cx-1}-\mathrm{i})^5}{2\,(\sqrt{cx+1}-1)^5}+\frac{671\,b\,(\sqrt{cx-1}-\mathrm{i})^7}{2\,(\sqrt{cx+1}-1)^7}+\frac{671\,b\,(\sqrt{cx-1}-\mathrm{i})^9}{2\,(\sqrt{cx+1}-1)^9}+\frac{333\,b\,(\sqrt{cx-1}-\mathrm{i})^{11}}{2\,(\sqrt{cx+1}-1)^{11}}+\frac{23\,b\,(\sqrt{cx-1}-\mathrm{i})^{13}}{2\,(\sqrt{cx+1}-1)^{13}}-}{c^5-\frac{8\,c^5\,(\sqrt{cx-1}-\mathrm{i})^2}{(\sqrt{cx+1}-1)^2}+\frac{28\,c^5\,(\sqrt{cx-1}-\mathrm{i})^4}{(\sqrt{cx+1}-1)^4}-\frac{56\,c^5\,(\sqrt{cx-1}-\mathrm{i})^6}{(\sqrt{cx+1}-1)^6}+\frac{70\,c^5\,(\sqrt{cx-1}-\mathrm{i})^8}{(\sqrt{cx+1}-1)^8}-\frac{56\,c^5\,(\sqrt{cx-1}-\mathrm{i})^{10}}{(\sqrt{cx+1}-1)^{10}}+\frac{28\,c^5\,(\sqrt{cx-1}-\mathrm{i})^{12}}{(\sqrt{cx+1}-1)^{12}}}$$
$$-\frac{\frac{14\,a\,(\sqrt{cx-1}-\mathrm{i})^3}{(\sqrt{cx+1}-1)^3}+\frac{14\,a\,(\sqrt{cx-1}-\mathrm{i})^5}{(\sqrt{cx+1}-1)^5}+\frac{2\,a\,(\sqrt{cx-1}-\mathrm{i})^7}{(\sqrt{cx+1}-1)^7}+\frac{2\,a\,(\sqrt{cx-1}-\mathrm{i})}{\sqrt{cx+1}-1}}{c^3-\frac{4\,c^3\,(\sqrt{cx-1}-\mathrm{i})^2}{(\sqrt{cx+1}-1)^2}+\frac{6\,c^3\,(\sqrt{cx-1}-\mathrm{i})^4}{(\sqrt{cx+1}-1)^4}-\frac{4\,c^3\,(\sqrt{cx-1}-\mathrm{i})^6}{(\sqrt{cx+1}-1)^6}+\frac{c^3\,(\sqrt{cx-1}-\mathrm{i})^8}{(\sqrt{cx+1}-1)^8}}$$
$$+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)}{c^3}+\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)}{2\,c^5}$$

input
```
int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((23*b*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*b*((c*
x - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*b*((c*x - 1)^(1/2
) - 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*b*((c*x - 1)^(1/2) - 1i)^9)/
(2*((c*x + 1)^(1/2) - 1)^9) + (333*b*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x +
 1)^(1/2) - 1)^11) + (23*b*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2)
- 1)^13) - (3*b*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) -
(3*b*((c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*
x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) -
 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x
+ 1)^(1/2) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
)^8 - (56*c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^
5*((c*x - 1)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)
/((c*x + 1)^(1/2) - 1)^16) - ((14*a*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(
1/2) - 1)^3 + (14*a*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2
*a*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*a*((c*x - 1)^(1/
2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/(
(c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2)
 - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*
((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((c*x -...
```

3.350. $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.351 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$

3.351.1 Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx = \frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}$$

output
```
1/3*(3*a*c^2+2*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/3*b*x^2*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^2
```

3.351.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(3ac^2+b(2+c^2x^2))}{3c^4}$$

input
```
Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4)
```

3.351.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)}{\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{3}\left(3a+\frac{2b}{c^2}\right)\int \frac{x}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3a+\frac{2b}{c^2}\right)}{3c^2}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

input
```
Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
((3*a + (2*b)/c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2) + (b*x^2*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(3*c^2)
```

3.351.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.351.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38

input
```
int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(bc^2x^2+3\,ac^2+2\,b)\sqrt{cx+1}\sqrt{cx-1}}{3\,c^4}$$

input
```
integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

3.351.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.11

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$

input
```
integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()
), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4,
 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), (
(-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) +
I*b*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2,
 -3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^2}{3\,c^2} + \frac{\sqrt{c^2x^2-1}a}{c^2} + \frac{2\,\sqrt{c^2x^2-1}b}{3\,c^4}$$

input
```
integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x
^2 - 1)*b/c^4
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{cx+1}\sqrt{cx-1}\left((cx+1)\left(\frac{(cx+1)b}{c^3}-\frac{2\,b}{c^3}\right)+\frac{3\,(ac^{11}+bc^9)}{c^{12}}\right)}{3\,c}$$

input
```
integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*((c*x + 1)*b/c^3 - 2*b/c^3) + 3
*(a*c^11 + b*c^9)/c^12)/c
```

3.351.9 Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c\,x-1}\left(\frac{3\,a\,c^2+2\,b}{3\,c^4}+\frac{b\,x^3}{3\,c}+\frac{b\,x^2}{3\,c^2}+\frac{x\,(3\,a\,c^3+2\,b\,c)}{3\,c^4}\right)}{\sqrt{c\,x+1}}$$

input
```
int((x*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((c*x - 1)^(1/2)*((2*b + 3*a*c^2)/(3*c^4) + (b*x^3)/(3*c) + (b*x^2)/(3*c^2
) + (x*(2*b*c + 3*a*c^3))/(3*c^4)))/(c*x + 1)^(1/2)
```

3.352 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.352.1 Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2}+\frac{(b+2ac^2)\operatorname{arccosh}(cx)}{2c^3}$$

output
```
1/2*(2*a*c^2+b)*arccosh(c*x)/c^3+1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

3.352.2 Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{bcx\sqrt{-1+cx}\sqrt{1+cx}+2(b+2ac^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{2c^3}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*(b + 2*a*c^2)*ArcTanh[Sqrt[(-1 + c
*x)/(1 + c*x)]])/(2*c^3)
```

3.352.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {646, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{cx-1}\sqrt{cx+1}}\,dx$$
$$\Big\downarrow \text{ 646}$$
$$\frac{(2ac^2+b)\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx}{2c^2}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$
$$\Big\downarrow \text{ 43}$$
$$\frac{(2ac^2+b)\operatorname{arccosh}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

input
```
Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/
(2*c^3)
```

3.352.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

3.352.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 4.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

method	result
risch	$\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}+\frac{(2c^2a+b)\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx-1)(cx+1)}}{2c^2\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\operatorname{csgn}(c)c\sqrt{c^2x^2-1}\,bx+2\ln\left(\left(\sqrt{c^2x^2-1}\,\operatorname{csgn}(c)+cx\right)\operatorname{csgn}(c)\right)a\,c^2+\ln\left(\left(\sqrt{c^2x^2-1}\,\operatorname{csgn}(c)+cx\right)\operatorname{csgn}(c)\right)b\right)\operatorname{csgn}(c)}{2c^3\sqrt{c^2x^2-1}}$

input
```
int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2+1/2*(2*a*c^2+b)/c^2*ln(c^2*x/(c^2)
^(1/2)+(c^2*x^2-1)^(1/2))/(c^2)^(1/2)*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{\sqrt{cx+1}\sqrt{cx-1}bcx-(2\,ac^2+b)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{2\,c^3}$$

input
```
integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)))/c^3
```

3.352.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
Timed out
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c}+\frac{\sqrt{c^2x^2-1}bx}{2\,c^2} + \frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{2\,c^3}$$

input
```
integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + 1/2*sqrt(c^2*x^2 - 1)*b*x/c^2 +
 1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{cx+1}\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2}-\frac{b}{c^2}\right)-\frac{2\,(2\,ac^2+b)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^2}}{2\,c}$$

input
```
integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 +
b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^2)/c
```

3.352.9 Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 6.23

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

input
```
int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
(2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c
*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) -
1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)
^(1/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 -
 (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x -
 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)
^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1
/2) - 1)^8) - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/(((c*x + 1)^(1/2) - 1)*
(-c^2)^(1/2))))/(-c^2)^(1/2)
```

3.352. $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.353 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.353.1 Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
a*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

3.353.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+2a\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

input
```
Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + 2*a*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x
)]]
```

3.353.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {960, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{960}$$

$$a\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

$$\Big\downarrow \text{103}$$

$$ac\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

$$\Big\downarrow \text{218}$$

$$a\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

input
```
Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*
x]]
```

3.353.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.353.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2+\sqrt{c^2x^2-1}\,b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}\,c^2}$	62

input
```
int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+(c^2*x^2-1)^(1/2)*b)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(c^2*x^2-1)^(1/2)/c^2
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

input
```
integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
(2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c
*x - 1)*b)/c^2
```

3.353.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.95 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

input
```
integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
 ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/
2)) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2,
 -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/
(c**2*x**2))/(4*pi**(3/2)*c**2)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}b}{c^2}$$

input
```
integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-a*arcsin(1/(c*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

input
```
integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
-2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b/c^2
```

3.353.9 Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{b\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c^2} - a\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1\mathrm{i}$$

input
```
int((a + b*x^2)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
(b*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c^2 - a*(log(((c*x - 1)^(1/2) - 1i)^2/
((c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2)
 - 1)))*1i
```

3.354 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.354.1 Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x}+\frac{b\operatorname{arccosh}(cx)}{c}$$

output
```
b*arccosh(c*x)/c+a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

3.354.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x}+\frac{2b\operatorname{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{c}$$

input
```
Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (2*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x
)]])/c
```

3.354.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {956, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^2\sqrt{cx-1}\sqrt{cx+1}}\,dx$$

$$\Big\downarrow \text{956}$$

$$b\int \frac{1}{\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}$$

$$\Big\downarrow \text{43}$$

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\operatorname{arccosh}(cx)}{c}$$

input
```
Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

3.354.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.354.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\operatorname{csgn}(c) c\sqrt{c^{2}x^{2}-1}\, a+\ln\left(\left(\sqrt{c^{2}x^{2}-1}\, \operatorname{csgn}(c)+cx\right) \operatorname{csgn}(c)\right) bx\right) \operatorname{csgn}(c)}{\sqrt{c^{2}x^{2}-1}\, xc}$	77
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\ln\left(\frac{c^{2}x}{\sqrt{c^{2}}}+\sqrt{c^{2}x^{2}-1}\right)\sqrt{(cx-1)(cx+1)}}{\sqrt{c^{2}}\,\sqrt{cx-1}\,\sqrt{cx+1}}$	78

input
```
int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(csgn(c)*c*(c^2*x^2-1)^(1/2)*a+ln(((c^2*x^2-1)
^(1/2)*csgn(c)+c*x)*csgn(c))*b*x)*csgn(c)/(c^2*x^2-1)^(1/2)/x/c
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{ac^2x+\sqrt{cx+1}\sqrt{cx-1}ac-bx\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{cx}$$

input
```
integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*
sqrt(c*x - 1)))/(c*x)
```

3.354.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{acG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iacG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

input
```
integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
-a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1)
, ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi*
*(3/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1,
 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/
4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x
**2))/(4*pi**(3/2)*c)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c} + \frac{\sqrt{c^2x^2-1}a}{x}$$

input
```
integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + sqrt(c^2*x^2 - 1)*a/x
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4} - b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

input
```
integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x +
1) - sqrt(c*x - 1))^4))/c
```

3.354.9 Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{x} - \frac{4\,b\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

input
```
int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
(a*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/x - (4*b*atan((c*((c*x - 1)^(1/2) - 1i
))/(((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

3.355 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.355.1 Optimal result

Integrand size = 29, antiderivative size = 60

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
1/2*(a*c^2+2*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*a*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/x^2
```

3.355.2 Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\left(2b+ac^2\right)\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

input
```
Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + (2*b + a*c^2)*ArcTan[Sqrt[(-1 +
 c*x)/(1 + c*x)]]
```

3.355.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {956, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{2}\left(ac^2+2b\right)\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}c\left(ac^2+2b\right)\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(ac^2+2b\right)\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

input
```
Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]])/2
```

3.355.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.355.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}-\frac{\left(b+\frac{c^2 a}{2}\right)\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{(cx-1)(cx+1)}}{\sqrt{cx-1}\,\sqrt{cx+1}}$	71
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2x^2+2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,x^2-\sqrt{c^2x^2-1}\,a\right)}{2\sqrt{c^2x^2-1}\,x^2}$	84

input
```
int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-(b+1/2*c^2*a)*arctan(1/(c^2*x^2-1)^(
1/2))*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\,(ac^2+2\,b)x^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}a}{2\,x^2}$$

input
```
integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt
(c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

3.355.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
Timed out
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{1}{2}\,ac^2\arcsin\left(\frac{1}{c|x|}\right) - b\arcsin\left(\frac{1}{c|x|}\right) + \frac{\sqrt{c^2x^2-1}a}{2\,x^2}$$

input
```
integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*c^2*arcsin(1/(c*abs(x))) - b*arcsin(1/(c*abs(x))) + 1/2*sqrt(c^2*x^
2 - 1)*a/x^2
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(48) = 96.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{(ac^3+2\,bc)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right) + \frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

input
```
integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
-((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3
*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1
))^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

3.355.9 Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 4.95

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{a\,c^2\,1\mathrm{i}}{32}+\frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,\left(\sqrt{c\,x+1}-1\right)^2}-\frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^4}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}} - b\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1\mathrm{i} - \frac{a\,c^2\,\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2} + \frac{a\,c^2\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}}{2} + \frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^2}$$

input
```
int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2)
- 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4
))/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2)
 - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1
/2) - 1)^6) - b*(log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)
 - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))*1i - (a*c^2*log(((c*
x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log(((c*x
 - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) -
 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

3.356 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.356.1 Optimal result

Integrand size = 29, antiderivative size = 62

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{(3b+2ac^2)\,\sqrt{-1+cx}\sqrt{1+cx}}{3x}$$

output
```
1/3*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^3+1/3*(2*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/x
```

3.356.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}(a+3bx^2+2ac^2x^2)}{3x^3}$$

input
```
Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3)
```

3.356.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {956, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{3}\left(2ac^2+3b\right)\int \frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

$$\Big\downarrow \text{106}$$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

input
```
Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])/(3*x)
```

3.356.3.1 Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.356.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	37
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	37
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{csgn}(c)^2(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	41

input
```
int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(2\,ac^3+3\,bc)x^3+((2\,ac^2+3\,b)x^2+a)\sqrt{cx+1}\sqrt{cx-1}}{3\,x^3}$$

input
```
integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(
c*x - 1))/x^3
```

3.356.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.35

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bcG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
-a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4,
 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2)
)/(4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/
2, 7/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4
, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c
**2*x**2))/(4*pi**(3/2))
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\sqrt{c^2x^2-1}ac^2}{3\,x} + \frac{\sqrt{c^2x^2-1}b}{x} + \frac{\sqrt{c^2x^2-1}a}{3\,x^3}$$

input
```
integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
 1)*a/x^3
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{8\left(3\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^8 + 24\,ac^4\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4 + 24\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4 + 32\,ac^4 +}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^3 c}$$

input
```
integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
 sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4
+ 48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c\,x-1}\left(\left(\frac{2\,a\,c^3}{3}+b\,c\right)x^3+\left(\frac{2\,a\,c^2}{3}+b\right)x^2+\frac{a\,c\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{c\,x+1}}$$

input
```
int((a + b*x^2)/(x^4*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((c*x - 1)^(1/2)*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) +
(a*c*x)/3))/(x^3*(c*x + 1)^(1/2))
```

3.356. $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.357 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.357.1 Optimal result

Integrand size = 29, antiderivative size = 99

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2} + \frac{1}{8}c^2(4b+3ac^2)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
1/8*c^2*(3*a*c^2+4*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*a*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/x^4+1/8*(3*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

3.357.2 Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{1}{8}\left(\frac{\sqrt{-1+cx}\sqrt{1+cx}(4bx^2+a(2+3c^2x^2))}{x^4} + (8bc^2+6ac^4)\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)$$

input
```
Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*b*x^2 + a*(2 + 3*c^2*x^2)))/x^4 + (8*b*c
^2 + 6*a*c^4)*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]])/8
```

3.357.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {956, 114, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^5\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{4}\left(3ac^2+4b\right)\int \frac{1}{x^3\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4}\left(3ac^2+4b\right)\left(\frac{1}{2}\int \frac{c^2}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(3ac^2+4b\right)\left(\frac{1}{2}c^2\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{4}\left(3ac^2+4b\right)\left(\frac{1}{2}c^3\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(3ac^2+4b\right)\left(\frac{1}{2}c^2\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}}{2x^2}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

input
```
Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
(a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*((Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(2*x^2) + (c^2*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2))
/4
```

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.357.4 Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

method	result
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3a\,c^2x^2+4b\,x^2+2a)}{8x^4}-\frac{c^2(3c^2a+4b)\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{(cx-1)(cx+1)}}{8\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^4x^4+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,c^2x^4-3\sqrt{c^2x^2-1}\,a\,c^2x^2-4\sqrt{c^2x^2-1}\,b\,x^2-2\sqrt{c^2x^2-1}\,a\right)}{8\sqrt{c^2x^2-1}\,x^4}$

input
```
int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*a*c^2*x^2+4*b*x^2+2*a)/x^4-1/8*c^2*(3*a
*c^2+4*b)*arctan(1/(c^2*x^2-1)^(1/2))*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$=\frac{2\,(3\,ac^4+4\,bc^2)x^4\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+((3\,ac^2+4\,b)x^2+2\,a)\sqrt{cx+1}\sqrt{cx-1}}{8\,x^4}$$

input
```
integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
+ ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
Timed out
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{3}{8}\,ac^4\arcsin\left(\frac{1}{c|x|}\right)-\frac{1}{2}\,bc^2\arcsin\left(\frac{1}{c|x|}\right) + \frac{3\sqrt{c^2x^2-1}ac^2}{8\,x^2}+\frac{\sqrt{c^2x^2-1}b}{2\,x^2}+\frac{\sqrt{c^2x^2-1}a}{4\,x^4}$$

input
```
integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*a*c^4*arcsin(1/(c*abs(x))) - 1/2*b*c^2*arcsin(1/(c*abs(x))) + 3/8*sqr
t(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2
- 1)*a/x^4
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.71

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{(3\,ac^5+4\,bc^3)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(3\,ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4\,bc^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44\,ac^5\left(\sqrt{cx\right.\right.}{}}{}}{}$$

input
```
integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) +
2*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - s
qrt(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x -
 1))^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x
+ 1) - sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((s
qrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

3.357.9 Mupad [B] (verification not implemented)

Time = 31.05 (sec) , antiderivative size = 650, normalized size of antiderivative = 6.57

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{b\,c^2\,1\mathrm{i}}{32}+\frac{b\,c^2\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{c\,x+1}-1\right)}^2}-\frac{b\,c^2\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{c\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{c\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{c\,x+1}-1\right)}^6}}$$
$$-\frac{\frac{a\,c^4\,1\mathrm{i}}{1024}-\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2\,3\mathrm{i}}{128\,{\left(\sqrt{c\,x+1}-1\right)}^2}-\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^4\,53\mathrm{i}}{512\,{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^6\,87\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^6}+\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^8\,657\mathrm{i}}{1024\,{\left(\sqrt{c\,x+1}-1\right)}^8}+\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{10}\,121\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^{10}}}{\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{c\,x+1}-1\right)}^6}+\frac{6\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{c\,x+1}-1\right)}^8}+\frac{4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{c\,x+1}-1\right)}^{10}}+\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{12}}{{\left(\sqrt{c\,x+1}-1\right)}^{12}}}$$
$$-\frac{a\,c^4\,\ln\left(\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{c\,x+1}-1\right)}^2}+1\right)\,3\mathrm{i}}{8}-\frac{b\,c^2\,\ln\left(\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{c\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}$$
$$+\frac{a\,c^4\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,3\mathrm{i}}{8}+\frac{b\,c^2\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2\,7\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^2}$$
$$-\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^4\,1\mathrm{i}}{1024\,{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{b\,c^2\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{c\,x+1}-1\right)}^2}$$

input
```
int((a + b*x^2)/(x^5*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
((b*c^2*1i)/32 + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2)
- 1)^2) - (b*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4
))/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2)
 - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1
/2) - 1)^6) - ((a*c^4*1i)/1024 - (a*c^4*((c*x - 1)^(1/2) - 1i)^2*3i)/(128*
((c*x + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*53i)/(512*((c*x
 + 1)^(1/2) - 1)^4) + (a*c^4*((c*x - 1)^(1/2) - 1i)^6*87i)/(256*((c*x + 1)
^(1/2) - 1)^6) + (a*c^4*((c*x - 1)^(1/2) - 1i)^8*657i)/(1024*((c*x + 1)^(1
/2) - 1)^8) + (a*c^4*((c*x - 1)^(1/2) - 1i)^10*121i)/(256*((c*x + 1)^(1/2)
 - 1)^10))/((((c*x - 1)^(1/2) - 1i)^4/((c*x + 1)^(1/2) - 1)^4 + (4*((c*x -
1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (6*((c*x - 1)^(1/2) - 1i)^8)/(
(c*x + 1)^(1/2) - 1)^8 + (4*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) -
1)^10 + ((c*x - 1)^(1/2) - 1i)^12/((c*x + 1)^(1/2) - 1)^12) - (a*c^4*log((
(c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (b*c^2*log(((
c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^4*log(((c
*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*3i)/8 + (b*c^2*log(((c*x - 1)^(
1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^4*((c*x - 1)^(1/2) - 1i)^2*
7i)/(256*((c*x + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*1i)/(1
024*((c*x + 1)^(1/2) - 1)^4) + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c
*x + 1)^(1/2) - 1)^2)
```

3.357. $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.358 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.358.1 Optimal result

Integrand size = 31, antiderivative size = 164

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{c^4(5bc^2+6ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^7}$$

output
```
1/8*c^4*(6*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7+1/16*c^
2*(6*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/24*(6*a*d^2+5*b*c^
2)*x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/6*b*x^5*(d*x-c)^(1/2)*(d*x+c)^(1/
2)/d^2
```

3.358.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{dx\sqrt{-c+dx}\sqrt{c+dx}(6ad^2(3c^2+2d^2x^2)+b(15c^4+10c^2d^2x^2+8d^4x^4))+6c^4(5bc^2+6ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c}}{\sqrt{c+}}\right.}{48d^7}$$

input
```
Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4
 + 10*c^2*d^2*x^2 + 8*d^4*x^4)) + 6*c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/(48*d^7)
```

3.358.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {960, 111, 27, 101, 27, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^2)}{\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\int \frac{x^4}{\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{\int \frac{3c^2x^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{3c^2\int \frac{x^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{3c^2\left(\frac{\int \frac{c^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{3c^2\left(\frac{c^2\int\frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \; 45$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{3c^2\left(\frac{c^2\int\frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(6a+\frac{5bc^2}{d^2}\right)\left(\frac{3c^2\left(\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{4d^2}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}\right)+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

input
```
Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*x^5*Sqrt[-c + d*x]*Sqrt[c + d*x])/(6*d^2) + ((6*a + (5*b*c^2)/d^2)*((x^
3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (3*c^2*((x*Sqrt[-c + d*x]*Sqrt[c
 + d*x])/(2*d^2) + (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3))/(4*d^
2)))/6
```

3.358.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.358.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x\left(8b\,d^4x^4+12a\,d^4x^2+10b\,c^2d^2x^2+18a\,c^2d^2+15b\,c^4\right)(-dx+c)\sqrt{dx+c}}{48d^6\sqrt{dx-c}}+\frac{c^4\left(6a\,d^2+5b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{16d^6\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\,\mathrm{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}+12\,\mathrm{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+10\,\mathrm{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}+18\,\mathrm{csgn}(d)d^3\sqrt{d^2x^2-c^2}\,a\right.}{48d}$

input
```
int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(8*b*d^4*x^4+12*a*d^4*x^2+10*b*c^2*d^2*x^2+18*a*c^2*d^2+15*b*c^4)*
(-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2)+1/16*c^4*(6*a*d^2+5*b*c^2)/d^6*ln
(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2
)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.70

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(8\,bd^5x^5+2\,(5\,bc^2d^3+6\,ad^5)x^3+3\,(5\,bc^4d+6\,ac^2d^3)x)\sqrt{dx+c}\sqrt{dx-c}-3\,(5\,bc^6+6\,ac^4d^2)\log\left(-dx+\right.}{48\,d^7}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^
2*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x
 + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^5}{6\,d^2} + \frac{5\,\sqrt{d^2x^2-c^2}bc^2x^3}{24\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax^3}{4\,d^2} + \frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^7} + \frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^5} + \frac{5\,\sqrt{d^2x^2-c^2}bc^4x}{16\,d^6} + \frac{3\,\sqrt{d^2x^2-c^2}ac^2x}{8\,d^4}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4
 + 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2
*x^2 - c^2)*d)/d^7 + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5
+ 5/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d
^4
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.24

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{5\,bc}{d^6}\right)+\frac{3\,(15\,bc^2d^{36}+2\,ad^{38})}{d^{42}}\right)-\frac{55\,bc^3d^{36}+18\,acd^{38}}{d^{42}}\right)(dx+c)+\frac{85\,bc^4d^{36}+54\,ac^2d}{d^{42}}\right.}{48\,d}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(((2*((d*x + c)*(4*(d*x + c)*((d*x + c)*b/d^6 - 5*b*c/d^6) + 3*(15*b*
c^2*d^36 + 2*a*d^38)/d^42) - (55*b*c^3*d^36 + 18*a*c*d^38)/d^42)*(d*x + c)
 + (85*b*c^4*d^36 + 54*a*c^2*d^38)/d^42)*(d*x + c) - 3*(11*b*c^5*d^36 + 10
*a*c^3*d^38)/d^42)*sqrt(d*x + c)*sqrt(d*x - c) - 6*(5*b*c^6 + 6*a*c^4*d^2)
*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)/d
```

3.358.9 Mupad [B] (verification not implemented)

Time = 58.97 (sec) , antiderivative size = 1682, normalized size of antiderivative = 10.26

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
int((x^4*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((5*b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
- (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1
/2))^3) + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x
- c)^(1/2))^5) + (8361*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2)
 - (d*x - c)^(1/2))^7) + (42259*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((
-c)^(1/2) - (d*x - c)^(1/2))^9) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1/2))
^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25295*b*c^6*((c + d*x)^(1/2)
 - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (42259*b*c^6*((c +
 d*x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (8361*b
*c^6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17)
 + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^19) - (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (
d*x - c)^(1/2))^21) + (5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1
/2) - (d*x - c)^(1/2))^23))/(d^7 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/
((-c)^(1/2) - (d*x - c)^(1/2))^2 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/
((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^6)
/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^8
)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^
10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^7*((c + d*x)^(1/2) - c^(1/2
))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^7*((c + d*x)^(1/2) - ...
```

3.359 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.359.1 Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{2c^2(4bc^2+5ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6}+\frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}$$

output
```
2/15*c^2*(5*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/15*(5*a*d^2+4
*b*c^2)*x^2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/5*b*x^4*(d*x-c)^(1/2)*(d*x+c
)^(1/2)/d^2
```

3.359.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\sqrt{-c+dx}\sqrt{c+dx}(5ad^2(2c^2+d^2x^2)+b(8c^4+4c^2d^2x^2+3d^4x^4))}{15d^6}$$

input
```
Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^
2*d^2*x^2 + 3*d^4*x^4)))/(15*d^6)
```

3.359.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {960, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)}{\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{5}\left(5a+\frac{4bc^2}{d^2}\right)\int \frac{x^3}{\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{5}\left(5a+\frac{4bc^2}{d^2}\right)\left(\frac{\int \frac{2c^2x}{\sqrt{dx-c}\sqrt{c+dx}}dx}{3d^2}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}\right)+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5a+\frac{4bc^2}{d^2}\right)\left(\frac{2c^2\int \frac{x}{\sqrt{dx-c}\sqrt{c+dx}}dx}{3d^2}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}\right)+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{5}\left(\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}}{3d^4}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}\right)\left(5a+\frac{4bc^2}{d^2}\right)+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

input
```
Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])/(5*d^2) + ((5*a + (4*b*c^2)/d^2)*((2*
c^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (x^2*Sqrt[-c + d*x]*Sqrt[c + d
*x])/(3*d^2)))/5
```

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.359.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3b\,d^{4}x^{4}+5a\,d^{4}x^{2}+4b\,c^{2}d^{2}x^{2}+10a\,c^{2}d^{2}+8b\,c^{4}\right)}{15d^{6}}$	68
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3b\,d^{4}x^{4}+5a\,d^{4}x^{2}+4b\,c^{2}d^{2}x^{2}+10a\,c^{2}d^{2}+8b\,c^{4}\right)}{15d^{6}}$	68
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\left(3b\,d^{4}x^{4}+5a\,d^{4}x^{2}+4b\,c^{2}d^{2}x^{2}+10a\,c^{2}d^{2}+8b\,c^{4}\right)}{15d^{6}\sqrt{dx-c}}$	74

input
```
int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.359. $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

output
```
1/15/d^6*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*
x^2+10*a*c^2*d^2+8*b*c^4)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+(4\,bc^2d^2+5\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^6}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*
sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

3.359.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.03

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{ac^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{bc^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6} + \frac{ibc^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}$$

input
```
integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4,
-1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-
2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**
2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/
4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/
(d**2*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4
, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**6)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^4}{5\,d^2} + \frac{4\,\sqrt{d^2x^2-c^2}bc^2x^2}{15\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax^2}{3\,d^2} + \frac{8\,\sqrt{d^2x^2-c^2}bc^4}{15\,d^6} + \frac{2\,\sqrt{d^2x^2-c^2}ac^2}{3\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4
 + 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6
+ 2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\left(\left(\left(dx+c\right)\left(3\left(dx+c\right)\left(\frac{(dx+c)b}{d^5}-\frac{4\,bc}{d^5}\right)+\frac{22\,bc^2d^{25}+5\,ad^{27}}{d^{30}}\right)-\frac{10\left(2\,bc^3d^{25}+acd^{27}\right)}{d^{30}}\right)(dx+c)+\frac{15\left(bc^4d^{25}+ac^2d^{27}\right)}{d^{30}}\right)\sqrt{\ldots}}{15\,d}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)*b/d^5 - 4*b*c/d^5) + (22*b*c^2*d^
25 + 5*a*d^27)/d^30) - 10*(2*b*c^3*d^25 + a*c*d^27)/d^30)*(d*x + c) + 15*(
b*c^4*d^25 + a*c^2*d^27)/d^30)*sqrt(d*x + c)*sqrt(d*x - c)/d
```

3.359.9 Mupad [B] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{8\,b\,c^5+10\,a\,c^3\,d^2}{15\,d^6}+\frac{x^3\,\left(4\,b\,c^2\,d^3+5\,a\,d^5\right)}{15\,d^6}+\frac{x\,\left(8\,b\,c^4\,d+10\,a\,c^2\,d^3\right)}{15\,d^6}+\frac{b\,x^5}{5\,d}+\frac{x^2\,\left(4\,b\,c^3\,d^2+5\,a\,c\,d^4\right)}{15\,d^6}+\frac{b\,c\,x^4}{5\,d^2}\right)}{\sqrt{c+d\,x}}$$

input
```
int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((d*x - c)^(1/2)*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*
c^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*
d) + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d
*x)^(1/2)
```

3.360 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.360.1 Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{c^2(3bc^2+4ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^5}$$

output
```
1/4*c^2*(4*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5+1/8*(4*
a*d^2+3*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/4*b*x^3*(d*x-c)^(1/2)*(
d*x+c)^(1/2)/d^2
```

3.360.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{dx\sqrt{-c+dx}\sqrt{c+dx}(3bc^2+4ad^2+2bd^2x^2)+(6bc^4+8ac^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}$$

input
```
Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + (6*b
*c^4 + 8*a*c^2*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

3.360.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {960, 101, 27, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)}{\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 960}$$

$$\frac{1}{4}\left(4a+\frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{4}\left(4a+\frac{3bc^2}{d^2}\right)\left(\frac{\int \frac{c^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(4a+\frac{3bc^2}{d^2}\right)\left(\frac{c^2\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

$$\Big\downarrow \text{ 45}$$

$$\frac{1}{4}\left(4a+\frac{3bc^2}{d^2}\right)\left(\frac{c^2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{4}\left(4a+\frac{3bc^2}{d^2}\right)\left(\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

input
```
Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*d^2) + ((4*a + (3*b*c^2)/d^2)*((x*
Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c
 + d*x]])/d^3))/4
```

3.360.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.360. $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.360.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x\left(2b\,d^{2}x^{2}+4a\,d^{2}+3b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{8d^{4}\sqrt{dx-c}}+\frac{c^{2}\left(4a\,d^{2}+3b\,c^{2}\right)\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{8d^{4}\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\,\mathrm{csgn}(d)b\,d^{3}x^{3}\sqrt{d^{2}x^{2}-c^{2}}+4\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)d^{3}ax+3\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)db\,c^{2}x+4\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)+dx\right)\right.\right.}{8d^{5}\sqrt{d^{2}x^{2}-c^{2}}}$

input
```
int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*b*d^2*x^2+4*a*d^2+3*b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1
/2)+1/8*c^2*(4*a*d^2+3*b*c^2)/d^4*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2)
)/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(2\,bd^3x^3+(3\,bc^2d+4\,ad^3)x)\sqrt{dx+c}\sqrt{dx-c}-(3\,bc^4+4\,ac^2d^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^5}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
 (3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{\sqrt{d^2x^2-c^2}bx^3}{4\,d^2} + \frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^5} \\ &\quad + \frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d^3} \\ &\quad + \frac{3\,\sqrt{d^2x^2-c^2}bc^2x}{8\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax}{2\,d^2}\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
 - c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/
8*sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

$$\begin{aligned}&\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\ &= \frac{\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^4}-\frac{3\,bc}{d^4}\right)+\frac{9\,bc^2d^{16}+4\,ad^{18}}{d^{20}}\right)-\frac{5\,bc^3d^{16}+4\,acd^{18}}{d^{20}}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{2\,(3\,bc^4+4\,ac^2d^2)\,\text{lo}}{}}{8\,d}\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16
 + 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(
d*x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c
)))/d^4)/d
```

3.360.9 Mupad [B] (verification not implemented)

Time = 33.91 (sec) , antiderivative size = 1048, normalized size of antiderivative = 8.88

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{\frac{2\,a\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}}$$

$$-\frac{\frac{23\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}-\frac{3\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{333\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{671\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}+\frac{671\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^9}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^9}+\frac{333}{2}\cdots}{d^5-\frac{8\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{28\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{56\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{70\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}-\frac{56\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^{10}}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^{10}}+\cdots}$$

$$-\frac{2\,a\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}-\frac{3\,b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^5}$$

input
```
int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (1
4*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 +
(14*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5
+ (2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7
)/(d^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/
2))^2 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2
))^4 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2)
)^6 + (d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
) - ((23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(
1/2))^3) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c
)^(1/2))) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*
x - c)^(1/2))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2
) - (d*x - c)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-
c)^(1/2) - (d*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(
1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2
) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c
+ d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c
+ d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c
+ d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*(...
```

3.360. $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.361 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.361.1 Optimal result

Integrand size = 29, antiderivative size = 72

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(2bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}$$

output
```
1/3*(3*a*d^2+2*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/3*b*x^2*(d*x-c)^(1
/2)*(d*x+c)^(1/2)/d^2
```

3.361.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(2bc^2+3ad^2+bd^2x^2)}{3d^4}$$

input
```
Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4)
```

3.361.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {960, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)}{\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int \frac{x}{\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

$$\Big\downarrow \text{83}$$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3a+\frac{2bc^2}{d^2}\right)}{3d^2}+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

input
```
Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
((3*a + (2*b*c^2)/d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^2) + (b*x^2*Sqrt
[-c + d*x]*Sqrt[c + d*x])/(3*d^2)
```

3.361.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.361.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4}$	43
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4}$	43
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4\sqrt{dx-c}}$	49

input
```
int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d^4*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{(bd^2x^2+2\,bc^2+3\,ad^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^4}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

3.361.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.10

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ibc^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

input
```
integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2
, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*
I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1,
 -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(
4*pi**(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)*d**4)
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^2}{3\,d^2} + \frac{2\sqrt{d^2x^2-c^2}bc^2}{3\,d^4} + \frac{\sqrt{d^2x^2-c^2}a}{d^2}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sq
rt(d^2*x^2 - c^2)*a/d^2
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^3}-\frac{2\,bc}{d^3}\right)+\frac{3\,(bc^2d^9+ad^{11})}{d^{12}}\right)}{3\,d}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^3 - 2*b*c/d^3) +
 3*(b*c^2*d^9 + a*d^11)/d^12)/d
```

3.361.9 Mupad [B] (verification not implemented)

Time = 7.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{2\,b\,c^3+3\,a\,c\,d^2}{3\,d^4}+\frac{b\,x^3}{3\,d}+\frac{x\,\left(2\,b\,c^2\,d+3\,a\,d^3\right)}{3\,d^4}+\frac{b\,c\,x^2}{3\,d^2}\right)}{\sqrt{c+d\,x}}$$

input
```
int((x*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((d*x - c)^(1/2)*((2*b*c^3 + 3*a*c*d^2)/(3*d^4) + (b*x^3)/(3*d) + (x*(3*a*
d^3 + 2*b*c^2*d))/(3*d^4) + (b*c*x^2)/(3*d^2)))/(c + d*x)^(1/2)
```

3.362 $\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.362.1 Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

output
```
(2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/2*b*x*(d*x-c)^(
1/2)*(d*x+c)^(1/2)/d^2
```

3.362.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{bdx\sqrt{-c+dx}\sqrt{c+dx}+2(bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] + 2*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c
+ d*x]/Sqrt[c + d*x]])/(2*d^3)
```

3.362.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {646, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{dx-c}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{646}$$

$$\frac{(2ad^2+bc^2)\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

$$\Big\downarrow \text{45}$$

$$\frac{(2ad^2+bc^2)\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d^2}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2ad^2+bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

input
```
Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sq
rt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

3.362.3.1 Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

3.362.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(-dx+c)\sqrt{dx+c}\,bx}{2d^2\sqrt{dx-c}}+\frac{(2a\,d^2+b\,c^2)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{2d^2\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\operatorname{csgn}(d)d\sqrt{d^2x^2-c^2}\,bx+\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2+2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^2\right)}{2d^3\sqrt{d^2x^2-c^2}}$

input
```
int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+c)*(d*x+c)^(1/2)*b*x/d^2/(d*x-c)^(1/2)+1/2*(2*a*d^2+b*c^2)/d^2*
ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1
/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}bdx-(bc^2+2\,ad^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{2\,d^3}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt
(d*x + c)*sqrt(d*x - c)))/d^3
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d^3} + \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d} + \frac{\sqrt{d^2x^2-c^2}bx}{2\,d^2}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + a*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}\left(\frac{(dx+c)b}{d^2}-\frac{bc}{d^2}\right)-\frac{2\left(bc^2+2\,ad^2\right)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}}{2\,d}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*b/d^2 - b*c/d^2) - 2*(b*c^2 +
2*a*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)/d
```

3.362.9 Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.13

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\frac{2\,b\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}} + \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}} - \frac{2\,b\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}$$

input
```
int((a + b*x^2)/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((2*b*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (1
4*b*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 +
(14*b*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5
+ (2*b*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7
)/(d^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/
2))^2 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2
))^4 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2)
)^6 + (d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
) + (4*a*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^
(1/2) - c^(1/2)))))/(-d^2)^(1/2) - (2*b*c^2*atanh(((c + d*x)^(1/2) - c^(1/
2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3
```

3.363 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.363.1 Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+\frac{a\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}$$

output
```
a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^
2
```

3.363.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+\frac{2a\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{c}$$

input
```
Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (2*a*ArcTan[Sqrt[-c + d*x]/Sqrt[c +
 d*x]])/c
```

3.363.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {960, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{960}$$

$$a\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{103}$$

$$ad\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{a\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

input
```
Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c +
d*x])/c])/c
```

3.363.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.363.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(48) = 96$.

Time = 4.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

method	result	size
default	$\frac{\left(-\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2+b\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)\sqrt{dx-c}\,\sqrt{dx+c}}{\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,d^2}$	108

input
```
int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/(-c^2)^(
1/2)/d^2
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\,ad^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}bc}{cd^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sq
rt(d*x - c)*b*c)/(c*d^2)
```

3.363.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.18

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bcG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

input
```
integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
 ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/
(4*pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4,
 0, 1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijer
g(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)),
 c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c} + \frac{\sqrt{d^2x^2-c^2}b}{d^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-a*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2 - c^2)*b/d^2
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c} + \frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqr
t(d*x - c)*b/d^2
```

3.363.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{b\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{d^2} - \frac{a\sqrt{-c}\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}$$

input
```
int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
(b*(c + d*x)^(1/2)*(d*x - c)^(1/2))/d^2 - (a*(-c)^(1/2)*(log(((c + d*x)^(1
/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1
/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2)
```

3.364 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.364.1 Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

output
```
2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c
^2/x
```

3.364.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

input
```
Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqr
t[c + d*x]])/d
```

3.364.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {956, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^2\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{956}$$

$$b\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}$$

$$\Big\downarrow \text{45}$$

$$2b\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{2b\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

input
```
Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqr
t[c + d*x]])/d
```

3.364.3.1 Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.364.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^{2}x+\operatorname{csgn}(d)\,d\sqrt{d^{2}x^{2}-c^{2}}\,a\right)\operatorname{csgn}(d)}{c^{2}\sqrt{d^{2}x^{2}-c^{2}}\,xd}$	97
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{c^{2}x\sqrt{dx-c}}+\frac{b\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	98

input
```
int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn
(d))*b*c^2*x+csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^2*x^2-c^2)^(1/2)/
x/d
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-ad^2x-\sqrt{dx+c}\sqrt{dx-c}\,cad}{c^2dx}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c
)*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

3.364.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.89

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

input
```
integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
-a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4,
 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*
x**2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0,
 1/4, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijer
g(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2
*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}a}{c^2x}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + sqrt(d^2*x^2 - c^2)*a/(c^2*x)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*
x + c) - sqrt(d*x - c))^4))/d
```

3.364.9 Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}}+\frac{a\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{c^2\,x}$$

input
```
int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
(4*b*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2
) - c^(1/2)))))/(-d^2)^(1/2) + (a*(c + d*x)^(1/2)*(d*x - c)^(1/2))/(c^2*x)
```

3.365 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.365.1 Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{(2bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

output
```
1/2*(a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+1/2*a*(d*x-c
)^(1/2)*(d*x+c)^(1/2)/c^2/x^2
```

3.365.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\frac{ac\sqrt{-c+dx}\sqrt{c+dx}}{x^2}+2(2bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2c^3}$$

input
```
Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
((a*c*Sqrt[-c + d*x]*Sqrt[c + d*x])/x^2 + 2*(2*b*c^2 + a*d^2)*ArcTan[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(2*c^3)
```

3.365.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {956, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^3\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{2}\left(\frac{ad^2}{c^2}+2b\right)\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}d\left(\frac{ad^2}{c^2}+2b\right)\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{ad^2}{c^2}+2b\right)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

input
```
Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b + (a*d^2)/c^2)*ArcTan
[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c)
```

3.365.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.365.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

method	result	size
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{2c^2x^2\sqrt{dx-c}}-\frac{\left(a\,d^2+2b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)\sqrt{(dx-c)(dx+c)}}{2c^2\sqrt{-c^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	123
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2+2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\right)}{2c^2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$	158

input
```
int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(-d*x+c)*(d*x+c)^(1/2)/c^2/x^2/(d*x-c)^(1/2)-1/2/c^2*(a*d^2+2*b*c^2
)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*
(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\,(2\,bc^2+ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}ac}{2\,c^3x^2}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

3.365.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c}-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}+\frac{\sqrt{d^2x^2-c^2}a}{2\,c^2x^2}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-b*arcsin(c/(d*abs(x)))/c - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c^3 + 1/2*sqrt(
d^2*x^2 - c^2)*a/(c^2*x^2)
```

3.365.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{\frac{(2\,bc^2d+ad^3)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3
+ 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c)
- sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

3.365.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 457, normalized size of antiderivative = 6.01

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}} - \frac{b\,\sqrt{-c}\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}} - \frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}} - \frac{\frac{a\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{a\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}} + \frac{a\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

input
```
int((a + b*x^2)/(x^3*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
(a*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c
)^(1/2))^2 + 1))/(2*c^(9/2)) - (b*(-c)^(1/2)*(log(((c + d*x)^(1/2) - c^(1/
2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2) - c^(1/
2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2) - (a*(-c)^(3/2)*d^2*log(((c
+ d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - ((a
*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (a*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/
2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*a*(-c)^(3/2)*d^
2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2
*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d
*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*d^2*((c + d*
x)^(1/2) - c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2
))^2)
```

3.365. $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.366 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.366.1 Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}$$

output
```
1/3*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^3+1/3*(2*a*d^2+3*b*c^2)*(d*x-c)^(1
/2)*(d*x+c)^(1/2)/c^4/x
```

3.366.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\sqrt{-c+dx}\sqrt{c+dx}(3bc^2x^2+a(c^2+2d^2x^2))}{3c^4x^3}$$

input
```
Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(3*c^4*
x^3)
```

3.366.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {956, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{3}\left(\frac{2ad^2}{c^2}+3b\right)\int \frac{1}{x^2\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

$$\Big\downarrow \text{106}$$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(\frac{2ad^2}{c^2}+3b\right)}{3c^2x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

input
```
Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b + (2*a*d^2)/c^2)*Sqrt
[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x)
```

3.366.3.1 Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.366.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2 a\,d^{2} x^{2}+3 b\,c^{2} x^{2}+c^{2} a\right)}{3 c^{4} x^{3}}$	49
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\operatorname{csgn}\left(d\right)^{2} \left(2 a\,d^{2} x^{2}+3 b\,c^{2} x^{2}+c^{2} a\right)}{3 c^{4} x^{3}}$	53
risch	$-\frac{\sqrt{dx+c}\,\left(-dx+c\right) \left(2 a\,d^{2} x^{2}+3 b\,c^{2} x^{2}+c^{2} a\right)}{3 x^{3} c^{4} \sqrt{dx-c}}$	55

input
```
int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c^4/x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3\,bc^2d+2\,ad^3)x^3+(ac^2+(3\,bc^2+2\,ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^4x^3}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*
x + c)*sqrt(d*x - c))/(c^4*x^3)
```

3.366.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.27

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{bdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} - \frac{ibdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

input
```
integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
-a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4,
 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2,
 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2)
- I*b*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1,
 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\sqrt{d^2x^2-c^2}b}{c^2x}+\frac{2\sqrt{d^2x^2-c^2}ad^2}{3\,c^4x}+\frac{\sqrt{d^2x^2-c^2}a}{3\,c^2x^3}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/
3*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)
```

3.366.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$=\frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x +
c) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*
c^4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\sqrt{d\,x-c}\left(\frac{a}{3\,c}+\frac{x^2\,\left(3\,b\,c^3+2\,a\,c\,d^2\right)}{3\,c^4}+\frac{x^3\,\left(3\,b\,c^2\,d+2\,a\,d^3\right)}{3\,c^4}+\frac{a\,d\,x}{3\,c^2}\right)}{x^3\,\sqrt{c+d\,x}}$$

input
```
int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
((d*x - c)^(1/2)*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*
a*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^(1/2))
```

3.366. $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.367 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.367.1 Optimal result

Integrand size = 31, antiderivative size = 123

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{d^2(4bc^2+3ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}$$

output
```
1/8*d^2*(3*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/4*a*
(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^4+1/8*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d
*x+c)^(1/2)/c^4/x^2
```

3.367.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{c\sqrt{-c+dx}\sqrt{c+dx}(2ac^2+4bc^2x^2+3ad^2x^2)+2d^2(4bc^2+3ad^2)\,x^4\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^5x^4}$$

input
```
Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2) + 2*
d^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^5*x
^4)
```

3.367.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {956, 114, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^5\sqrt{dx-c}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{4}\left(\frac{3ad^2}{c^2}+4b\right)\int \frac{1}{x^3\sqrt{dx-c}\sqrt{c+dx}}dx+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4}\left(\frac{3ad^2}{c^2}+4b\right)\left(\frac{\int \frac{d^2}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{2c^2}+\frac{\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}\right)+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{3ad^2}{c^2}+4b\right)\left(\frac{d^2\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{2c^2}+\frac{\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}\right)+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{4}\left(\frac{3ad^2}{c^2}+4b\right)\left(\frac{d^3\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)}{2c^2}+\frac{\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}\right)+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{3ad^2}{c^2}+4b\right)\left(\frac{d^2\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}\right)+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

input
```
Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

output
```
(a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b + (3*a*d^2)/c^2)*((Sq
rt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + (d^2*ArcTan[(Sqrt[-c + d*x]*Sqrt
[c + d*x])/c])/(2*c^3)))/4
```

3.367.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.367. $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.367.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\left(3a\,d^2x^2+4b\,c^2x^2+2c^2a\right)}{8c^4x^4\sqrt{dx-c}}-\frac{d^2\left(3a\,d^2+4b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)\sqrt{(dx-c)(dx+c)}}{8c^4\sqrt{-c^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4+4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4-3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2x^2-4\right.}{8c^4\sqrt{d^2x^2-c^2}\,x^4\sqrt{-c^2}}$

input
```
int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x+c)^(1/2)*(-d*x+c)*(3*a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/c^4/x^4/(d*x
-c)^(1/2)-1/8*d^2*(3*a*d^2+4*b*c^2)/c^4/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(
1/2)*(d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)
^(1/2)
```

3.367.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\left(4\,bc^2d^2+3\,ad^4\right)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\left(2\,ac^3+\left(4\,bc^3+3\,acd^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^5x^4}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x -
 c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c
))/(c^5*x^4)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3} - \frac{3\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^5} + \frac{\sqrt{d^2x^2-c^2}b}{2\,c^2x^2} + \frac{3\,\sqrt{d^2x^2-c^2}ad^2}{8\,c^4x^2} + \frac{\sqrt{d^2x^2-c^2}a}{4\,c^2x^4}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*d^2*arcsin(c/(d*abs(x)))/c^3 - 3/8*a*d^4*arcsin(c/(d*abs(x)))/c^5 +
 1/2*sqrt(d^2*x^2 - c^2)*b/(c^2*x^2) + 3/8*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*
x^2) + 1/4*sqrt(d^2*x^2 - c^2)*a/(c^2*x^4)
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(105) = 210.

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.64

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{\frac{\left(4\,bc^2d^3+3\,ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} + \frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+\right.}{}}{}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2
/c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqr
t(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x -
c))^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(s
qrt(d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x
- c))^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*
(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4
*c^2)^4*c^4))/d
```

3.367.9 Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 1005, normalized size of antiderivative = 8.17

$$
\begin{aligned}
&\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{3\,a\,\sqrt{-c}\,d^4\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{11/2}} - \frac{\frac{b\,(-c)^{3/2}\,d^2}{32\,c^{9/2}} + \frac{b\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} - \frac{15\,b\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} + \frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}} \\
&\quad - \frac{\frac{a\,\sqrt{-c}\,d^4}{1024\,c^{11/2}} - \frac{3\,a\,\sqrt{-c}\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{11/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} - \frac{53\,a\,\sqrt{-c}\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{11/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{87\,a\,\sqrt{-c}\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{11/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6} + \frac{657\,a\,\sqrt{-c}\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{1024\,c^{11/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6} + \frac{6\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8} + \frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}} + \frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^1}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}} \\
&\quad - \frac{b\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}} - \frac{3\,a\,\sqrt{-c}\,d^4\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{8\,c^{11/2}} \\
&\quad + \frac{b\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}} - \frac{7\,a\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{256\,\sqrt{-c}\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2} \\
&\quad + \frac{a\,d^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{1024\,\sqrt{-c}\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4} + \frac{b\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}
\end{aligned}
$$

input
```
int((a + b*x^2)/(x^5*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

output
```
(3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c
)^(1/2))))/(8*c^(11/2)) - ((b*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (b*(-c)^(3/2)
*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1
/2))^2) - (15*b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1
/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) -
 (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c
)^(1/2))^6) - ((a*(-c)^(1/2)*d^4)/(1024*c^(11/2)) - (3*a*(-c)^(1/2)*d^4*((
c + d*x)^(1/2) - c^(1/2))^2)/(128*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^
2) - (53*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(11/2)*((-
c)^(1/2) - (d*x - c)^(1/2))^4) + (87*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c
^(1/2))^6)/(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) + (657*a*(-c)^(
1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(11/2)*((-c)^(1/2) - (d*x
- c)^(1/2))^8) + (121*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(25
6*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2)
)^4/((-c)^(1/2) - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/(
(-c)^(1/2) - (d*x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^
(1/2) - (d*x - c)^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2
) - (d*x - c)^(1/2))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*
x - c)^(1/2))^12) - (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/(...
```

3.367. $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

3.368 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.368.1 Optimal result

Integrand size = 31, antiderivative size = 161

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6}+\frac{3c^2(5bc^2+4ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7}$$

output
```
3/4*c^2*(4*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7-1/4*(4*
a*d^2+5*b*c^2)*x^3/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*b*x^5/d^2/(d*x-c)^(
1/2)/(d*x+c)^(1/2)+3/8*(4*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

3.368.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{4ad^3x(-3c^2+d^2x^2)+bdx(-15c^4+5c^2d^2x^2+2d^4x^4)+6c^2(5bc^2+4ad^2)\sqrt{}}{8d^7\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4
) + 6*c^2*(5*b*c^2 + 4*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x]*ArcTanh[Sqrt[c
+ d*x]/Sqrt[-c + d*x]])/(8*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

3.368.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {960, 109, 27, 101, 27, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^2)}{(dx-c)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(-\frac{\int -\frac{3cx^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{cd^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(\frac{3\int \frac{x^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{d^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(\frac{3\left(\frac{\int \frac{c^2}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{d^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(\frac{3\left(\frac{c^2\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{2d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{d^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{45}$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(\frac{3\left(\frac{c^2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d^2}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{d^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(4a+\frac{5bc^2}{d^2}\right)\left(\frac{3\left(\frac{c^2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{x\sqrt{dx-c}\sqrt{c+dx}}{2d^2}\right)}{d^2}-\frac{x^3}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(b*x^5)/(4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((4*a + (5*b*c^2)/d^2)*(-(x
^3/(d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])) + (3*((x*Sqrt[-c + d*x]*Sqrt[c + d*
x])/(2*d^2) + (c^2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3))/d^2))/4
```

3.368.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

3.368. $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.368.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{x\left(2b\,d^{2}x^{2}+4a\,d^{2}+7b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{8d^{6}\sqrt{dx-c}}+\frac{c^{2}\left(\frac{12a\,d^{2}\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)}{\sqrt{d^{2}}}+\frac{15b\,c^{2}\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)}{\sqrt{d^{2}}}-\frac{4\left(a\,d^{2}+b\,c^{2}\right)\sqrt{d^{2}\left(x- \cdots\right.}}{d^{2}\left(x-\frac{c}{d} \cdots\right.}\right.}{8d^{6}\sqrt{dx-c}\,\sqrt{dx \cdots}}$
default	$\frac{\sqrt{dx-c}\left(-2\,\mathrm{csgn}(d)b\,d^{5}x^{5}\sqrt{d^{2}x^{2}-c^{2}}-4\,\mathrm{csgn}(d)a\,d^{5}x^{3}\sqrt{d^{2}x^{2}-c^{2}}-5\,\mathrm{csgn}(d)b\,c^{2}d^{3}x^{3}\sqrt{d^{2}x^{2}-c^{2}}-12\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)+dx\right)\cdots\right.\right.}{\cdots}$

input
```
int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*b*d^2*x^2+4*a*d^2+7*b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1
/2)+1/8*c^2/d^6*(12*a*d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^
(1/2)+15*b*c^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)-4*(a*
d^2+b*c^2)/d^2/(x-c/d)*(d^2*(x-c/d)^2+2*c*d*(x-c/d))^(1/2)-4*(a*d^2+b*c^2)
/d^2/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/
(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.18

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{8\,bc^6+8\,ac^4d^2-8\,(bc^4d^2+ac^2d^4)x^2+(2\,bd^5x^5+(5\,bc^2d^3+4\,ad^5)x^3-3\,(}{}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5
+ (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x +
c)*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*
x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^5}{4\sqrt{d^2x^2-c^2}d^2} + \frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}d^4} + \frac{ax^3}{2\sqrt{d^2x^2-c^2}d^2} - \frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}d^6} - \frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}d^4} + \frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{8\,d^7} + \frac{3\,ac^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{2\,d^5}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d
^4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c
^2)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x
+ 2*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
c^2)*d)/d^5
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.33

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^7}-\frac{5\,bc}{d^7}\right)+\frac{25\,bc^2d^{35}+4\,ad^{37}}{d^{42}}\right)-\frac{35\,bc^3d^{35}+12\,acd^{37}}{d^{42}}\right)}{8\sqrt{dx-c}} - \frac{3\left(5\,bc^4+4\,ac^2d^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{8\,d^7} - \frac{2\left(bc^5+ac^3d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^7}$$

input
```
integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^7 - 5*b*c/d^7) + (25*b*c^2*d^3
5 + 4*a*d^37)/d^42) - (35*b*c^3*d^35 + 12*a*c*d^37)/d^42)*(d*x + c) + 2*(7
*b*c^4*d^35 + 2*a*c^2*d^37)/d^42)*sqrt(d*x + c)/sqrt(d*x - c) - 3/8*(5*b*c
^4 + 4*a*c^2*d^2)*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^7 - 2*(b*c^5 +
a*c^3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{x^4\,(b\,x^2+a)}{(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.369 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

3.369.1 Optimal result

Integrand size = 31, antiderivative size = 115

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx = -\frac{(4bc^2+3ad^2)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}$$

output
```
-1/3*(3*a*d^2+4*b*c^2)*x^2/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*b*x^4/d^2/(
d*x-c)^(1/2)/(d*x+c)^(1/2)+2/3*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/
2)/d^6
```

3.369.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx = \frac{-8bc^4-6ac^2d^2+4bc^2d^2x^2+3ad^4x^2+bd^4x^4}{3d^6\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^
6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

3.369.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {960, 109, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2)}{(dx-c)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{3}\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\left(3a+\frac{4bc^2}{d^2}\right)\left(-\frac{\int -\frac{2cx}{\sqrt{dx-c}\sqrt{c+dx}}dx}{cd^2}-\frac{x^2}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3a+\frac{4bc^2}{d^2}\right)\left(\frac{2\int \frac{x}{\sqrt{dx-c}\sqrt{c+dx}}dx}{d^2}-\frac{x^2}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{3}\left(\frac{2\sqrt{dx-c}\sqrt{c+dx}}{d^4}-\frac{x^2}{d^2\sqrt{dx-c}\sqrt{c+dx}}\right)\left(3a+\frac{4bc^2}{d^2}\right)+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(b*x^4)/(3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((3*a + (4*b*c^2)/d^2)*(-(x
^2/(d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*Sqrt[-c + d*x]*Sqrt[c + d*x])/
d^4))/3
```

3.369.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.369.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{-b\,d^{4}x^{4}-3a\,d^{4}x^{2}-4b\,c^{2}d^{2}x^{2}+6a\,c^{2}d^{2}+8b\,c^{4}}{3d^{6}\sqrt{dx-c}\,\sqrt{dx+c}}$	68
default	$\frac{\sqrt{dx-c}\,\left(-b\,d^{4}x^{4}-3a\,d^{4}x^{2}-4b\,c^{2}d^{2}x^{2}+6a\,c^{2}d^{2}+8b\,c^{4}\right)}{3(-dx+c)d^{6}\sqrt{dx+c}}$	76
risch	$-\frac{\left(b\,d^{2}x^{2}+3a\,d^{2}+5b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{3d^{6}\sqrt{dx-c}}-\frac{c^{2}\left(a\,d^{2}+b\,c^{2}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{d^{6}\sqrt{-\left(dx+c\right)\left(-dx+c\right)}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	115

3.369. $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

input
```
int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x
^2+6*a*c^2*d^2+8*b*c^4)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(bd^4x^4 - 8\,bc^4 - 6\,ac^2d^2 + (4\,bc^2d^2 + 3\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,(d^8x^2 - c^2d^6)}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^4}{3\,\sqrt{d^2x^2-c^2}d^2} + \frac{4\,bc^2x^2}{3\,\sqrt{d^2x^2-c^2}d^4} + \frac{ax^2}{\sqrt{d^2x^2-c^2}d^2} - \frac{8\,bc^4}{3\,\sqrt{d^2x^2-c^2}d^6} - \frac{2\,ac^2}{\sqrt{d^2x^2-c^2}d^4}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d
^4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6
) - 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

3.369.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.74

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\left(2\,(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{4\,bc}{d^6}\right)+\frac{10\,bc^2d^{24}+3\,ad^{26}}{d^{30}}\right)-\frac{3\,(9\,bc^3d^{24}+5\,acd^{26})}{d^{30}}\right)\cdots}{6\,\sqrt{dx-c}}$$
$$+\frac{2\,(b^2c^8+2\,abc^6d^2+a^2c^4d^4)}{\left(bc^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^5+2\,ac^3d^2\right)d^6}$$

input
```
integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/6*(2*(d*x + c)*((d*x + c)*((d*x + c)*b/d^6 - 4*b*c/d^6) + (10*b*c^2*d^24
 + 3*a*d^26)/d^30) - 3*(9*b*c^3*d^24 + 5*a*c*d^26)/d^30)*sqrt(d*x + c)/sqr
t(d*x - c) + 2*(b^2*c^8 + 2*a*b*c^6*d^2 + a^2*c^4*d^4)/((b*c^4*(sqrt(d*x +
 c) - sqrt(d*x - c))^2 + a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b
*c^5 + 2*a*c^3*d^2)*d^6)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{d\,x-c}\left(\frac{x^2\,(4\,b\,c^2\,d^2+3\,a\,d^4)}{3\,d^7}-\frac{8\,b\,c^4+6\,a\,c^2\,d^2}{3\,d^7}+\frac{b\,x^4}{3\,d^3}\right)}{x\,\sqrt{c+d\,x}-\frac{c\,\sqrt{c+d\,x}}{d}}$$

input
```
int((x^3*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
((d*x - c)^(1/2)*((x^2*(3*a*d^4 + 4*b*c^2*d^2))/(3*d^7) - (8*b*c^4 + 6*a*c
^2*d^2)/(3*d^7) + (b*x^4)/(3*d^3)))/(x*(c + d*x)^(1/2) - (c*(c + d*x)^(1/2
))/d)
```

3.370 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} \, dx$

3.370.1 Optimal result

Integrand size = 31, antiderivative size = 152

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} \, dx = -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}$$

output
```
(2*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5-1/2*c*(2*a*d^2+
3*b*c^2)/d^5/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*b*x^3/d^2/(d*x-c)^(1/2)/(d*x+
c)^(1/2)-1/2*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/2)/d^5/(d*x+c)^(1/2)
```

3.370.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} \, dx = \frac{-3bc^2dx-2ad^3x+bd^3x^3+2(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}\,\text{arctanh}\left(\frac{\sqrt{\cdots}}{\sqrt{-\cdots}}\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + 2*(3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*
x]*Sqrt[c + d*x]*ArcTanh[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^5*Sqrt[-c + d
*x]*Sqrt[c + d*x])
```

3.370.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {960, 100, 27, 87, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^2)}{(dx-c)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{960}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\int \frac{x^2}{(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\left(\frac{\int \frac{cd^2x}{\sqrt{dx-c}(c+dx)^{3/2}}dx}{cd^3}-\frac{c}{d^3\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\left(\frac{\int \frac{x}{\sqrt{dx-c}(c+dx)^{3/2}}dx}{d}-\frac{c}{d^3\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\left(\frac{\frac{\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{d}-\frac{\sqrt{dx-c}}{d^2\sqrt{c+dx}}}{d}-\frac{c}{d^3\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{45}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\left(\frac{\frac{2\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d}-\frac{\sqrt{dx-c}}{d^2\sqrt{c+dx}}}{d}-\frac{c}{d^3\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2a+\frac{3bc^2}{d^2}\right)\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^2}-\frac{\sqrt{dx-c}}{d^2\sqrt{c+dx}}}{d}-\frac{c}{d^3\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

3.370. $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}dx$

input
```
Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(b*x^3)/(2*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((2*a + (3*b*c^2)/d^2)*(-(c
/(d^3*Sqrt[-c + d*x]*Sqrt[c + d*x])) + (-(Sqrt[-c + d*x]/(d^2*Sqrt[c + d*x
])) + (2*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^2)/d))/2
```

3.370.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.370.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{bx(-dx+c)\sqrt{dx+c}}{2d^4\sqrt{dx-c}}+\frac{\left(\frac{2a\,d^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}+\frac{3b\,c^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}-\frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\frac{c}{d}\right)}}{d^2\left(x-\frac{c}{d}\right)}-\frac{\left(a\,d^2+b\,c^2\right)}{}\right.}{2d^4\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-\operatorname{csgn}(d)b\,d^3x^3\sqrt{d^2x^2-c^2}-2\ln\left(\left(\sqrt{d^2x^2-c^2}\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^4x^2-3\ln\left(\left(\sqrt{d^2x^2-c^2}\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2d\right.}{}$

input
```
int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x*(-d*x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)+1/2/d^4*(2*a*d^2*ln(x*d^
2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)+3*b*c^2*ln(x*d^2/(d^2)^(1/2
)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)-(a*d^2+b*c^2)/d^2/(x-c/d)*(d^2*(x-c/d)^
2+2*c*d*(x-c/d))^(1/2)-(a*d^2+b*c^2)/d^2/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c
/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int\frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\frac{2\,bc^4+2\,ac^2d^2-2\,(bc^2d^2+ad^4)x^2+(bd^3x^3-(3\,bc^2d+2\,ad^3)x)\sqrt{dx+c}\sqrt{}}{2\,(d^7x}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b
*c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2
- (3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d
^7*x^2 - c^2*d^5)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^3}{2\sqrt{d^2x^2-c^2}d^2} - \frac{3\,bc^2x}{2\sqrt{d^2x^2-c^2}d^4} - \frac{ax}{\sqrt{d^2x^2-c^2}d^2} + \frac{3\,bc^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{2\,d^5} + \frac{a\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{d^3}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4
) - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2
 - c^2)*d)/d^5 + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{dx+c}\left((dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{3\,bc}{d^5}\right)+\frac{bc^2d^{15}-ad^{17}}{d^{20}}\right)}{2\sqrt{dx-c}} - \frac{(3\,bc^2+2\,ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{2\,d^5} - \frac{2\,(bc^3+acd^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^5}$$

input
```
integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x + c)*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^15 -
 a*d^17)/d^20)/sqrt(d*x - c) - 1/2*(3*b*c^2 + 2*a*d^2)*log((sqrt(d*x + c)
- sqrt(d*x - c))^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt(d*x
- c))^2 + 2*c)*d^5)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{x^2\,(b\,x^2+a)}{(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.371 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.371.1 Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{(2bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{c^2d^4}$$

output
```
-(a/c^2+b/d^2)*x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2)+(a*d^2+2*b*c^2)*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/c^2/d^4
```

3.371.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-2bc^2-ad^2+bd^2x^2}{d^4\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

3.371.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {958, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)}{(dx-c)^{3/2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 958$$

$$\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{dx-c}\sqrt{c+dx}}dx-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \; 83$$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)}{d^2}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
-(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((a/c^2 + (2*b)/
d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2
```

3.371.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 958
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
 + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e,
m, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !I
ntegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[
p + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

3.371.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{-b\,d^{2} x^{2}+a\,d^{2}+2 b\,c^{2}}{d^{4} \sqrt{d x -c}\,\sqrt{d x +c}}$	43
default	$\frac{\sqrt{d x -c}\,\left(-b\,d^{2} x^{2}+a\,d^{2}+2 b\,c^{2}\right)}{\sqrt{d x +c}\,d^{4} \left(-d x +c \right)}$	50
risch	$-\frac{b \left(-d x +c \right) \sqrt{d x +c}}{d^{4} \sqrt{d x -c}}-\frac{\left(a\,d^{2}+b\,c^{2}\right) \sqrt{\left(d x -c \right) \left(d x +c \right)}}{d^{4} \sqrt{-\left(d x +c \right) \left(-d x +c \right)}\,\sqrt{d x -c}\,\sqrt{d x +c}}$	92

input
```
int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-b*d^2*x^2+a*d^2+2*b*c^2)
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(bd^2x^2-2\,bc^2-ad^2)\sqrt{dx+c}\sqrt{dx-c}}{d^6x^2-c^2d^4}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d
^4)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^2}{\sqrt{d^2x^2-c^2}d^2} - \frac{2\,bc^2}{\sqrt{d^2x^2-c^2}d^4} - \frac{a}{\sqrt{d^2x^2-c^2}d^2}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(s
qrt(d^2*x^2 - c^2)*d^2)
```

3.371.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(68) = 136.

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{dx+c}\left(\frac{2\,(dx+c)b}{d^4} - \frac{5\,bc^2d^8+ad^{10}}{cd^{12}}\right)}{2\,\sqrt{dx-c}} + \frac{2\,(b^2c^4+2\,abc^2d^2+a^2d^4)}{\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + 2\,bc^3 + 2\,acd^2\right)d^4}$$

input
```
integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^10)/(c*d^12))/sq
rt(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(sqrt(d*x + c)
 - sqrt(d*x - c))^2 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^3 +
2*a*c*d^2)*d^4)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a\,d^2\,\sqrt{d\,x-c}+2\,b\,c^2\,\sqrt{d\,x-c}-b\,d^2\,x^2\,\sqrt{d\,x-c}}{d^4\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

input
```
int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
(a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/
2))/(d^4*(c + d*x)^(1/2)*(c - d*x))
```

3.372 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.372.1 Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{2b\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

output
```
2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3-(a/c^2+b/d^2)*x/(d*x-c)^(1/2)
/(d*x+c)^(1/2)
```

3.372.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-\frac{2(bc^2d+ad^3)x}{c^2\sqrt{-c+dx}\sqrt{c+dx}}+4b\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
((-2*(b*c^2*d + a*d^3)*x)/(c^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*b*ArcTanh
[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^3)
```

3.372.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {645, 45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(dx-c)^{3/2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{645}$$

$$\frac{b\int \frac{1}{\sqrt{dx-c}\sqrt{c+dx}}dx}{d^2}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{45}$$

$$\frac{2b\int \frac{1}{d-\frac{d(dx-c)}{c+dx}}d\frac{\sqrt{dx-c}}{\sqrt{c+dx}}}{d^2}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
-(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/d^3
```

3.372.3.1 Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 645
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[(b*c*e - a*d*f)*x*(c + d*x)^(m + 1)*((e + f*x)^(n +
1)/(2*c*d*e*f*(m + 1))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(2*c*d*e*f*(m
+ 1))   Int[(c + d*x)^(m + 1)*(e + f*x)^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && LtQ[m, -1]
```

3.372.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.63

method	result
default	$\frac{\sqrt{dx-c}\left(-\ln\left(\left(\sqrt{-(dx+c)(-dx+c)}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^{2}d^{2}x^{2}+\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)d^{3}ax+\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)db\,c^{2}x+\ln\left(\left(\sqrt{-\right.\right.}{(-dx+c)\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}d^{3}\sqrt{dx+c}}$

input
```
int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x-c)^(1/2)*(-ln(((-(d*x+c)*(-d*x+c))^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^2*
d^2*x^2+(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x+(d^2*x^2-c^2)^(1/2)*csgn(d)*d*
b*c^2*x+ln(((-(d*x+c)*(-d*x+c))^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)
/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/c^2/d^3/(d*x+c)^(1/2)
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bc^4+ac^2d^2-(bc^2d+ad^3)\sqrt{dx+c}\sqrt{dx-c}\,x-(bc^2d^2+ad^4)x^2-(bc^2d^2x^2}{c^2d^5x^2-c^4d^3}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*
c^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sq
rt(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=-\frac{ax}{\sqrt{d^2x^2-c^2}c^2}-\frac{bx}{\sqrt{d^2x^2-c^2}d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d^3}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d
^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

3.372.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.79

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=-\frac{b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3}-\frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)cd^3}-\frac{(bc^2d^3+ad^5)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^2d^6}$$

input
```
integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(
d*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d
*x + c)/(sqrt(d*x - c)*c^2*d^6)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\int \frac{b\,x^2+a}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.373 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.373.1 Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}}-\frac{a\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3}$$

output
```
-a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+(-a/c^2-b/d^2)/(d*x-c)^(1/2)/
(d*x+c)^(1/2)
```

3.373.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\frac{-\frac{2(bc^3+acd^2)}{d^2\sqrt{-c+dx}\sqrt{c+dx}}+4a\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^3}$$

input
```
Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
((-2*(b*c^3 + a*c*d^2))/(d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*a*ArcTan[Sq
rt[c + d*x]/Sqrt[-c + d*x]])/(2*c^3)
```

3.373.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {958, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x(dx-c)^{3/2}(c+dx)^{3/2}}\,dx$$
$$\Big\downarrow \; 958$$
$$-\frac{a\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{c^2}-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}$$
$$\Big\downarrow \; 103$$
$$-\frac{ad\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)}{c^2}-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}$$
$$\Big\downarrow \; 218$$
$$-\frac{a\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
-((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d
*x]*Sqrt[c + d*x])/c])/c^3
```

3.373.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 958
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
 + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e,
m, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !I
ntegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[
p + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

3.373.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(58) = 116$.

Time = 4.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.98

method	result
default	$\frac{\sqrt{dx-c}\left(-\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^2+\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2d^2+\sqrt{-c^2}\sqrt{d^2x^2-c^2}\,a\,d^2+b\,c^2\sqrt{-c^2}\sqrt{d^2x^2-}\right.}{c^2(-dx+c)\sqrt{-c^2}\sqrt{d^2x^2-c^2}\,d^2\sqrt{dx+c}}$

input
```
int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x-c)^(1/2)/c^2*(-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^4*
x^2+ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2)*a*d^2+b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-d*x+c
)/(-c^2)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(d*x+c)^(1/2)
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^3+acd^2)\sqrt{dx+c}\sqrt{dx-c}+2\left(ad^4x^2-ac^2d^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2
)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c^3} - \frac{a}{\sqrt{d^2x^2-c^2}c^2} - \frac{b}{\sqrt{d^2x^2-c^2}d^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
a*arcsin(c/(d*abs(x)))/c^3 - a/(sqrt(d^2*x^2 - c^2)*c^2) - b/(sqrt(d^2*x^2
 - c^2)*d^2)
```

3.373.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3} - \frac{(bc^2+ad^2)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^3d^2} + \frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

input
```
integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d
^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x
+ c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x\,(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.374 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.374.1 Optimal result

Integrand size = 31, antiderivative size = 67

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(bc^2+2ad^2)\,x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}$$

output
```
a/c^2/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)-(2*a*d^2+b*c^2)*x/c^4/(d*x-c)^(1/2)/(d
*x+c)^(1/2)
```

3.374.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-bc^2x^2+a(c^2-2d^2x^2)}{c^4x\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

3.374.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {956, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^2(dx-c)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{956}$$

$$\left(\frac{2ad^2}{c^2}+b\right)\int \frac{1}{(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{41}$$

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(\frac{2ad^2}{c^2}+b\right)}{c^2\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b + (2*a*d^2)/c^2)*x)/(c^2*Sqrt
[-c + d*x]*Sqrt[c + d*x])
```

3.374.3.1 Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.374.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{-2a\,d^2x^2-b\,c^2x^2+c^2a}{c^4x\sqrt{dx-c}\,\sqrt{dx+c}}$	48
default	$\frac{(2a\,d^2x^2+b\,c^2x^2-c^2a)\sqrt{dx-c}\,\operatorname{csgn}(d)^2}{c^4(-dx+c)x\sqrt{dx+c}}$	60
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{c^4x\sqrt{dx-c}}-\frac{(a\,d^2+b\,c^2)x\sqrt{(dx-c)(dx+c)}}{\sqrt{-(dx+c)(-dx+c)}\,c^4\sqrt{dx-c}\,\sqrt{dx+c}}$	95

input
```
int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^4/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-2*a*d^2*x^2-b*c^2*x^2+a*c^2)
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2d^2+2\,ad^4)x^3-(ac^2d-(bc^2d+2\,ad^3)x^2)\sqrt{dx+c}\sqrt{dx-c}-(bc^4+2\,ac^2d^2)x}{c^4d^3x^3-c^6dx}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{bx}{\sqrt{d^2x^2-c^2}c^2} - \frac{2\,ad^2x}{\sqrt{d^2x^2-c^2}c^4} + \frac{a}{\sqrt{d^2x^2-c^2}c^2x}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-b*x/(sqrt(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) + a/(
sqrt(d^2*x^2 - c^2)*c^2*x)
```

3.374.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(59) = 118.

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2+ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}c^4d}$$
$$-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+4\,bc^4+12\,a\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+8\,c^3\right)c^3d}$$

input
```
integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(
d*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*
a*c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqr
t(d*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*
c^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

3.374.9 Mupad [B] (verification not implemented)

Time = 7.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{2\,a\,d^2\,x^2\,\sqrt{d\,x-c}-a\,c^2\,\sqrt{d\,x-c}+b\,c^2\,x^2\,\sqrt{d\,x-c}}{c^4\,x\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

input
```
int((a + b*x^2)/(x^2*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
(2*a*d^2*x^2*(d*x - c)^(1/2) - a*c^2*(d*x - c)^(1/2) + b*c^2*x^2*(d*x - c)
^(1/2))/(c^4*x*(c + d*x)^(1/2)*(c - d*x))
```

3.375 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.375.1 Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5}$$

output
```
-1/2*(3*a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/2*(-3*a
*d^2-2*b*c^2)/c^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*a/c^2/x^2/(d*x-c)^(1/2)/
(d*x+c)^(1/2)
```

3.375.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\frac{-2bc^3x^2+a(c^3-3cd^2x^2)}{x^2\sqrt{-c+dx}\sqrt{c+dx}} + (4bc^2+6ad^2)\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^5}$$

input
```
Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
((-2*b*c^3*x^2 + a*(c^3 - 3*c*d^2*x^2))/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
 + (4*b*c^2 + 6*a*d^2)*ArcTan[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*c^5)
```

3.375.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {956, 115, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^3(dx-c)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{2}\left(\frac{3ad^2}{c^2}+2b\right)\int \frac{1}{x(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{115}$$

$$\frac{1}{2}\left(\frac{3ad^2}{c^2}+2b\right)\left(-\frac{\int \frac{d}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{c^2d}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3ad^2}{c^2}+2b\right)\left(-\frac{\int \frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{c^2}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}\left(\frac{3ad^2}{c^2}+2b\right)\left(-\frac{d\int \frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)}{c^2}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{3ad^2}{c^2}+2b\right)\left(-\frac{\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
a/(2*c^2*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((2*b + (3*a*d^2)/c^2)*(-(1/(
c^2*Sqrt[-c + d*x]*Sqrt[c + d*x])) - ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])
/c]/c^3))/2
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.375. $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.375.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(99) = 198$.

Time = 4.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.04

method	result
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2c^4x^2\sqrt{dx-c}} - \frac{\left(-\frac{\left(3a\,d^2+2b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{\sqrt{-c^2}} + \frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\frac{c}{d}\right)}}{dc\left(x-\frac{c}{d}\right)} - \frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x+\frac{c}{d}\right)^2-}}{dc\left(x+\frac{c}{d}\right)}}{2c^4\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4+3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2\right.}{2c^4(-dx+c)\sqrt{-c^2}\,x^2\sqrt{}}$

input
```
int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(-d*x+c)*(d*x+c)^(1/2)/c^4/x^2/(d*x-c)^(1/2)-1/2/c^4*(-(3*a*d^2+2*b*
c^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)+(a*d^2
+b*c^2)/d/c/(x-c/d)*(d^2*(x-c/d)^2+2*c*d*(x-c/d))^(1/2)-(a*d^2+b*c^2)/d/c/
(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-
c)^(1/2)/(d*x+c)^(1/2)
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(ac^3-(2\,bc^3+3\,acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}-2\,((2\,bc^2d^2+3\,ad^4)x^4-(2}{2\,(c^5d^2x^4-c^7x^2)}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*(
(2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x -
sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c^3} + \frac{3\,ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5} - \frac{b}{\sqrt{d^2x^2-c^2}c^2} - \frac{3\,ad^2}{2\,\sqrt{d^2x^2-c^2}c^4} + \frac{a}{2\,\sqrt{d^2x^2-c^2}c^2x^2}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
b*arcsin(c/(d*abs(x)))/c^3 + 3/2*a*d^2*arcsin(c/(d*abs(x)))/c^5 - b/(sqrt(
d^2*x^2 - c^2)*c^2) - 3/2*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4) + 1/2*a/(sqrt(d^
2*x^2 - c^2)*c^2*x^2)
```

3.375.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.80

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(2\,bc^2+3\,ad^2)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} - \frac{(bc^2+ad^2)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^5} + \frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^4} + \frac{2\left(ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^4}$$

input
```
integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
(2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 -
1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/
(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c)
- sqrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqr
t(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x^3\,{(c+d\,x)}^{3/2}\,{(d\,x-c)}^{3/2}}\,dx$$

input
```
int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.376 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.376.1 Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{2d^2(3bc^2+4ad^2)\,x}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}$$

output
```
1/3*a/c^2/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*(4*a*d^2+3*b*c^2)/c^4/x/(d*x
-c)^(1/2)/(d*x+c)^(1/2)-2/3*d^2*(4*a*d^2+3*b*c^2)*x/c^6/(d*x-c)^(1/2)/(d*x
+c)^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3bc^2x^2(c^2-2d^2x^2)+a(c^4+4c^2d^2x^2-8d^4x^4)}{3c^6x^3\sqrt{-c+dx}\sqrt{c+dx}}$$

input
```
Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c
^6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

3.376.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {956, 114, 27, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4(dx-c)^{3/2}(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{3}\left(\frac{4ad^2}{c^2}+3b\right)\int \frac{1}{x^2(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\left(\frac{4ad^2}{c^2}+3b\right)\left(\frac{\int \frac{2d^2}{(dx-c)^{3/2}(c+dx)^{3/2}}dx}{c^2}+\frac{1}{c^2x\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{4ad^2}{c^2}+3b\right)\left(\frac{2d^2\int \frac{1}{(dx-c)^{3/2}(c+dx)^{3/2}}dx}{c^2}+\frac{1}{c^2x\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{41}$$

$$\frac{1}{3}\left(\frac{1}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{2d^2x}{c^4\sqrt{dx-c}\sqrt{c+dx}}\right)\left(\frac{4ad^2}{c^2}+3b\right)+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((3*b + (4*a*d^2)/c^2)*(1/(c^
2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*x)/(c^4*Sqrt[-c + d*x]*Sqrt[c +
 d*x])))/3
```

3.376.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.376.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4}{3c^6x^3\sqrt{dx-c}\,\sqrt{dx+c}}$	73
default	$-\frac{\sqrt{dx-c}\ \operatorname{csgn}(d)^2\left(-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4\right)}{3c^6(-dx+c)x^3\sqrt{dx+c}}$	85
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(5a\,d^2x^2+3b\,c^2x^2+c^2a\right)}{3c^6x^3\sqrt{dx-c}}-\frac{d^2\left(a\,d^2+b\,c^2\right)x\sqrt{(dx-c)(dx+c)}}{\sqrt{-(dx+c)(-dx+c)}\,c^6\sqrt{dx-c}\,\sqrt{dx+c}}$	122

input
```
int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c^6/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*
c^2*d^2*x^2+3*b*c^4*x^2+a*c^4)
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\,(3\,bc^2d^3+4\,ad^5)x^5-2\,(3\,bc^4d+4\,ac^2d^3)x^3-(ac^4-2\,(3\,bc^2d^2+4\,ad^4)x^4+(3\,bc^4+4\,ac^2d^2)x^2)\sqrt{dx+c}}{3\,(c^6d^2x^5-c^8x^3)}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a
*c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d
*x + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

3.376.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2\,bd^2x}{\sqrt{d^2x^2-c^2}c^4} - \frac{8\,ad^4x}{3\,\sqrt{d^2x^2-c^2}c^6} + \frac{b}{\sqrt{d^2x^2-c^2}c^2x} + \frac{4\,ad^2}{3\,\sqrt{d^2x^2-c^2}c^4x} + \frac{a}{3\,\sqrt{d^2x^2-c^2}c^2x^3}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2*b*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) - 8/3*a*d^4*x/(sqrt(d^2*x^2 - c^2)*c^
6) + b/(sqrt(d^2*x^2 - c^2)*c^2*x) + 4/3*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*x)
 + 1/3*a/(sqrt(d^2*x^2 - c^2)*c^2*x^3)
```

3.376.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(101) = 202.

Time = 0.40 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.03

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2d+ad^3)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^6} - \frac{2\,(bc^2d+ad^3)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^5} - \frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,ac^2 \cdots\right.}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3c^4}$$

input
```
integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*
d^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqr
t(d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8
+ 24*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x +
c) - sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt
(d*x - c))^4 + 4*c^2)^3*c^4)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c^2\,d}+\frac{x^2\,\left(3\,b\,c^4+4\,a\,c^2\,d^2\right)}{3\,c^6\,d}-\frac{x^4\,\left(6\,b\,c^2\,d^2+8\,a\,d^4\right)}{3\,c^6\,d}\right)}{x^4\,\sqrt{c+d\,x}-\frac{c\,x^3\,\sqrt{c+d\,x}}{d}}$$

input
```
int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
((d*x - c)^(1/2)*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) -
(x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^(1/2) - (c*x^3*(c
 + d*x)^(1/2))/d)
```

3.377 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.377.1 Optimal result

Integrand size = 31, antiderivative size = 166

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{3d^2(4bc^2+5ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^7}$$

output
```
-3/8*d^2*(5*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^7-3/8*d
^2*(5*a*d^2+4*b*c^2)/c^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*a/c^2/x^4/(d*x-c)
^(1/2)/(d*x+c)^(1/2)+1/8*(5*a*d^2+4*b*c^2)/c^4/x^2/(d*x-c)^(1/2)/(d*x+c)^(
1/2)
```

3.377.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\frac{4bc^3x^2(c^2-3d^2x^2)+a(2c^5+5c^3d^2x^2-15cd^4x^4)}{x^4\sqrt{-c+dx}\sqrt{c+dx}} + 6d^2(4bc^2+5ad^2)\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8c^7}$$

input
```
Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
((4*b*c^3*x^2*(c^2 - 3*d^2*x^2) + a*(2*c^5 + 5*c^3*d^2*x^2 - 15*c*d^4*x^4)
)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 6*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[Sq
rt[c + d*x]/Sqrt[-c + d*x]])/(8*c^7)
```

3.377.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {956, 114, 27, 115, 27, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^5(dx-c)^{3/2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{956}$$

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\int \frac{1}{x^3(dx-c)^{3/2}(c+dx)^{3/2}}dx+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{\int \frac{3d^2}{x(dx-c)^{3/2}(c+dx)^{3/2}}dx}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{3d^2\int \frac{1}{x(dx-c)^{3/2}(c+dx)^{3/2}}dx}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{115}$$

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{3d^2\left(-\frac{\int \frac{d}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{c^2d}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{3d^2\left(-\frac{\int\frac{1}{x\sqrt{dx-c}\sqrt{c+dx}}dx}{c^2}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$\Big\downarrow$ 103

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{3d^2\left(-\frac{d\int\frac{1}{dc^2+d(dx-c)(c+dx)}d\left(\sqrt{dx-c}\sqrt{c+dx}\right)}{c^2}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

$\Big\downarrow$ 218

$$\frac{1}{4}\left(\frac{5ad^2}{c^2}+4b\right)\left(\frac{3d^2\left(-\frac{\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}-\frac{1}{c^2\sqrt{dx-c}\sqrt{c+dx}}\right)}{2c^2}+\frac{1}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}\right)+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
a/(4*c^2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + ((4*b + (5*a*d^2)/c^2)*(1/(2*
c^2*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*d^2*(-(1/(c^2*Sqrt[-c + d*x]*Sq
rt[c + d*x])) - ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]/c^3))/(2*c^2)))/4
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 956
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1))   Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
 /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
 a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

3.377. $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

3.377.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.61

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)(7a\,d^2x^2+4b\,c^2x^2+2c^2a)}{8c^6x^4\sqrt{dx-c}} - \frac{d^2\left(-\frac{(15a\,d^2+12b\,c^2)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{\sqrt{-c^2}}+\frac{4(a\,d^2+b\,c^2)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\right.}}{dc\left(x-\frac{c}{d}\right)}\right.}{8c^6\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-15\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^6x^6-12\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^4x^6+15\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)\right.}{}$

input
```
int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x+c)^(1/2)*(-d*x+c)*(7*a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/c^6/x^4/(d*x-
c)^(1/2)-1/8/c^6*d^2*(-(15*a*d^2+12*b*c^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2
)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)+4*(a*d^2+b*c^2)/d/c/(x-c/d)*(d^2*(x-c/d)^2
+2*c*d*(x-c/d))^(1/2)-4*(a*d^2+b*c^2)/d/c/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+
c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(2\,ac^5-3\,(4\,bc^3d^2+5\,acd^4)x^4+(4\,bc^5+5\,ac^3d^2)x^2)\sqrt{dx+c}\sqrt{dx-c}-}{8\,(c^7d^2x^6}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*
x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c
^4*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))
/(c^7*d^2*x^6 - c^9*x^4)
```

3.377.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3\,bd^2 \arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5} + \frac{15\,ad^4 \arcsin\left(\frac{c}{d|x|}\right)}{8\,c^7} - \frac{3\,bd^2}{2\,\sqrt{d^2x^2-c^2}c^4} - \frac{15\,ad^4}{8\,\sqrt{d^2x^2-c^2}c^6} + \frac{b}{2\,\sqrt{d^2x^2-c^2}c^2x^2} + \frac{5\,ad^2}{8\,\sqrt{d^2x^2-c^2}c^4x^2} + \frac{a}{4\,\sqrt{d^2x^2-c^2}c^2x^4}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
 3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6
) + 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c
^4*x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

3.377.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(142) = 284$.

Time = 0.41 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.42

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3\,(4\,bc^2d^2+5\,ad^4)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7} - \frac{(bc^2d^2+ad^4)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^7} + \frac{2\,(bc^2d^2+ad^4)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6} + \frac{4\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+7\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+60\,ac^2}{}$$

input
```
integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c
)/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c
^2*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b
*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqr
t(d*x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^
2*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*
c^8*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) -
 sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x^5\,(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

output
```
int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.378 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

3.378.1 Optimal result

Integrand size = 31, antiderivative size = 40

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{-1+cx}\sqrt{1+cx} + \arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

output
```
arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

3.378.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{-1+cx}\sqrt{1+cx} + 2\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

input
```
Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]]
```

3.378.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {960, 103, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2x^2+1}{x\sqrt{cx-1}\sqrt{cx+1}}dx$$

$$\Big\downarrow \text{960}$$

$$\int \frac{1}{x\sqrt{cx-1}\sqrt{cx+1}}dx+\sqrt{cx-1}\sqrt{cx+1}$$

$$\Big\downarrow \text{103}$$

$$c\int \frac{1}{(cx-1)(cx+1)c+c}d\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}$$

$$\Big\downarrow \text{218}$$

$$\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}$$

input
```
Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

output
```
Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

3.378.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 960
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Simp[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1))   Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n
/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
 n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.378.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}}$	53

input
```
int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c^2*x^2-1)^(1/2)-arctan(1/(c^2*x^2-1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/(c^2*x^2-1)^(1/2)
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{cx+1}\sqrt{cx-1}+2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)$$

input
```
integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

3.378.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.83 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.70

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

input
```
integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

output
```
meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
 1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/
2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1
, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_p
olar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1,
 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2))
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{c^2x^2-1} - \arcsin\left(\frac{1}{c|x|}\right)$$

input
```
integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(c^2*x^2 - 1) - arcsin(1/(c*abs(x)))
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{cx+1}\sqrt{cx-1} - 2\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)$$

input
```
integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)
```

3.378.9 Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{c\,x-1}\,\sqrt{c\,x+1} - \ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i} + \ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}$$

input
```
int((c^2*x^2 + 1)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

output
```
log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i - log(((c*x - 1)^(1/2
) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i + (c*x - 1)^(1/2)*(c*x + 1)^(1/2
)
```

3.379 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

3.379.1 Optimal result

Integrand size = 57, antiderivative size = 53

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\sqrt{a+bx}$$

output
```
(c/a^2+d/b^2)*(b*x-a)^(1/2)*(b*x+a)^(1/2)/(x^(b^2*c/(a^2*d+b^2*c)))
```

3.379.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.60

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\,x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{1-\frac{b^2x^2}{a^2}}\left(-\left((b^2c+2a^2d)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{b^2c}{2(b^2c+a^2d)},\frac{b^2c+2a^2d}{2b^2c+2a^2d},\frac{b^2x^2}{a^2}\right)\right)+b\cdots}{b^2\,(b^2c+2a^2d)\sqrt{-a+bx}\sqrt{a+bx}}$$

input
```
Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x
]*Sqrt[a + b*x]),x]
```

output
```
((b^2*c + a^2*d)*Sqrt[1 - (b^2*x^2)/a^2]*(-((b^2*c + 2*a^2*d)*Hypergeometr
ic2F1[1/2, -1/2*(b^2*c)/(b^2*c + a^2*d), (b^2*c + 2*a^2*d)/(2*b^2*c + 2*a^
2*d), (b^2*x^2)/a^2]) + b^2*d*x^2*Hypergeometric2F1[1/2, (b^2*c + 2*a^2*d)
/(2*b^2*c + 2*a^2*d), (3*b^2*c + 4*a^2*d)/(2*b^2*c + 2*a^2*d), (b^2*x^2)/a
^2]))/(b^2*(b^2*c + 2*a^2*d)*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sq
rt[a + b*x])
```

3.379.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = $\{952\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right) x^{-\frac{a^2 d+2 b^2 c}{a^2 d+b^2 c}}}{\sqrt{bx-a}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 952}$$

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right) x^{-\frac{b^2 c}{a^2 d+b^2 c}}$$

input
```
Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt
[a + b*x]),x]
```

output
```
((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

3.379.3.1 Defintions of rubi rules used

rule 952
```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
), 0] && NeQ[m, -1]
```

3.379.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

method	result	size
gosper	$\frac{x\sqrt{bx-a}\,\sqrt{bx+a}\,(a^2d+b^2c)x^{-\frac{a^2d+2b^2c}{a^2d+b^2c}}}{a^2b^2}$	66

input
```
int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
x/a^2/b^2*(b*x-a)^(1/2)*(b*x+a)^(1/2)*(a^2*d+b^2*c)/(x^((a^2*d+2*b^2*c)/(a
^2*d+b^2*c)))
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\sqrt{bx+a}\sqrt{bx-a}x}{a^2b^2x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}$$

input
```
integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x
+a)^(1/2),x, algorithm="fricas")
```

output
```
(b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d
)/(b^2*c + a^2*d)))
```

3.379.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 36.68 (sec) , antiderivative size = 1867, normalized size of antiderivative = 35.23

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/
2)/(b*x+a)**(1/2),x)
```

output
```
-a**(-a**2*d/(a**2*d + b**2*c) - 2*b**2*c/(a**2*d + b**2*c))*b**(a**2*d/(a
**2*d + b**2*c) + 2*b**2*c/(a**2*d + b**2*c) - 1)*c*meijerg(((a**2*d/(2*a*
*2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4, a**2*d/(2*a**2*d + 2*b*
*2*c) + b**2*c/(a**2*d + b**2*c) + 3/4, 1), (a**2*d/(2*a**2*d + 2*b**2*c)
+ b**2*c/(a**2*d + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a
**2*d + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**
2*c) + 1)), ((a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c), a**
2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4, a**2*d/(2*a**2
*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2
*c) + b**2*c/(a**2*d + b**2*c) + 3/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*
c/(a**2*d + b**2*c) + 1), (0,)), a**2/(b**2*x**2))/(4*pi**(3/2)) - I*a**(-
a**2*d/(a**2*d + b**2*c) - 2*b**2*c/(a**2*d + b**2*c))*b**(a**2*d/(a**2*d
+ b**2*c) + 2*b**2*c/(a**2*d + b**2*c) - 1)*c*meijerg(((a**2*d/(2*a**2*d +
 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1/2, a**2*d/(2*a**2*d + 2*b**2*c)
+ b**2*c/(a**2*d + b**2*c) - 1/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a
**2*d + b**2*c), a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) +
 1/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/2, 1), (
)), ((a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1/4, a**2*
d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4), (a**2*d/(2*a**2
*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1/2, a**2*d/(2*a**2*d + 2*b...
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\sqrt{bx+a}\sqrt{bx-a}\,xe^{\left(-\frac{2\,b^2c\log(x)}{b^2c+a^2d}-\frac{a^2d\log(x)}{b^2c+a^2d}\right)}}{a^2b^2}$$

input
```
integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x
+a)^(1/2),x, algorithm="maxima")
```

output
```
(b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c +
a^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

3.379. $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

3.379.8 Giac [F]

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \int \frac{dx^2+c}{\sqrt{bx+a}\sqrt{bx-a}\,x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}\,dx$$

input
```
integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x
+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b
^2*c + a^2*d))), x)
```

3.379.9 Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.81

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = -\frac{\frac{x\,(d\,a^4+c\,a^2\,b^2)}{a^2\,b^2}-\frac{x^3\,(d\,a^2\,b^2+c\,b^4)}{a^2\,b^2}}{x^{\frac{d\,a^2+2\,c\,b^2}{d\,a^2+c\,b^2}}\,\sqrt{a+b\,x}\,\sqrt{b\,x-a}}$$

input
```
int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*
x - a)^(1/2)),x)
```

output
```
-((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))
/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))
```

3.380 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx$

3.380.1 Optimal result

Integrand size = 32, antiderivative size = 36

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\frac{\sqrt{1-x}\arcsin(x)}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}$$

output
```
arcsin(x)*(1-x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)
```

3.380.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=-i\log\left(-x+i\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}\right)$$

input
```
Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

output
```
(-I)*Log[-x + I*Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]]
```

3.380.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2038, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 2038}$$

$$\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

$$\Big\downarrow \text{ 39}$$

$$\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x^2}}dx}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\sqrt{1-x}\arcsin(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

input
```
Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

output
```
(Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

3.380.3.1 Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

3.380.4 Maple [F]

$$\int \frac{1}{\sqrt{1+x}\,\sqrt{-1-\sqrt{x}}\,\sqrt{\sqrt{x}-1}}dx$$

input
```
int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(x^(1/2)-1)^(1/2),x)
```

output
```
int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(x^(1/2)-1)^(1/2),x)
```

3.380.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = -i\,\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+i\,x-1}{x}\right) + i\,\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-i\,x-1}{x}\right)$$

input
```
integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm
="fricas")
```

output
```
-I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I
*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

3.380.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

3.380.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

3.380.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{x+1}}\,dx$$

input
```
int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.381 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$

3.381.1 Optimal result

Integrand size = 41, antiderivative size = 75

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx=-\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

output
```
-2*arctan((-b^2*x+a^2)^(1/2)/(b^2*x+a^2)^(1/2))*(-b^2*x+a^2)^(1/2)/b^2/(a-
b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2)
```

3.381.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx=-\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

input
```
Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

output
```
(-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sq
rt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

3.381.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {2038, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}dx$$

$$\Big\downarrow \text{2038}$$

$$\frac{\sqrt{a^2-b^2x}\int \frac{1}{\sqrt{a^2-b^2x}\sqrt{a^2+b^2x}}dx}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

$$\Big\downarrow \text{45}$$

$$\frac{2\sqrt{a^2-b^2x}\int \frac{1}{-\frac{(a^2-b^2x)b^2}{a^2+b^2x}-b^2}d\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

input
```
Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

output
```
(-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sq
rt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

3.381.3.1 Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

3.381.4 Maple [F]

$$\int \frac{1}{\sqrt{b^2x+a^2}\,\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}dx$$

input
```
int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

output
```
int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = -\frac{2\,\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\,\sqrt{b\sqrt{x}+a}\,\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

input
```
integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, a
lgorithm="fricas")
```

output
```
-2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) +
a))/(b^2*x))/b^2
```

3.381.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

input
```
integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/
2),x)
```

output
```
Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)),
x)
```

3.381.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

input
```
integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),
x)
```

3.381.8 Giac [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

input
```
integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, a
lgorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),
x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{a+b\,\sqrt{x}}\,\sqrt{a-b\,\sqrt{x}}\,\sqrt{a^2+x\,b^2}}\,dx$$

input
```
int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)), x
)
```

3.382 $\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx$

3.382.1 Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} (c+dx^{2n})^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2n},-p,-q,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

output
```
x*(a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q*AppellF1(1/2/n,-p,-q,1+1/2/n,b^2
*x^(2*n)/a^2,-d*x^(2*n)/c)/((1-b^2*x^(2*n)/a^2)^p)/((1+d*x^(2*n)/c)^q)
```

3.382.2 Mathematica [F]

$$\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx = \int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx$$

input
```
Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

output
```
Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

3.382.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2038, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx$$

$$\Big\downarrow \text{ 2038}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p} \int \left(a^2-b^2x^{2n}\right)^p \left(dx^{2n}+c\right)^q dx$$

$$\Big\downarrow \text{ 937}$$

$$(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(dx^{2n}+c\right)^q dx$$

$$\Big\downarrow \text{ 937}$$

$$(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(\frac{dx^{2n}}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 936}$$

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2n},-p,-q,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{d}{}\right.$$

input
```
Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
 (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))
/a^2)^p*(1 + (d*x^(2*n))/c)^q)
```

3.382.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

3.382.4 Maple [F]

$$\int \left(a-b\,x^{n}\right)^{p} \left(a+b\,x^{n}\right)^{p} \left(c+d\,x^{2 n}\right)^{q}d x$$

input
```
int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

output
```
int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

3.382.5 Fricas [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int \left(dx^{2\,n}+c\right)^{q}(bx^{n}+a)^{p}(-bx^{n}+a)^{p}\,dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="fricas")
```

output
```
integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.382.6 Sympy [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \,dx = \text{Timed out}$$

input
```
integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
```

output
```
Timed out
```

3.382.7 Maxima [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \,dx = \int \left(dx^{2n}+c\right)^q (bx^n+a)^p(-bx^n+a)^p \,dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="maxima")
```

output
```
integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.382.8 Giac [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \,dx = \int \left(dx^{2n}+c\right)^q (bx^n+a)^p(-bx^n+a)^p \,dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="giac")
```

output
```
integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx = \int \left(c+d\,x^{2\,n}\right)^q (a+b\,x^n)^p (a-b\,x^n)^p\,dx$$

input
```
int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p,x)
```

output
```
int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.383 $\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx$

3.383.1 Optimal result

Integrand size = 35, antiderivative size = 87

$$\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx = x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n},-p,\frac{1}{4}\left(4+\frac{1}{n}\right),\frac{b^4x^{4n}}{a^4}\right)$$

output
```
x*(a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p*hypergeom([-p, 1/4/n],[1+1/4
/n],b^4*x^(4*n)/a^4)/((1-b^4*x^(4*n)/a^4)^p)
```

3.383.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx = x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n},-p,1+\frac{1}{4n},\frac{b^4x^{4n}}{a^4}\right)$$

input
```
Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(
4*n), -p, 1 + 1/(4*n), (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

3.383.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2038, 785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{ 2038}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p} \int \left(a^2-b^2x^{2n}\right)^p \left(b^2x^{2n}+a^2\right)^p dx$$

$$\Big\downarrow \text{ 785}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(a^4-b^4x^{4n}\right)^{-p} \int \left(a^4-b^4x^{4n}\right)^p dx$$

$$\Big\downarrow \text{ 779}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} \int \left(1-\frac{b^4x^{4n}}{a^4}\right)^p dx$$

$$\Big\downarrow \text{ 778}$$

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n},-p,\frac{1}{4}\left(4+\frac{1}{n}\right),\frac{b^4x^{4n}}{a^4}\right)$$

input
```
Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(
4*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

3.383.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(Eq
Q[n, 2] && IGtQ[q, 0])
```

3.383.4 Maple [F]

$$\int (a - b\,x^n)^p\,(a + b\,x^n)^p\,\left(a^2 + b^2x^{2n}\right)^p\,dx$$

input
```
int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

output
```
int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

3.383.5 Fricas [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (b^2x^{2n}+a^2)^p (bx^n+a)^p (-bx^n+a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="fricas
")
```

output
```
integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
```

output
```
Timed out
```

3.383.7 Maxima [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (b^2x^{2n}+a^2)^p (bx^n+a)^p (-bx^n+a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="maxima
")
```

output
```
integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.383.8 Giac [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int \left(b^2x^{2n}+a^2\right)^p (bx^n+a)^p (-bx^n+a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (a+b\,x^n)^p \, (a-b\,x^n)^p \, \left(a^2+b^2\,x^{2\,n}\right)^p \, dx$$

input
```
int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x)
```

output
```
int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x)
```

3.384 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

3.384.1 Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \frac{x(c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2n},1,-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

output
```
x*(c+d*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,b^2*x^(2*n)/a^2,-d*x^(2*n)/c
)/a^2/((1+d*x^(2*n)/c)^p)
```

3.384.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(76) = 152.

Time = 0.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.39

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \frac{a^2c(1+2n)x(c+dx^{2n})^p\operatorname{AppellF1}\left(\frac{1}{2n},-p,1\right.}{(a^2-b^2x^{2n})\left(2a^2dnpx^{2n}\operatorname{AppellF1}\left(1+\frac{1}{2n},1-p,1,2+\frac{1}{2n},-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}\operatorname{AppellF1}\left(1+\frac{1}{2n},-\right.\right.}$$

input
```
Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

output
```
(a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n),
 -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x
^(2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2
*x^(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n
), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n)
, -p, 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

3.384.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2036, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}dx$$

$$\Big\downarrow \text{2036}$$

$$\int \frac{(c+dx^{2n})^p}{a^2-b^2x^{2n}}dx$$

$$\Big\downarrow \text{937}$$

$$(c+dx^{2n})^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}\int \frac{\left(\frac{dx^{2n}}{c}+1\right)^p}{a^2-b^2x^{2n}}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(c+dx^{2n})^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},1,-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

input
```
Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

output
```
(x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n)
)/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

3.384.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

3.384.4 Maple [F]

$$\int \frac{(c+d\,x^{2n})^p}{(a-b\,x^n)\,(a+b\,x^n)}dx$$

input
```
int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

output
```
int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

3.384.5 Fricas [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = \int -\frac{(dx^{2\,n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

input
```
integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

3.384.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.384.7 Maxima [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \int -\frac{(dx^{2n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

input
```
integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

3.384.8 Giac [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \int -\frac{(dx^{2n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

input
```
integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = -\int -\frac{(c+d\,x^{2\,n})^p}{a^2-b^2\,x^{2\,n}}\,dx$$

input
```
int((c + d*x^(2*n))^p/((a + b*x^n)*(a - b*x^n)),x)
```

output
```
-int(-(c + d*x^(2*n))^p/(a^2 - b^2*x^(2*n)), x)
```

3.385 $\int (a-bx^{n/2})^p (a+bx^{n/2})^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)$

3.385.1 Optimal result

Integrand size = 76, antiderivative size = 96

$$\int (a-bx^{n/2})^p (a+bx^{n/2})^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx = -\frac{b^2(1+n+np)x(a-bx^{n/2})^{1+p}(a+bx^{n/2})^{1+p}\left(-\frac{a^2dn(1+p)}{b^2(1+n+np)}+dx^n\right)^{-\frac{1+n+np}{n}}}{a^4dn(1+p)}$$

output
```
-b^2*(n*p+n+1)*x*(a-b*x^(1/2*n))^(p+1)*(a+b*x^(1/2*n))^(p+1)/a^4/d/n/(p+1)
/((-a^2*d*n*(p+1)/b^2/(n*p+n+1)+d*x^n)^((n*p+n+1)/n))
```

3.385.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int (a-bx^{n/2})^p (a+bx^{n/2})^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx = -\frac{b^2(1+n+np)x(a-bx^{n/2})^{p}(a+bx^{n/2})^{p}\left(d\left(-\frac{a^2n(1+p)}{b^2(1+n+np)}+x^n\right)\right)^{-\frac{1+n+np}{n}}(a^2-b^2x^n)}{a^4dn(1+p)}$$

input
```
Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (
-1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

output
```
-((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n)
)/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1
+ n + n*p)/n)))
```

3.385.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2038, 906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2 d(p+1)}{b^2 \left(\frac{n(-p)-2n-1}{n}+1\right)}+dx^n\right)^{\frac{n(-p)-2n-1}{n}} dx$$

$$\Big\downarrow \text{2038}$$

$$\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(a^2-b^2x^n\right)^{-p} \int \left(a^2-b^2x^n\right)^p \left(dx^n-\frac{a^2 dn(p+1)}{b^2(pn+n+1)}\right)^{-p-\frac{1}{n}-2} dx$$

$$\Big\downarrow \text{906}$$

$$-\frac{b^2 x(np+n+1) \left(a^2-b^2x^n\right) \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(dx^n-\frac{a^2 dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{1}{n}-p-1}}{a^4 dn(p+1)}$$

input
```
Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2
*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

output
```
-((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n)
*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^(-1 - n^(-1) - p))/(a^
4*d*n*(1 + p)))
```

3.385.3.1 Defintions of rubi rules used

rule 906
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

3.385.4 Maple [F]

$$\int \left(a-b\,x^{\frac{n}{2}}\right)^{p} \left(a+b\,x^{\frac{n}{2}}\right)^{p} \left(\frac{a^{2} d (1+p)}{b^{2} \left(1+\frac{-np-2n-1}{n}\right)}+d\,x^{n}\right)^{\frac{-np-2n-1}{n}}dx$$

input
```
int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n
)+d*x^n)^((-n*p-2*n-1)/n),x)
```

output
```
int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n
)+d*x^n)^((-n*p-2*n-1)/n),x)
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \left(a-bx^{n/2}\right)^p \left(a +bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx = \frac{((b^4np+b^4n+b^4)xx^{2n}-(2\,a^2b^2np+2\,a^2b^2n+a^2b^2)xx}{(a^4np+a^4n)\left(-\frac{a^2dnp+a^2dn-(b^2d}{b^2np+b}\right.}$$

input
```
integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*
n-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

output
```
((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^
2)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(
(a^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x
^n)/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

3.385.6 Sympy [F(-1)]

Timed out.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \text{Timed out}$$

input
```
integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-
n*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

output
```
Timed out
```

3.385.7 Maxima [F]

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

input
```
integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*
n-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

output
```
integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/
(b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

3.385.8 Giac [F]

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}\,dx=\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

input
```
integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*
n-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

output
```
integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/
(b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}\,dx=\int \frac{\left(a+b\,x^{n/2}\right)^p\left(a-b\,x^{n/2}\right)^p}{\left(d\,x^n-\frac{a^2\,d\,(p+1)}{b^2\left(\frac{2\,n+n\,p+1}{n}-1\right)}\right)^{\frac{2\,n+n\,p+1}{n}}}\,dx$$

input
```
int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((
2*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n),x)
```

output
```
int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((
2*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
        else:
            return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```